

Governance through Threats of Intervention and Exit[☆]

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Abstract

This paper studies a model in which an activist shareholder has the possibility of disciplining management through the threat of intervention and threat of exit. Intervention is the equilibrium form of governance when the activist has a toehold almost sufficient for exercising voice, when the activist is effective in restoring firm value, and when the temptation for misbehavior by the management is large. Intervention plays a positive disciplinary role only when the activist can punish the manager for misbehavior. A stronger disciplinary role played by the threat of intervention leads to fewer ex-post interventions, and a stronger disciplinary role played by the threat of exit leads to fewer block sales, suggesting that corporate governance is most effective when activists rarely act. When the activist chooses the size of the toehold, he effectively determines the equilibrium form of governance. Forces that lead to the adoption of the most effective governance structure are analyzed.

One of the fundamental issues in modern corporate finance is the problem of separation of firm ownership from control. The gap between management and shareholders is potentially wide and the danger is great for agency problems to divert a widely-held firm’s resources from their efficient use. Therefore it is important to understand what mechanisms are available for reconciling these interests, to what extent they are used, and to what extent they are effective.

If a shareholder decides he does not like what a firm’s management is doing, he has two alternatives: He can *intervene* or he can *exit*—that is, he can work on changing the firm’s manager’s behavior directly or he can sell his shares. Intervention, sometimes referred to as “voice,” includes a variety of possible actions to compel changes in behavior by management: replacement of boards of directors, support for takeover bids, and proxy initiatives to limit management discretion or to affect management compensation. Exit, sometimes referred to as the “Wall Street walk” is the sale of shares in a firm when the shareholder disapproves of management actions but does not choose to engage in direct intervention.

However, both exit and intervention can also have indirect effects, because the foreknowledge by managers of the possible reactions of dissatisfied shareholders can alter managerial behavior. Thus we are not only interested in exit and intervention as behaviors by the blockholder, we are also interested in how they affect managerial behavior. That is, we are also interested in the incentive effects on managers of the *threats* of shareholder exit or intervention.

Beginning with Admati and Pfleiderer (2009) and Edmans (2009), a se-

ries of recent articles have shown that, provided management compensation is tied in the short run to share price, the threat of exit and the resultant reduction of share price, can serve as a disciplinary device. Despite empirical investigations of each of these strands of governance, surprisingly little theoretical attention has been paid to the question of comparing the factors that lead to the adoption of exit or of intervention. Moreover, no theoretical attention has been paid to the question of how *the threat of intervention* affects the choice of these two governance mechanisms in equilibrium.

In this paper we ask the following research questions. What type of shareholder action—intervention or exit—can exist as an equilibrium response to managerial misbehavior? Under what circumstances can intervention and exit play stronger disciplinary roles? Do more frequent ex post interventions mean that corporate governance is functioning well? Under what circumstances will an activist shareholder choose a form of governance that leads to the highest firm value as an equilibrium response to managerial misbehavior?

To address these important questions, we provide a simple model in which an activist shareholder can accumulate a toehold and then discipline management through the threat of exit and threat of intervention. Once the toehold is established, the activist can decide to sell shares on observing a negative signal (i.e., exit). Alternatively, the activist can decide to extend the toehold and intervene when there is a potential for value creation. In anticipation of these actions, the management is more restrained in consumption of private benefits.

The extent to which the activist's information about managerial behav-

ior is revealed through market prices plays an important role in the model. If market prices are unaffected, there is no channel through which a blockholder's actions can influence the manager in the exit equilibrium. On the other hand, if the market fully reveals the activist's private information, the activist has no incentive to sell rather than hold on to the assets. Moreover, when prices are fully revealing the activist has no incentive to accumulate the toehold in the first place. For this reason it is important to consider the effect of liquidity trading on the mechanisms; the presence of liquidity trades enables the activist to a certain degree to hide his information. We focus in this paper on liquidity trades by the activist himself, since these are the most effective sources of information clouding, and we consider both buying and selling shocks.

Our research framework is relevant in modern financial markets, because most publicly traded firms can be subject to either type of governance. Because the effects of these mechanisms on managerial behavior are often unobservable to empiricists, theoretical analysis of what form of governance disciplines the manager can enhance our understanding of how financial markets operate. We therefore investigate the circumstances that encourage the use of one of the mechanisms rather than the other in equilibrium. We also consider the cases where two equilibria exist, one with each mechanism, and consider the factors that lead to the greater effectiveness of one or the other mechanism.

The model reveals several key results. First, we ask what type of shareholder action—intervention or exit—can exist as an equilibrium response to managerial misbehavior. We find that the intervention equilibrium is more

likely to exist when the costs of intervention are low – in particular, when the activists builds an initial toehold almost big enough to be an effective stake for exercising voice – or when the activist is particularly effective in restoring firm value. Moreover, increases in the temptation for misbehavior by the management increase the chances for an intervention equilibrium. In all of these cases the effect on existence of an exit equilibrium is reversed. Thus, variations in the size of the initial toehold, the activist’s effectiveness in restoring firm value, and the temptation for misbehavior by the management generate a *substitution* between two governance mechanisms.¹

Second, when intervention is the equilibrium form of governance, disciplinary pressure on the manager is greater as the activist has greater power to penalize deviating managers and has smaller power to restore firm value. The effect of liquidity shocks that force him to sell the block on manager’s incentives can be either positive or negative. Importantly, the stronger disciplinary role played by the intervention mechanism leads to *fewer* ex post interventions in equilibrium. In the extreme, one would not observe any intervention events if the threat of intervention were so powerful as to prevent the manager from taking the bad action in any state of the world.²

Third, we analyze the robustness of governance mechanisms to changes in the structure of the manager’s compensation. We find that if the man-

¹In addition, we establish conditions in which the lone equilibrium outcome is a mixed strategy, including probabilities of both exit and intervention. This result is reminiscent of several papers which have shown in more complicated contexts that exit and voice can be complementary (e.g., Dasgupta and Piacentino, 2014); in our model it can be thought of as occurring in cases where neither mechanism is strong enough to survive on its own.

²In this extreme case the situation bears a similarity to the theory of contestable markets, where potential competition, even though unobserved, manages to provide market discipline against temptations toward inefficient behavior (Baumol et al., 1988).

ager's compensation were more aligned with the long-term firm value, the disciplinary role played by the intervention (exit) mechanism would increase (decrease). This result suggests that exit becomes more preferred as a form of governance the stronger the exogenous (i.e., un-modeled) factors that lead to short-termism.

Fourth, several interesting findings come from the analysis of liquidity shocks. Typically, bid-ask spreads are used empirically as a measure of stock liquidity. We find that in the exit equilibrium firm value is lower and the bid-ask spread is narrower when the activist is more likely to face sell-side liquidity shocks. Thus, the variation in the sell-side liquidity shock implies a negative association between measured stock liquidity and firm value. In contrast, the variation in the buy-side liquidity shock implies a positive association between measured stock liquidity and firm value. When we study the role of liquidity shocks in a disciplinary intervention equilibrium, we find that firm value is lower and the bid-ask spread is wider when the activist is more likely to face sell-side liquidity shocks. Overall, the results suggest that both the source of the change in measured stock liquidity and the equilibrium form of governance play key roles in determining the relation between endogenously determined stock liquidity and firm value.

In the final section of the paper, we endogenize the activist's choice of initial toehold. The size of the toehold is increasing in stock market liquidity and decreasing in the cost associated with holding the toehold. We conclude the analysis by asking under what conditions the activist's private choice leads to the most effective form of governance (i.e., one that leads to the highest firm value). We show that there can be a discrepancy between the

activist's private choice and the effective outcome, so that, for example, when the exit form of governance is more effective, conditions which encourage amassing a large initial toehold actually decrease the chances of introducing the effective form of governance.

Related Literature

This paper is related to several strands of the corporate governance literature that studies the role of blockholders in reducing agency costs.³

First, the paper contributes to the strand of literature that studies how shareholder intervention can increase firm value *ex post* (e.g., Shleifer and Vishny, 1986; Kyle and Vila, 1991; Admati et al., 1994; Maug, 1998; Bolton and von Thadden, 1998; Kahn and Winton, 1998; Noe, 2002; Faure-Grimaud and Gromb, 2004; Collin-Dufresne and Fos, 2014). For example, in their classic paper Shleifer and Vishny (1986) show that the presence of a large minority shareholder provides a partial solution to the free-rider problem and therefore reduces the agency costs. In this strand of literature, intervention does not play a disciplinary role. Intervention occurs in the absence of managerial action; more effective monitoring does not change the manager's incentives and therefore is beneficial for shareholders only because it increases firm value *ex post*.

Second, the paper contributes to the literature that studies how corporate governance can affect management's incentives. Grossman and Hart (1980) were the first to argue that managers face trade offs between a high profit action with an associated low chance of being raided and a low profit

³Edmans (2013) surveys theoretical and empirical literature on the role of blockholders in corporate governance.

(but high managerial-utility) action which leads to a successful takeover bid. In their model managers are more reluctant to take self-serving actions that lower firm value and increase the probability of a takeover. Scharfstein (1988) explicitly models the source of contractual inefficiencies which was not studied by Grossman and Hart (1980). He explores the conditions under which the takeover threat plays a genuine role (beyond incentive contracts) in disciplining management.⁴ The literature has also studied the governance role of exit and showed that a large shareholder can alleviate conflicts of interest between managers and shareholders through the credible threat of exit on the basis of private information (e.g., Admati and Pfleiderer, 2009; Edmans, 2009). Our paper is mainly related to Admati and Pfleiderer (2009) because we adopt their modeling setup of the exit form of governance.

