

Project Update – Michael Tehranchi

The topic of my research most related to my fellowship proposal is on the development of polynomial models. As described in my key findings to date, easy-to-check necessary and sufficient conditions have been found (in joint work with my PhD student Si Cheng) for the existence of a factor process in the one dimensional case. This result bridges the practical calculations of derivative prices with the economic assumptions underlying the modelling framework.

Spectral models, which appear in the work of Brody & Hughston and Siegel, can be regarded as generalisations of polynomial models. Working in a rather general framework with possibly a continuum of degrees of freedom, sufficient conditions have been found (again in joint work with Si Cheng) for the existence of a unique solution to the pricing equation.

I should also mention that Si Cheng has now finished his PhD, having had a successful viva this past July, with Steve Satchell as the internal examiner and Martin Keller-Ressel of the University of Dresden the external examiner.

I am still working on implied volatility. I have submitted a revision of the paper on uniform bounds on implied volatility. In the process of preparing the revision, I have found an interesting long-dated/short-dated symmetry which complements the more well-known put-call symmetry of the Black—Scholes model. This new symmetry simplifies some calculations that appear in the revised paper. There is reason to hope that this symmetry might find other applications.

I am also working on a research monograph on implied volatility together with Antoine Jacquier of Imperial College London. One of the goals of my coming sabbatical leave is to complete this project.

I am continuing my work with my PhD student, David Driver, connecting certain optimisation problems – such as certain optimal investment problems found in finance – with classical reaction-diffusion equations. This past July, David Driver has presented our joint work at the World Congress in Probability and Statistics in Toronto.

Finally, I am working on the underpinnings of arbitrage theory, expressing the duality between portfolios and prices. This analysis leads to a new fundamental theorem of asset pricing which holds without the assumption of the existence of a numeraire portfolio.

Since the renewal of my CERF fellowship, I have submitted the following paper.

Uniform bounds for Black—Scholes implied volatility. Revision requested from *SIAM Journal on Financial Mathematics*. Available at <http://arxiv.org/abs/1512.06812>