Our paper contributes to that literature not only by jointly considering the effect of two main corporate governance mechanisms in improving management's incentives, but also by studying how the threat of intervention affects the choice of these two governance mechanisms in equilibrium. Very few papers consider the disciplinary roles of both exit and the intervention mechanisms in resolving an agency conflict between shareholders and management. Dasgupta and Piacentino (2014) show that when money managers compete for investor capital, the threat of exit loses credibility, weakening its governance role. When they allow funds to engage in activist measures, they

⁴While the above papers show that takeover plays a positive disciplinary role, several other papers have highlighted some negative aspects of the threat of intervention (e.g., Stein, 1988; Zwiebel, 1996; Burkart et al., 1997). For example, Stein (1988) develops a model in which takeover pressure can be damaging because it leads managers to sacrifice long-term interests in order to boost current profit.

find that the threat of exit and intervention are complementary in generating good governance, because blockholders will use intervention if and only if they can credibly threaten to exit.⁵ Levit (2012) interprets voice as a strategic transmission of information from an activist investor to an opportunistic manager. He shows that this type of voice and exit exhibit complementarity. Edmans and Manso (2011) also investigates a structure in which both trade and direct action are available to blockholders. In their model the manager does not expend enough effort because prices do not fully reflect the impact of manager's action on firm value. Their focus is on the role of multiple small blockholders and show that while such a structure generates free-rider problems that hinder intervention, the same coordination difficulties strengthen a second governance mechanism: disciplining the manager through trading.

1. Setup

In the basic model there are three periods 0, 1, and 2 and three types of agents: the manager, whom we denote by \mathcal{M} , an activist shareholder, \mathcal{A} , who owns φ shares in the firm, and a continuum of uninformed traders (the “market makers”). Markets for shares in the firm occur in periods one and

⁵Our model differs from Dasgupta and Piacentino (2014) on several key dimensions. First, in Dasgupta and Piacentino (2014) the blockholder does not have to choose between two mechanisms. Instead, the blockholder can engage in exit *after* an intervention attempt. In our model, the activist faces a choice between the two, generally substitute, mechanisms. Second, while in our model the activist can improve firm value and impose a private cost on the manager if the bad action is taken, in Dasgupta and Piacentino (2014) he can only impose a private cost on the manager. Our modeling assumption is consistent with robust empirical evidence on value creation by activist shareholders (e.g., Brav et al., 2008). Finally, in Dasgupta and Piacentino (2014) the activist owns a sufficient number of shares and therefore does not need to trade in order to become active. In our model the activist is required to trade in order to become active.

two. In Section 4 we analyze a period prior to period 0 in which \mathcal{A} 's choice of initial holding φ is endogenized. The choice of optimal φ will take into account the impact of φ on \mathcal{M} 's incentives (and therefore firm value and \mathcal{A} 's profits) and the cost of holding φ shares (e.g., lack of diversification, effort spent on gathering information).

In period 0, \mathcal{M} decides whether or not to take a particular action. An agency problem arises because \mathcal{M} and the shareholders have conflicting preferences with respect to the action. Specifically, we assume the action is “bad” in the sense that it reduces the value of the firm, but provides a private benefit to \mathcal{M} . The benefit has the positive value β , known with certainty by all participants.⁶ The cost of the damage to the firm is $\tilde{\delta}$, a random value which \mathcal{M} learns privately immediately before making his decision.⁷ Let the decision be denoted a (either zero or one); then the value of the firm in period 2 will be $v - a\tilde{\delta}$, in the absence of intervention by the activist. The value v is common knowledge. All agents know that the value $\tilde{\delta}$ is drawn from a continuous distribution $F(\cdot)$ with density $f(\cdot)$ and support $[0, \bar{\delta}]$, where $\bar{\delta}$ is sufficiently large. To illustrate some results we will further assume that the distribution of δ is exponential with $F(\delta) = 1 - e^{-\delta\lambda}$. To simplify notation we write \tilde{a} rather than $a(\tilde{\delta})$.

\mathcal{M} 's strategy can be described by defining the set $\Delta \subseteq [0, \bar{\delta}]$, such that

⁶Fos and Jiang (2015) document evidence consistent with a manager's value of private benefits of control being 5%-20% of the stock price when the company is targeted in proxy contest.

⁷In a supplement to this paper we also consider the case where the manager's action is “good” in that it increases the firm's value at a private cost to the manager. For the most part that G version of the model (to use the terminology of Admati and Pfleiderer) provides results parallel to the B version adopted here.

$a = 1$ if and only if δ is in the set Δ . Let $\Phi = \Pr\{\delta \in \Delta\}$, the ex ante probability that \mathcal{M} chooses $a = 1$.

In period 1, \mathcal{A} observes the action taken by \mathcal{M} . Given \mathcal{M} 's strategy, observing \mathcal{M} 's actions provides \mathcal{A} with a noisy signal of $\tilde{\delta}$. Then \mathcal{A} must decide whether to buy, sell, or hold his shares at the period 1 market. If \mathcal{A} buys shares at the period 1 market, he may be able to *intervene* in period 2, reducing the benefit to \mathcal{M} of taking the bad action, and reduces the damage of the action to the firm. Specifically, if \mathcal{A} chooses to intervene, then the benefit to \mathcal{M} is reduced to $\beta\gamma$ and the value of the firm is restored to $v - a\tilde{\delta}\kappa$, where $0 < (1 - \gamma) < 1$ measures the effectiveness of \mathcal{A} in reducing the private benefits of control and $0 < (1 - \kappa) < 1$ measures his effectiveness in restoring firm value. Let $b \in \{0, 1\}$ represent the decision to intervene. Then the ultimate value of the firm is $v - a\tilde{\delta}(\kappa b + 1 - b)$. We assume this value is publicly revealed before the market at the end of period 2, so that trade in the final market occurs at this price.

As we are going to see later on, both \mathcal{A} 's ability to reduce the benefit to \mathcal{M} of taking the bad action, $(1 - \gamma)$, and \mathcal{A} 's ability to reduce the damage of the action to the firm, $(1 - \kappa)$, will play an important role in the model. Whereas $(1 - \kappa)$ will be one of key parameters to determine what governance mechanism exists in equilibrium (intervention or exit), both $(1 - \gamma)$ and $(1 - \kappa)$ will determine the degree of discipline imposed on \mathcal{M} through the threat of intervention. Consistently with $(1 - \gamma) > 0$, Fos and Tsoutsoura (2014) show that activist shareholders are able to impose a significant career cost on directors of targeted companies. Directors of companies that experience a proxy contest lose seats not only on boards of targeted companies, but also

on boards of other companies. Several pieces of evidence motivate $(1 - \kappa) > 0$. For example, Brav et al. (2008) show that firm value increases upon intervention by activist hedge funds. Similarly, Fos (2015) shows that firm value increases upon announcements of a proxy contest, which is probably the most hostile type of shareholder activism.

We next characterize \mathcal{A} 's trading in period 1. If \mathcal{A} decides to sell shares after observing \mathcal{M} 's action, he sells the entire position φ . (As we are going to see later on, in any equilibrium in which \mathcal{A} decide to sell shares, it is optimal to sell the entire position.) If \mathcal{A} decides to buy shares, he buys $\alpha - \varphi$ shares. (In our simple information structure this will be the only size of purchase which will allow him to disguise his intentions when purchasing.) Both α and φ are publicly known in period 1. \mathcal{A} 's choice of φ is endogenized in Section 4. Empirically, φ could correspond to stockholder's ownership disclosed in Schedule 13F filings.

Several factors could affect α . For example, a larger α could correspond to cases when \mathcal{A} needs more voting power to make the intervention effective. A larger α could also correspond to a higher direct cost of intervention. Instead of modelling one specific channel that determines α , we leave it as an exogenous parameters in the model. The difference between α and φ therefore captures the degree of \mathcal{A} 's dependence on financial markets. High $(\alpha - \varphi)$ means that \mathcal{A} would need to purchase many shares in order to intervene. Interventions by activist hedge funds are a good example of this scenario because activist hedge funds often start accumulating shares when they own about 2-3% of outstanding shares (φ) and then they end up with about 7% of outstanding shares (α) (Brav et al., 2008; Collin-Dufresne and Fos, 2015).

Small ($\alpha - \varphi$) means that \mathcal{A} would need to purchase only a few shares in order to intervene. Activism campaigns aimed at passing shareholder proposals sponsored by pension funds are a good example of this scenario because such campaigns involve little (if any) trading by activists.⁸

\mathcal{A} 's trades in period 1 may reveal information both about \mathcal{M} 's actions and about \mathcal{A} 's own intentions for period 2. We will assume that this information will be somewhat obscured by additional liquidity needs of \mathcal{A} . For example, these liquidity needs could come from either a positive or a negative shock to \mathcal{A} 's capital. Specifically, we assume that \mathcal{A} in period 1 will with probability θ suffer a liquidity shock which requires him to divest himself of any holdings of firm shares and which prevents him from purchasing any shares of the firm. With probability ζ he will suffer a liquidity shock which requires him to buy additional shares. And with probability $1 - \theta - \zeta > 0$, he will suffer no shock. In order to make sure that \mathcal{A} 's decision to purchase shares is not fully revealing, we assume that the amount he will need to buy due to liquidity shock will be identical to the amount he would need to buy for intervention purposes. If he does not suffer a liquidity shock, then his purchases and sales will be based on his information and his strategy for future intervention. Other participants in the market are unable to observe the liquidity shock of \mathcal{A} , and so the price prevailing will take into account

⁸We have also considered a case when there is a direct cost of intervention. In that case, all solutions become more complicated, because prices reflect not only the probability that the bad action is taken, but also the probability the damage to firm value is large enough. When \mathcal{M} knows that the damage of his action to firm value is large enough to justify the intervention, there is no material change in the results. However, when \mathcal{M} 's action implies a small damage to firm value, \mathcal{M} realizes that \mathcal{A} won't intervene because the intervention will not recover enough damage to make the intervention profitable. Intervention plays no disciplinary role in this case. Results are available upon request.

their expectation of the relative likelihood of the shock.

\mathcal{M} 's compensation is assumed to be linear in the realized market price of the firm in periods 1 and 2, P_1 and P_2 . Specifically, we assume that compensation is equal to $\omega_1 P_1 + \omega_2 P_2$, where ω_1 and ω_2 are positive coefficients representing the dependence of the compensation on the firm's short-term ("Period 1") and long-term ("Period 2") price performance, respectively.⁹ \mathcal{M} chooses whether to take the action or not to maximize his expected utility for every realization of $\tilde{\delta}$.

When \mathcal{A} is not present, \mathcal{M} 's preferred cutoff point is equal to $\delta_{BM} = \beta/\omega_2$. That is, \mathcal{M} takes the action when $\tilde{\delta} \leq \delta_{BM} = \beta/\omega_2$ and the value of the firm is p_{BM} . \mathcal{A} 's role in governance will be measured by his impact on firm value.

Next consider the case when \mathcal{A} is present. If \mathcal{M} does not take the action, then \mathcal{M} 's utility is simply his compensation, $\omega_1 P_1 + \omega_2 P_2$. If he takes the action, then his utility depends on \mathcal{A} 's period 1 trading decision and on decision to intervene in period 2. If intervention does not occur, \mathcal{M} 's utility is equal to the sum of his compensation and the private benefit β . If intervention occurs, \mathcal{M} 's utility is equal to the sum of his compensation and the private benefit $\beta\gamma$. Note that in both cases prices will reflect \mathcal{A} 's period 1 trading decision and the decision to intervene in period 2.

When the intervention mechanism operates, the potential impact of \mathcal{A}

⁹ \mathcal{M} 's sensitivity to short-term prices is taken as exogenous in this paper. It can be motivated, for example, by takeover threats and concern for managerial reputation (Edmans, 2009). Neither existence nor the disciplinary role of the Intervention equilibrium is affected if we set $\omega_1 = 0$. However, whereas the Exit equilibrium can exist when $\omega_1 = 0$, it plays no disciplinary role.

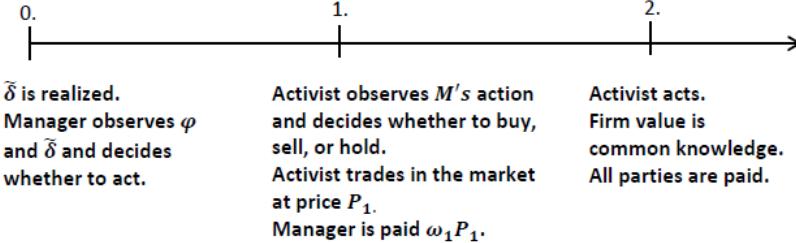


Figure 1: **Time-line.**

on M 's decision comes about through the impact of his trading decisions on P_1 , through his impact on firm value at period 2, and through his impact on private benefits of control. When the exit mechanism operates, the potential impact of A on M 's decision comes about only through the impact of his trading decisions on P_1 . We assume that prices are set by risk-neutral, competitive market makers and therefore reflect all of the information publicly available. This means, as noted before, that P_2 equals $v - a\delta(\kappa b + 1 - b)$. In period 1, P_1 reflects the information contained in A 's trading decision. The timing of events is given in Figure 1.

2. Solving the Model

We assume A is restricted to three actions $T \in \{B, H, S\}$ in period 1: buy enough to get the level to the required amount for intervention (B); sell all holdings (S); or keep holdings unchanged (H for “hold”).

It is useful to introduce notation for the prices that would occur if uninformed agents observed M 's action (denote these as p_T^a). If $a = 0$, $p_T^0 = v$ for any T . If $a = 1$, $p_H^1 = p_S^1 = v - \Lambda$, where $\Phi \equiv \Pr(\delta \in \Delta)$ is the probability

of \mathcal{M} taking the action and $\Lambda \equiv \Phi^{-1} E[\mathbb{1}_{\delta \in \Delta} \delta]$ is the expected damage to firm value, conditional on \mathcal{M} taking the action. Note that the price if held is the same as the price if sold, because without having enough of a holding to intervene, \mathcal{A} adds no value to the asset. Finally, $p_B^1 = v - \kappa\Lambda > p_H^1 = p_S^1$, reflecting the benefit from intervention.¹⁰

We next consider the value of \mathcal{A} 's position. The value from standing pat is $\pi_0^H = \varphi v$ if $a = 0$ and $\pi_1^H = \varphi(v - \Lambda)$ if $a = 1$. The value from selling the lot is $\pi_a^S = \varphi p_S$ (note this does not actually depend on a). The value from buying is $\pi_0^B = \alpha v - (\alpha - \varphi)p_B$ if $a = 0$ and $\pi_1^B = \alpha(v - \kappa\Lambda) - (\alpha - \varphi)p_B$ if $a = 1$. Hereafter, we will refer to the value *net* of the initial holding φv as ‘ \mathcal{A} 's profits.’

A *market equilibrium* for period 1 specifies the probability mixture for \mathcal{A} between buy, hold, and sell $(\sigma_a^B, \sigma_a^H, \sigma_a^S)$, for $a = 1$ or 0 , *conditional on no liquidity shock* and market prices p_B, p_S , such that the probabilities are maximizing choices given prices, and prices are consistent with the probabilities.

$$\sum_{T=B,H,S} \sigma_a^T \pi_a^T \geq \pi_a^{T'} \text{ for all } T' \in \{B, H, S\}, \text{ for } a = 0, 1.$$

$$p_T^1 \leq p_T \leq p_T^0, \text{ for } T \in B, S$$

¹⁰We know that $\Phi > 0$, because for any fixed values of ω_1 and ω_2 , for δ sufficiently close to zero, \mathcal{M} would prefer to take the action, even if it were publicly observable, and therefore reduced prices in both periods 1 and 2.

Table 1: \mathcal{A} 's Profits. This table describes profits of \mathcal{A} , as measured by $\pi - \varphi v$, adjusted to results of Lemma 1. Column (1) reports profits if \mathcal{M} does not take the action and column (2) reports profits if \mathcal{M} takes the action.

	$a = 0$ (no damage) (1)	$a = 1$ (damage) (2)
Buy	$(\alpha - \varphi)\kappa\Lambda \frac{\Phi\sigma_1^B + \bar{\zeta}\Phi}{\Phi\sigma_1^B + (1 - \Phi) + \bar{\zeta}}$	$-\alpha\kappa\Lambda + (\alpha - \varphi)\kappa\Lambda \frac{\Phi\sigma_1^B + \bar{\zeta}\Phi}{\Phi\sigma_1^B + (1 - \Phi) + \bar{\zeta}}$
Hold	0	$-\varphi\Lambda$
Sell	$-\varphi\Lambda \frac{\Phi(1 - \sigma_1^B) + \bar{\theta}\Phi}{\Phi(1 - \sigma_1^B) + \bar{\theta}}$	$-\varphi\Lambda \frac{\Phi(1 - \sigma_1^B) + \bar{\theta}\Phi}{\Phi(1 - \sigma_1^B) + \bar{\theta}}$

$$\begin{aligned}
p_S &= \frac{(1 - \theta - \zeta)[p_S^1\Phi\sigma_1^S + p_S^0(1 - \Phi)\sigma_0^S] + \theta[p_S^1\Phi + p_S^0(1 - \Phi)]}{(1 - \theta - \zeta)[\Phi\sigma_1^S + (1 - \Phi)\sigma_0^S] + \theta} \\
&= v - \Lambda \frac{\Phi\sigma_1^S + \bar{\theta}\Phi}{\Phi\sigma_1^S + (1 - \Phi)\sigma_0^S + \bar{\theta}} \\
p_B &= \frac{(1 - \theta - \zeta)[p_B^1\Phi\sigma_1^B + p_B^0(1 - \Phi)\sigma_0^B] + \zeta[p_B^1\Phi + p_B^0(1 - \Phi)]}{(1 - \theta - \zeta)[\Phi\sigma_1^B + (1 - \Phi)\sigma_0^B] + \zeta} \\
&= v - \kappa\Lambda \frac{\Phi\sigma_1^B + \bar{\zeta}\Phi}{\Phi\sigma_1^B + (1 - \Phi)\sigma_0^B + \bar{\zeta}},
\end{aligned}$$

where $\bar{\theta} \equiv \theta/(1 - \theta - \zeta)$ and $\bar{\zeta} \equiv \zeta/(1 - \theta - \zeta)$. The following Lemma shows that we can put more structure on equilibrium beliefs.

Lemma 1. *In equilibrium, $\sigma_0^B = 1$, $\sigma_0^H = 0$, $\sigma_0^S = 0$, and $\sigma_1^H = 0$.*¹¹

\mathcal{A} 's profits, as measured by $\pi - \varphi v$, are presented in Table 1.

2.1. Equilibrium with Intervention

We begin by characterizing equilibria in which \mathcal{A} intervenes with a positive probability, i.e., when $\sigma_1^B > 0$. Let $\Phi_I, \Lambda_I, \delta_I$ denote the equilibrium values of Φ, Λ, δ .

¹¹All proofs are in the appendix.

Proposition 1. *There exists a unique equilibrium with $\sigma_1^B = 1$ if and only if*

$$\frac{\varphi}{\alpha} > \frac{\kappa}{(1-\kappa)} \frac{(1-\Phi_I)}{\Phi_I}, \quad (1)$$

where $\delta_I = \frac{\beta \theta + (1-\theta)\gamma}{\omega_2 \theta + (1-\theta)\kappa}$, $\Phi_I = F(\delta_I)$, and $\Lambda_I = \Phi_I^{-1} E[\mathbb{1}_{\delta < \delta_I} \delta]$. Equilibrium beliefs are $(\sigma_0^B = 1, \sigma_0^H = 0, \sigma_0^S = 0; \sigma_1^B = 1, \sigma_1^H = 0, \sigma_1^S = 0)$. Equilibrium prices are $p_B = v - \kappa \Lambda_I \Phi_I$ and $p_S = v - \Lambda_I \Phi_I$. The equilibrium is disciplinary if and only if $\gamma < \kappa$.

Columns (1) and (2) in Table 2 summarize the effects of the various parameters on equilibrium existence and effectiveness in disciplining the manager, defined as $\frac{\delta_{BM}}{\delta_I}$. The impact of the intervention on \mathcal{M} 's incentives is illustrated in Figure 2. Whether the impact is positive or negative depends on the relation between γ and κ . When $\gamma < \kappa$ (\mathcal{A} is more effective in reducing the private benefits than in restoring firm value), the intervention plays a *positive* disciplinary role ($\delta_I < \delta_{BM}$). In contrast, when $\gamma > \kappa$ (\mathcal{A} is more effective in restoring firm value than in reducing the private benefits), the intervention plays a *negative* disciplinary role ($\delta_{BM} < \delta_I$)—that is, the manager chooses the bad action more frequently than he would if \mathcal{A} were absent.

Panel A in Figure 2 depicts the case when $\gamma < \kappa$. When $\delta > \delta_{BM}$, \mathcal{M} does not take the bad action even when \mathcal{A} is not present. In this case the damage to firm value is so large that \mathcal{M} prefers to forego the private benefit β . In the intermediate region $\delta_{BM} > \delta > \delta_I$, \mathcal{A} 's presence prevents \mathcal{M} from taking the bad action. This is the disciplinary role of the intervention. Finally, when $\delta < \delta_I$, \mathcal{M} takes the bad action and ex post intervention

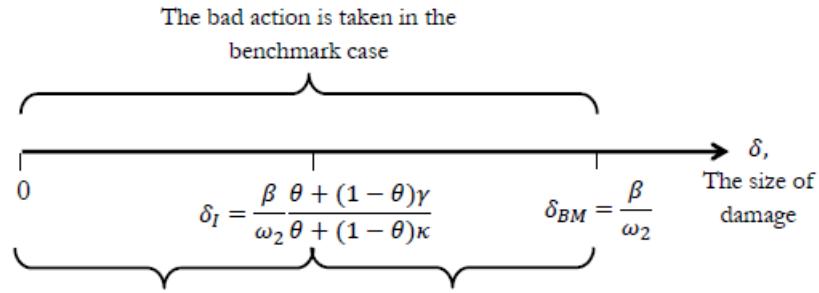
Table 2: **Intervention and Exit: Summary.** This table summarizes effects of key parameters on existence and effectiveness of each type of equilibrium. Columns 1 shows the effect of parameters on condition (1). Column 2 shows the effect of parameters on the disciplinary role of the intervention equilibrium, as measured by $\frac{\delta_{BM}}{\delta_I}$. Columns 3 shows the effect of parameters on condition (2). Column 4 shows the effect of parameters on the disciplinary role of the exit equilibrium, as measured by $\frac{\delta_{BM}}{\delta_E}$.

Intervention		Exit	
Existence	Effectiveness	Existence	Effectiveness
(1)	(2)	(3)	(4)
\mathcal{M}'s Characteristics			
$1/\lambda$	-	0	+
β/ω_2	+	0	-
ω_1/ω_2	0	0	+
\mathcal{A}'s Characteristics			
φ/α	+	0	-
$(1 - \kappa)$	+	-	-
$(1 - \gamma)$	-	+	+
θ	+/-	+/-	+/-
ζ	0	0	+/-

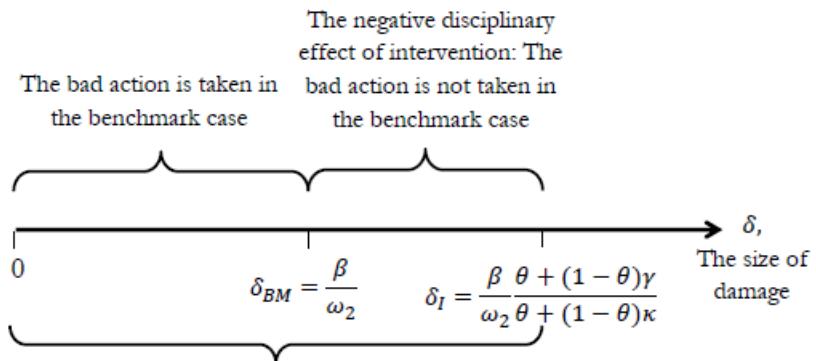
takes place. Only in this region will market participants observe incidents of intervention in the case $\gamma < \kappa$.

Panel B in Figure 2 depicts the case when $\gamma > \kappa$. When $\delta > \delta_I$, \mathcal{M} does not take the bad action even when \mathcal{A} is present. When $\delta < \delta_I$, \mathcal{M} takes the bad action and ex post intervention takes place. Notice that \mathcal{M} 's bad action is induced by the presence of \mathcal{A} when $\delta_{BM} < \delta < \delta_I$.

What is the overall effect of \mathcal{A} 's effectiveness at restoring firm value $(1 - \kappa)$ on firm value in equilibrium? On one side, a higher $(1 - \kappa)$ increases \mathcal{M} 's incentive to take the bad action because the damage to firm value will



(a) $\gamma < \kappa$: Positive disciplinary role of Intervention.



The bad action is taken in
the intervention equilibrium.

Ex-post intervention
happens.

(b) $\gamma > \kappa$: Negative disciplinary role of Intervention.

Figure 2: The disciplinary role of Intervention.

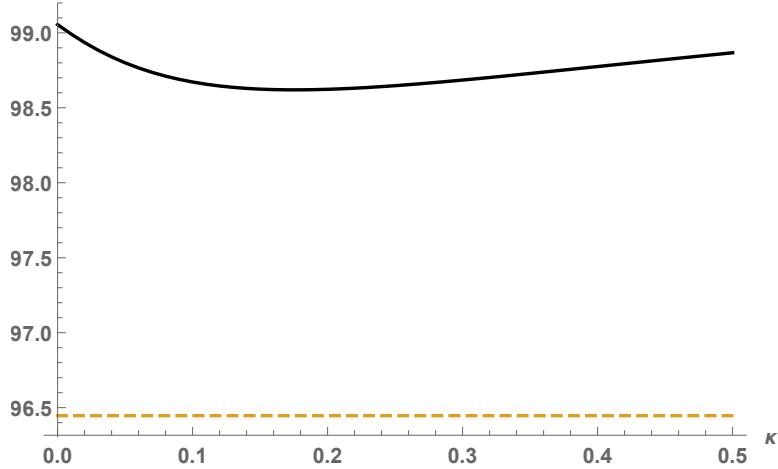


Figure 3: **The effect of κ on firm value in the Intervention equilibrium.** The black line plots the expected period 1 price in the Intervention equilibrium, $p_1 = (1 - \theta)p_B + \theta p_S$. The dashed line plots the expected period 1 price in the benchmark case. We assume $v=100$, $\beta=25$, $\omega_1=1$, $\omega_2=2$, $\theta=0.1$, $\zeta=0.1$, $\gamma=0.3$; $\frac{\varrho}{\alpha}=0.5$, $f[x] = \lambda \exp(-\lambda x)$, and $\lambda=0.1$.

be partially recovered. This is the negative impact of $(1 - \kappa)$ on firm value. On the other side, higher $(1 - \kappa)$ means that ex post intervention will create more value. The overall effect therefore depends on strength of these two effects. Figure 3 shows that the response is not monotonic in κ . When \mathcal{A} 's effectiveness in restoring firm value is high ($(1 - \kappa)$ is high), the positive effect that operates through ex post value creation dominates the negative effect that operates through \mathcal{M} 's incentives. In this case, *more* productive \mathcal{A} will result in higher firm value. In contrast, when \mathcal{A} 's effectiveness in restoring firm value is small ($(1 - \kappa)$ is low), the negative effect dominates the positive effect, and *less* productive \mathcal{A} will result in higher firm value. Note, however, that in this example firm value is always higher when in the benchmark case than when \mathcal{A} is not present.

The intuition behind the disciplinary effect of intervention is that \mathcal{M} takes into account the impact of \mathcal{A} 's intervention on private benefits of control and firm value. Whereas the negative impact of \mathcal{A} 's intervention on private benefits discourages \mathcal{M} from taking the bad action, the positive impact of \mathcal{A} 's intervention on firm value encourages \mathcal{M} to take the bad action. The disciplinary role of the intervention, as measured by $\frac{\delta_{BM}}{\delta_I}$, is increasing in \mathcal{A} 's effectiveness in reducing \mathcal{M} 's private benefits of control, $(1 - \gamma)$, and is decreasing \mathcal{A} 's effectiveness in restoring the damage to firm value, $(1 - \kappa)$. Higher probability of the sell-side liquidity shock, θ , shifts δ_I toward δ_{BM} and therefore decreases the impact of intervention on \mathcal{M} 's incentives. That is, higher θ has a negative (positive) impact on \mathcal{M} 's incentives when the intervention has a positive (negative) impact on \mathcal{M} 's incentives.

We next consider the existence of the intervention equilibrium. The equilibrium is more likely to exist when $\frac{\varphi}{\alpha}$ is higher (\mathcal{A} needs to purchase fewer shares in the open market) and $(1 - \kappa)$ is closer to one (\mathcal{A} is effective in restoring the damage). The equilibrium is also more likely to exist as Φ_I increases, that is, when \mathcal{M} is more likely to take the bad action. This happens when β is large (the agency problem is severe), $(1 - \gamma)$ is small (\mathcal{A} is less effective in reducing \mathcal{M} 's private benefits of control), $(1 - \kappa)$ is closer to one (\mathcal{A} is effective in restoring the damage), and when the distribution of δ shifts left. Note that $(1 - \kappa)$ positively affects the existence of the intervention equilibrium through two channels (condition (1) and Φ_I).

The impact of the probability of a sell-side liquidity shock on the existence depends on whether the equilibrium plays a positive or a negative disciplinary role. If $\gamma < \kappa$ and the equilibrium plays a positive disciplinary

role, as the probability of a sell-side liquidity shock increases, δ_I increases toward δ_{BM} and therefore Φ_I increases as well. As a result, condition (1) is less restrictive. If $\gamma > \kappa$ and the equilibrium plays a negative disciplinary role, as the probability of a sell-side liquidity shock increases, δ_I decreases toward δ_{BM} and therefore Φ_I decreases as well. As a result, condition (1) is more restrictive.

Finally, note that the existence and the disciplinary role of this equilibrium does not depend on ω_1 , which is one of key parameters that will drive existence and the effectiveness of the exit equilibrium. Also, note that buy-side liquidity shocks ζ affect neither existence nor the disciplinary role of this equilibrium. This is because when \mathcal{A} is forced to buy, he intervenes if \mathcal{M} takes the bad action.

2.2. Equilibrium with Exit

Next we construct equilibria in which \mathcal{A} does not intervene when \mathcal{M} takes the bad action (i.e., $\sigma_1^B = 0$). Again, $\Phi_E, \Lambda_E, \delta_E$ represent values of the endogenous variables in the equilibrium.

Proposition 2. *There exists a unique equilibrium with $\sigma_1^B = 0$ if and only if*

$$\frac{\varphi}{\alpha} < \frac{\kappa \left(1 - \Phi_E \frac{\bar{\zeta}}{(1-\Phi_E)+\bar{\zeta}} \right)}{\Phi_E \left(\frac{1+\bar{\theta}}{\Phi_E+\bar{\theta}} - \frac{\kappa \bar{\zeta}}{(1-\Phi_E)+\bar{\zeta}} \right)}, \quad (2)$$

where $\delta_E = \frac{\beta}{\omega_2} \frac{\gamma\zeta + (1-\zeta)}{\kappa\zeta + (1-\zeta)} - (1 - \theta - \zeta) \frac{\omega_1}{\omega_2} \frac{p_B - p_S}{\kappa\zeta + (1-\zeta)}$, $\Phi_E = F(\delta_E)$, $\Lambda_E = \Phi_E^{-1} E[\mathbb{1}_{\delta < \delta_E} \delta]$ and equilibrium prices are $p_B = v - \kappa \Lambda_E \Phi_E \frac{\bar{\zeta}}{(1-\Phi_E)+\bar{\zeta}}$ and $p_S = v - \Lambda_E \frac{\Phi_E + \bar{\theta} \Phi_E}{\Phi_E + \bar{\theta}}$. Equilibrium beliefs are

$(\sigma_0^B = 1, \sigma_0^H = 0, \sigma_0^S = 0; \sigma_1^B = 0, \sigma_1^H = 0, \sigma_1^S = 1)$. The equilibrium is disciplinary if $\gamma < \kappa$.

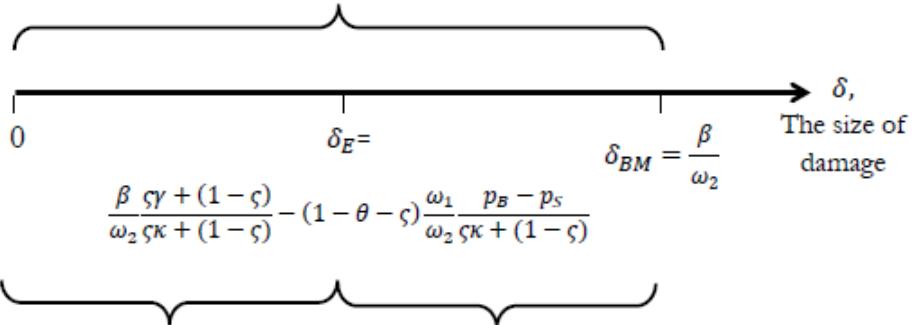
Columns (3) and (4) in Table 2 summarize the results. The impact of exit on \mathcal{M} 's incentives is summarized in Figure 4. The condition $\gamma < \kappa$ (\mathcal{A} is more effective in reducing the private benefits than in restoring firm value), is a sufficient condition exit to play a *positive* disciplinary role ($\delta_E < \delta_{BM}$). When $\gamma > \kappa$ (\mathcal{A} is more effective in restoring firm value than in reducing the private benefits), exit may play a *negative* disciplinary role.

Panel A in Figure 4 describes the case when $\delta_E < \delta_{BM}$. When $\delta > \delta_{BM}$, \mathcal{M} does not take the bad action even when \mathcal{A} is not present. In this case the damage to firm value is so large that \mathcal{M} prefers to forego the private benefit β . In the intermediate region $\delta_{BM} > \delta > \delta_E$, \mathcal{A} 's presence prevents \mathcal{M} from taking the bad action. This is the disciplinary role of the exit. Finally, when $\delta < \delta_E$, \mathcal{M} takes the bad action and \mathcal{A} sells his stake.

Panel B in Figure 4 describes the case when $\delta_E > \delta_{BM}$. When $\delta > \delta_E$, \mathcal{M} does not take the bad action even when \mathcal{A} is present. In this case the damage to firm value is so large that \mathcal{M} prefers to forego the private benefit β . When $\delta < \delta_E$, \mathcal{M} takes the bad action and \mathcal{A} sells his stake. Notice that \mathcal{M} 's bad action is induced by the presence of \mathcal{A} when $\delta_{BM} < \delta < \delta_E$.

Several parameters affect the disciplinary role of this equilibrium. \mathcal{M} is less likely to take the bad action when ω_1 is large (\mathcal{M} 's compensation is more dependent on period 1 prices), $(1 - \kappa)$ is small (\mathcal{A} is less effective in restoring the damage), $(1 - \gamma)$ is large (\mathcal{A} is more effective in reducing \mathcal{M} 's private benefits of control), and when β is small (the agency problem is not severe). Interestingly, the number of shares owned by \mathcal{A} does not affect the

The bad action is taken in the benchmark case



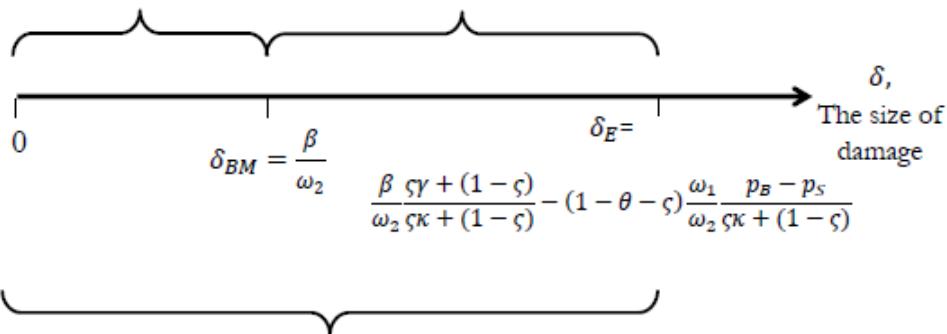
The bad action is taken in
the exit equilibrium.
No strategic ex-post
intervention.

The positive disciplinary
effect of exit: the bad
action is not taken in the
exit equilibrium.

(a) $\gamma < \kappa$: Positive disciplinary role of Exit.

The bad action is
taken in the
benchmark case

The negative disciplinary
effect of exit: the bad action is
taken in the exit equilibrium.



The bad action is taken in the exit equilibrium.
No strategic ex-post intervention.

(b) $\gamma > \kappa$: Negative disciplinary role of Exit.

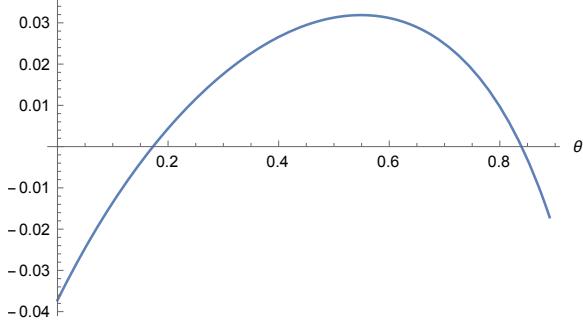
Figure 4: The disciplinary role of Exit.

disciplinary role of the exit and intervention equilibria.

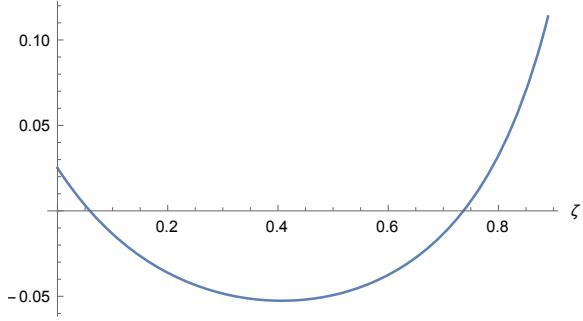
Liquidity shocks also affect the disciplinary role of the exit equilibrium. Sell-side liquidity shocks have a positive impact on the probability that \mathcal{M} takes the bad action because they make prices less informative. On the other side, the effect of buy-side liquidity shocks on the disciplinary role of the exit equilibrium can be either positive or negative. On the one hand buy-side liquidity shocks make prices less informative. On the other hand, buy-side liquidity shocks lead to “accidental” interventions, which may enhance the disciplinary role of the exit equilibrium.

The main intuition behind the exit form of governance is that the mere possibility of \mathcal{A} ’s exit after bad action makes period 1 prices sensitive to \mathcal{M} ’s decision to take the bad action. That is, short-term prices become more sensitive to the long-term firm value. Moreover, \mathcal{M} ’s incentives to take the bad action are affected by the possibility that \mathcal{A} intervenes if he faces a positive liquidity shock.

We next consider the existence of the exit equilibrium. The equilibrium is more likely to exist when $\frac{\varrho}{\alpha}$ is small (\mathcal{A} needs to purchase many shares in the open market in order to intervene) and $(1 - \kappa)$ is close to zero (\mathcal{A} is not efficient in restoring the damage). The equilibrium is also more likely to exist as Φ_E decreases, that is, when \mathcal{M} is less likely to take the bad action. This happens when β is small (the agency problem is not severe), ω_2 is large (\mathcal{M} has strong incentive to maximize the terminal firm value), $(1 - \gamma)$ is large (\mathcal{A} is more effective in reducing \mathcal{M} ’s private benefits of control), $(1 - \kappa)$ is small (\mathcal{A} is less effective in restoring the damage to firm value), and when the distribution of δ shifts right.



(a) The effect of θ on existence of the exit equilibrium.



(b) The effect of ζ on existence of the exit equilibrium.

Figure 5: When the solid line is above (below) x-axis condition 2 holds (does not hold). We assume $v=100$, $\beta=25$, $\omega_1=1$, $\omega_2=2$, $\theta=0.1$, $\zeta=0.1$, $\gamma=0.3$; $\frac{\phi}{\alpha}=0.5$, $\kappa=0.5$, $f[x] = \lambda \exp(-\lambda x)$, and $\lambda=0.1$.

The existence of the exit equilibrium is positively affected by ω_1 because when ω_1 increases, \mathcal{M} is less likely to take the bad action. Thus, when \mathcal{M} is more sensitive to the period 1 prices, the exit equilibrium is more likely to exist. Note that the existence of the intervention equilibrium does not depend on ω_1 .

The effect of θ on the existence of the exit equilibrium could be either positive or negative. For example, as is evident from Figure 5, when θ is high,

an increase in θ makes condition (2) less likely to hold. In contrast, when θ is low, an increase in θ makes condition (2) more likely to hold. Similarly, ζ has an ambiguous effect on the existence of exit equilibrium. Figure 5 shows that when ζ is low, an increase in ζ makes condition (2) less likely to hold. In contrast, when ζ is high, an increase in ζ makes condition (2) more likely to hold.

2.3. Mixed Strategy Equilibrium

We next show that there can exist an equilibrium in which both intervention and exit have positive probabilities.

Proposition 3. *There is a unique mixed strategy equilibrium if both conditions (1) and (2) are violated. Equilibrium beliefs are $(\sigma_0^B = 1, \sigma_0^H = 0, \sigma_0^S = 0; \sigma_1^B > 0, \sigma_1^H = 0, \sigma_1^S > 1)$. In equilibrium, $p_B = v - \kappa \Lambda \Phi \frac{\sigma_1^B + \bar{\zeta}}{\Phi \sigma_1^B + (1-\Phi) + \bar{\zeta}}$ and $p_S = v - \Lambda \Phi \frac{(1-\sigma_1^B) + \bar{\theta}}{\Phi(1-\sigma_1^B) + \bar{\theta}}$, where $\delta_{Mixed} = \frac{B(\theta, \zeta, \sigma_1^B, \gamma)/\omega_2 - (\omega_1/\omega_2)(1-\theta-\zeta)(1-\sigma_1^B)(p_B-p_S)}{((1-\theta-\zeta)\sigma_1^B + \bar{\zeta})\kappa + ((1-\theta-\zeta)(1-\sigma_1^B) + \bar{\theta})}$, $B(\theta, \zeta, \sigma_1^S, \gamma) = \gamma \beta [(1-\theta-\zeta)\sigma_1^B + \zeta] + \beta [(1-\theta-\zeta)(1-\sigma_1^B) + \theta]$, $\Phi = F(\delta_{Mixed})$, and $\Lambda = \Phi^{-1} E[\mathbb{1}_{\delta < \delta_{Mixed}} \delta]$.*

This result is reminiscent of several papers which have shown in more complicated contexts that exit and voice can be complementary (e.g., Dasgupta and Piacentino, 2014); in our model it can be thought of as occurring in cases where neither mechanism is strong enough to survive on its own.

2.4. Multiple Equilibria

Multiple equilibria are also possible. One possibility is that both (1) and (2) are satisfied for their respective values of Φ , in which case we will have

one pure strategy equilibrium of each sort. The next proposition provides sufficient conditions for this to occur.

Proposition 4. *Consider a convergent sequence of parameter values with $\gamma \rightarrow 1$ and $\zeta \rightarrow 0$. Suppose for this sequence (2) is satisfied, so that there is a sequence of exit equilibria. Suppose as well that for the same values of Φ (1) is satisfied. Then for parameters sufficiently far in the sequence, there are also intervention equilibria.*

In the sequence as constructed, the intervention equilibria are less disciplinary than the exit equilibria; however sequences can also be constructed such that the reverse is true. In section 3.2, we compare the effectiveness of the two equilibria when both occur.¹²

Another possibility is that there is one stable mixed strategy equilibrium and one stable exit equilibrium. This happens when condition (1) is violated (there is no pure intervention equilibrium) and condition (2) holds (there is pure exit equilibrium).¹³ If condition (1) is violated, then condition (2) will be violated for sufficiently high values of $\bar{\zeta}$ and $\bar{\theta}$.

3. Discussion and Implications

In this section we characterize circumstances when each type of governance is possible and discuss several empirical implications.

¹²When there are two pure strategy equilibria, there will also generally be a mixed strategy equilibrium, but it will be unstable.

¹³In this case $G(\sigma_1^B)$, defined in the proof of Proposition 3, crosses X axis for two values of σ_1^B . The solution with the higher value of σ_1^B is a stable equilibrium.

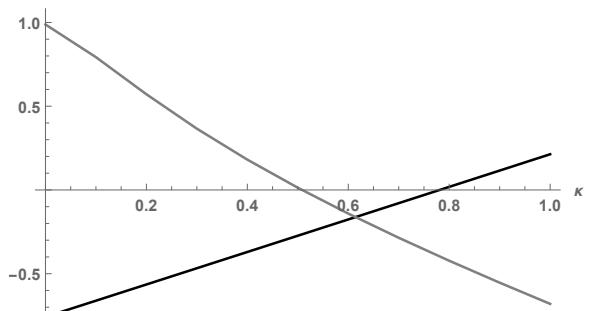
3.1. When does only one type of governance work?

We consider circumstances when intervention is the sole equilibrium form of governance.¹⁴ This is the case when condition (1) holds and condition (2) is violated. Three key parameters affect these conditions: \mathcal{A} 's effectiveness in restoring firm value ($1 - \kappa$), \mathcal{A} 's ownership relative to what is needed for intervention ($\frac{\varphi}{\alpha}$), and the probability of the bad action (Φ).

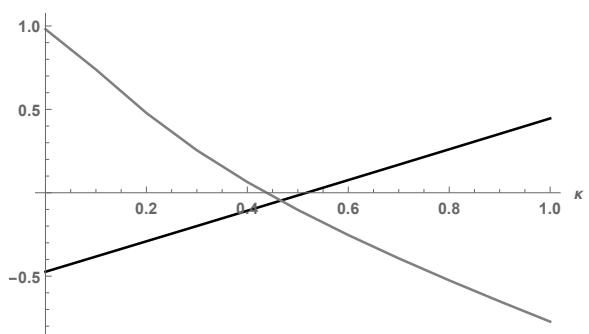
First, consider the role of \mathcal{A} 's effectiveness in restoring firm value. As Panel B in Figure 6 shows, Intervention is the only form of governance when $\kappa < \min(\kappa_I, \kappa_E)$, where κ_I (κ_E) is the threshold level of \mathcal{A} 's effectiveness in restoring the damage for the existence of the intervention (exit) equilibrium (see conditions (1) and (2)). In other words, if \mathcal{A} is very effective in restoring firm value (($1 - \kappa$) is high), then intervention will be the only form governance. Empirically, this result suggests that when it is common knowledge that there is a shareholder capable of restoring firm value, \mathcal{M} 's incentives cannot be affected by \mathcal{A} 's threat to liquidate his stake. Brav et al. (2008) document the emergence of activist hedge funds as a class of institutional investors who specializes in intervention and are often effective in increase the value of firms they target. This empirical regularity is consistent with the prevalence of the Intervention type of governance.

As Figure 6 shows, the intervention type of governance is more likely to prevail in equilibrium when \mathcal{A} 's ownership is high (as measured by $\frac{\varphi}{\alpha}$). Empirically, it implies that \mathcal{M} 's incentives will be affected by the threat of intervention and not the threat of exit when he learns that an activist

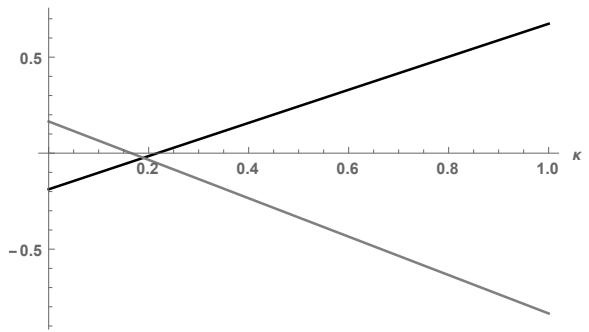
¹⁴Circumstances when the exit is the sole equilibrium form of governance can be analyzed in a similar way.



(a) $\frac{\varphi}{\alpha} = 0.8$



(b) $\frac{\varphi}{\alpha} = 0.5$



(c) $\frac{\varphi}{\alpha} = 0.2$

Figure 6: When grey (dark) line is above X-axis, the Intervention (Exit) equilibrium exists. We assume $v=100$, $\beta=25$, $\omega_1=1$, $\omega_2=2$, $\theta=0.1$, $\zeta=0.1$, $\gamma=0.3$, $f[x] = \lambda \exp(-\lambda x)$, and $\lambda=0.1$.

shareholder has amassed a toehold. \mathcal{M} can obtain this information through either Schedule 13F, Schedule 13D, or Schedule 13G filings.

It's worth noting that while $(1 - \kappa)$ and $\frac{\varrho}{\alpha}$ increase the chances that the Intervention form of governance is the only form of governance in equilibrium, these two parameters have different effects on \mathcal{M} 's incentives to take the bad action. Higher $(1 - \kappa)$ increases \mathcal{M} 's incentives to take the bad action and $\frac{\varrho}{\alpha}$ does not affect \mathcal{M} 's incentives to take the bad action.

Finally, the intervention type of governance is more likely to be the equilibrium form of governance when \mathcal{M} is more likely to take the bad action in the intervention equilibrium (therefore leading to higher κ_I) and when \mathcal{M} is more likely to take the bad action in the exit equilibrium (therefore leading to higher κ_E).

3.2. When both Exit and Intervention discipline the manager

We next characterize circumstances when both exit and Intervention types of governance can discipline \mathcal{M} . Both types of governance happen when condition (1) and condition (2) hold, i.e., when $\kappa_E < \kappa < \kappa_I$. In those circumstances where both types governance can happen, what considerations increase effectiveness of intervention vs. exit?

Effectiveness of each governance mechanism is measured by expected period 1 price in each equilibrium:

$$p_I = (1 - \theta)p_{BI} + \theta p_{SI} = p_{BI} - \theta(p_{BI} - p_{SI}) \quad (3)$$

and

$$\begin{aligned} p_E &= (1 - \theta - \zeta)(\Phi_E p_{SE} + (1 - \Phi_E)p_{BE}) + \theta p_{SE} + \zeta p_{BE} \\ &= p_{BE} - (\theta + (1 - \theta - \zeta\Phi_E))(p_{BE} - p_{SE}), \end{aligned} \quad (4)$$

where $p_{BI} = v - \kappa\Lambda_I\Phi_I$, $p_{SI} = v - \Lambda_I\Phi_I$ (see Proposition 1 for further details), $p_{BE} = v - \kappa\Lambda_E\Phi_E \frac{\bar{\zeta}}{(1 - \Phi_E) + \zeta}$, and $p_{SE} = v - \Lambda_E \frac{\Phi_E + \bar{\theta}\Phi_E}{\Phi_E + \theta}$ (see Proposition 2 for further details).

Thus, in order to compare the effectiveness of two governance mechanisms one needs to analyze equilibrium prices and stock liquidity. In the next section we perform such analysis.

3.3. Equilibrium prices, stock liquidity, and corporate governance

In this section we compare endogenously determined bid and ask prices as well as expected period 1 prices in each model, analyze forces that affect bid-ask spread, and study the relation between measured stock liquidity and two corporate governance mechanisms.

In the intervention equilibrium, $p_B = v - \kappa\Lambda_I\Phi_I$ and $p_S = v - \Lambda_I\Phi_I$. Both prices decline when \mathcal{M} is more likely to take the bad action (higher Φ_I) and when the expected damage to firm value is large (higher Λ_I). Figure 7 plots equilibrium prices as function of θ when intervention plays a positive disciplinary role ($\gamma < \kappa$). We see that prices decrease when \mathcal{A} is more likely to experience a sell-side liquidity shock. Thus, firm value is decreasing in θ when $\gamma < \kappa$.

In the intervention equilibrium, the bid-ask spread is $p_B - p_S = (1 - \kappa)\Lambda_I\Phi_I$. It is positive as long as \mathcal{A} is effective in restoring the damage,

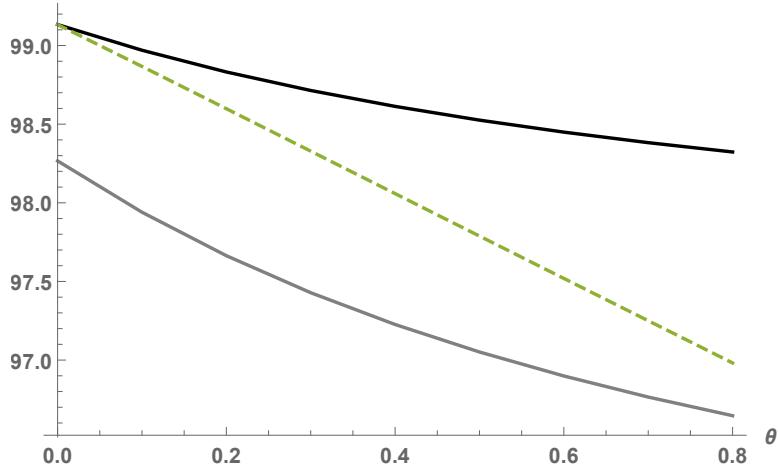


Figure 7: **The effect of θ on prices in the Intervention equilibrium.** The black line plots period 1 buy price, p_B . The grey line plots period 1 sell price, p_S . The dashed line plots the expected period 1 price, $p_1 = (1 - \theta)p_B + \theta p_S$. We assume $v=100$, $\beta=25$, $\omega_1=1$, $\omega_2=2$, $\zeta=0.1$, $\gamma=0.3$; $\frac{\varphi}{\alpha}=0.8$, $\kappa=0.5$, $f[x] = \lambda \exp(-\lambda x)$, and $\lambda=0.1$.

$(1-\kappa) > 0$. Thus, \mathcal{A} 's activism skills are the source of information asymmetry in this equilibrium (see also Collin-Dufresne and Fos, 2014). Interestingly, the bid-ask spread is wider when \mathcal{A} is more likely to experience a sell-side liquidity shock. Thus, Figure 7 reveals that lower likelihood of sell-side liquidity shocks leads to higher measured stock liquidity (narrower bid-ask spread) and higher firm value (higher p_1), implying a positive correlation between these two endogenously determined values.

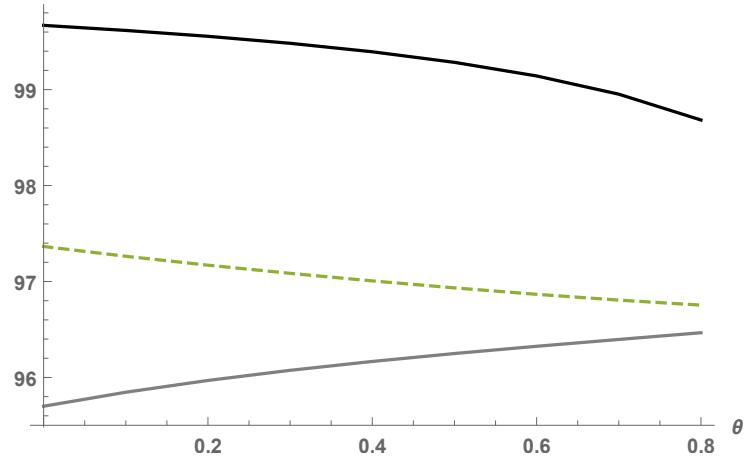
Corollary 1. *In a disciplinary intervention equilibrium ($\gamma < \kappa$), lower θ leads to higher measured stock liquidity (narrower bid-ask spread) and higher firm value (higher p_1).*

The model shows that if the intervention equilibrium is disciplinary

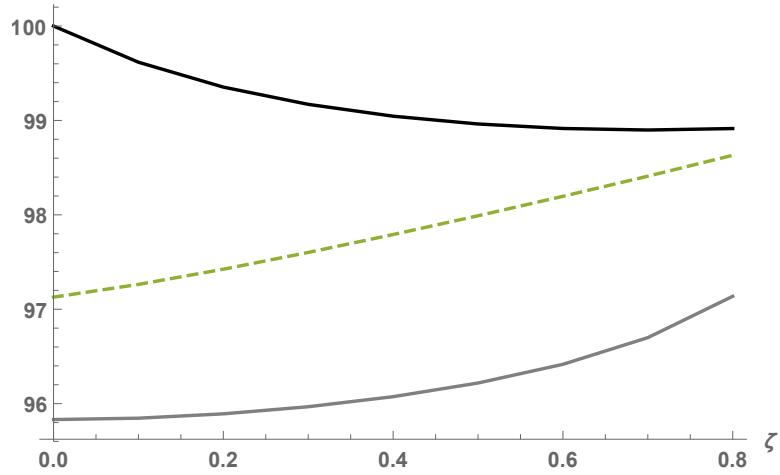
(panel A in Figure 4) then, conditional on having a block, \mathcal{A} may be *less* likely to intervene when θ decreases and as a result the measured liquidity increases. This happens because when θ decreases, the intervention mechanism becomes more disciplinary (δ_I is lower) and therefore fewer interventions take place in equilibrium. This prediction finds support in Edmans et al. (2013), who show that conditional on having a large block (i.e., high $\frac{\varrho}{\alpha}$), activists are more likely to file (passive) Schedule 13G vs. (active) Schedule 13D when measured liquidity increases. Interestingly, while the authors interpret the evidence as “*...liquidity reduces the likelihood that the blockholder governs through voice,*” the evidence is also consistent with an equilibrium in which the *threat* of intervention plays a disciplinary role:

Corollary 2. *In a disciplinary intervention equilibrium ($\gamma < \kappa$), lower θ leads to narrower bid-ask spreads and higher probability of intervention.*

In the exit equilibrium, the formulas in Proposition 2 show that when $\zeta = 0$, $p_B = v$, reflecting that in equilibrium \mathcal{A} buys when the bad action is not taken. When $\zeta > 0$, $p_B < v$ because buy-side liquidity shocks force \mathcal{A} to buy not only when the bad action is not taken, but also when the bad action is taken (accidental interventions). Thus, even though a higher ζ leads to more frequent accidental interventions, it has a negative effect on the buy price because it increases the chances of buying when the bad action is taken. When $\theta = 0$, $p_S = v - \Lambda_E$, reflecting that in equilibrium \mathcal{A} sells when the bad action is taken. When $\theta > 0$, $p_S > v - \Lambda_E$ because sell-side liquidity shocks force \mathcal{A} to sell not only when the bad action is taken, but also when the bad action is not taken.



(a) The effect of θ on prices in the Exit equilibrium.



(b) The effect of ζ on prices in the Exit equilibrium.

Figure 8: **The effect of θ and ζ on prices in the Exit equilibrium.** The black line plots period 1 buy price, p_B . The grey line plots period 1 sell price, p_S . The dashed line plots the expected period 1 price, $p_1 = (1 - \theta - \zeta)(\Phi_E p_S + (1 - \Phi_E)p_B) + \theta p_S + \zeta p_B$. We assume $v=100$, $\beta=25$, $\omega_1=1$, $\omega_2=2$, $\theta=0.1$, $\zeta=0.1$, $\gamma=0.3$, $\frac{\varphi}{\alpha}=0.2$, $\kappa=0.5$, $f[x] = \lambda \exp(-\lambda x)$, and $\lambda=0.1$.

Figure 8 shows the effects of θ and ζ on equilibrium prices in the exit equilibrium. When the chances of forced liquidity sale increase, the price at which \mathcal{A} can buy (sell) decreases (increases). The difference between the prices (i.e., bid-ask spread) narrows, corresponding to higher measured stock liquidity. Interestingly, when θ is high the equilibrium has lower bid-ask spread as well as lower firm value, implying a negative correlation between endogenously determined firm value and bid-ask spread. When related to the recent empirical literature on the positive relation between measured stock liquidity and firm value (e.g., Edmans et al., 2013; Bharath et al., 2013; Fos, 2015), this finding indicates that forces other than discipline through exit may drive the relation between firm value and stock liquidity.

While Figure 8 reveals similar relation between equilibrium prices and the chances of forced purchase, the relation between the expected period 1 price and ζ is positive and not negative as in case of θ . The variation in ζ implies that lower bid-ask spread (when ζ is high) leads to higher firm value because it increases the possibility that \mathcal{A} will purchase shares in period 1. Thus, the nature of the liquidity shock plays a key role when the effectiveness of the exit equilibrium is concerned.

Corollary 3. *In the exit equilibrium, higher θ (ζ) implies a negative (positive) correlation between endogenously determined bid-ask spread and firm value.*

4. Initial toehold

In this section we discuss the formation of the initial toehold, φ . The initial toehold plays an important role in the model because it determines

what type of corporate governance will prevail in equilibrium. To examine this issue, we provide a simple extension to the model which endogenizes the activist’s choice of size of toehold. In this section we will use the term “effective” to refer to the governance structure that yields the higher expected value of the firm in equilibrium.¹⁵

Suppose at the beginning of period 0 \mathcal{A} chooses how many shares φ to buy. The market does not observe \mathcal{A} ’s purchase or its size; instead there is an expectation that with probability $1 - \nu$ a purchase will be made by \mathcal{A} and with probability ν a purchase will be made by a non-monitoring investor. If \mathcal{A} does not purchase, the value of the firm is p_{BM} . In effect, higher values ν correspond to a more liquid period 0 market; \mathcal{A} ’s purchase can be hidden more effectively in the sea of non-monitor purchases. The actual price on the market in period 0, denoted p_0 , will reflect the activism role played by \mathcal{A} as well as the market’s expectation that he is participating. In other words, $p_0(\nu) = (1 - \nu)p_1(\varphi) + \nu p_{BM}$, where $p_1(\varphi)$ is the expected price in period 1.

After the initial trade occurs, \mathcal{A} ’s ownership stake (φ) becomes common knowledge. This assumption is motivated by the fact that market participants can use Schedule 13F, Schedule 13D, and Schedule 13G filings to infer changes in stock ownership. It is also assumed that holding φ shares involves private cost $C(\varphi) = -\phi \frac{\varphi^2}{2}$ for \mathcal{A} . For example, this cost could correspond to lower diversification of \mathcal{A} ’s portfolio.¹⁶

¹⁵While “effectiveness” is a natural criterion, it does not necessarily equate to Pareto optimality, since we are ignoring not only the costs of the activist to acquiring the initial toehold (discussed below) but also the payoff to the manager.

¹⁶The analysis can be extended to consider the possibility that higher φ increases the likelihood that \mathcal{A} faces a sell-side liquidity shock in period 1.

Recall that conditions (1) and (2) determine the existence of intervention and exit equilibrium, respectively. If φ is close to α , condition (1) is more likely to hold and condition (2) is less likely to hold. Similarly, if φ is close to zero, condition (1) is less likely to hold and condition (2) is more likely to hold. Given α , let φ_I^* and φ_E^* represent the boundary values for the two conditions, so that values φ above $\max(\varphi_I^*, \varphi_E^*)$ lead to the intervention equilibrium, and values below $\min(\varphi_I^*, \varphi_E^*)$ lead to the exit equilibrium. In equilibrium, then, φ affects firm value by determining what type of governance will prevail. We will focus on the case where $\varphi_I^* < \varphi_E^*$, so that for intermediate values of φ both types of equilibria exist.¹⁷

What size of the initial toehold will \mathcal{A} choose? For simplicity assume that when φ is consistent with multiple equilibria, the equilibrium selected is the one most preferred by \mathcal{A} .¹⁸ \mathcal{A} 's choice of φ is affected by several factors. \mathcal{A} 's expected profits are (after dropping factors not affected by ϕ):

$$V(\varphi; p_1) = \varphi\nu(p_1(\varphi) - p_{BM}) - \phi\frac{\varphi^2}{2}. \quad (5)$$

Note that as long as $\nu = 0$, $\varphi^* = 0$. This is because if \mathcal{A} needs to purchase φ shares in the open market at price that reflects \mathcal{A} 's impact of firm value, privately-optimal initial stake size will be zero in the absence of liquidity trading. When $\nu > 0$, \mathcal{A} profits from liquidity trading because prices do not fully reflect \mathcal{A} 's impact of firm value.

¹⁷The opposite possibility leads to mixed strategy equilibria when φ is in the intermediate range; this possibility can be analyzed in a similar fashion.

¹⁸An interesting possible technical extension would be to use the structure of global games to develop a selection criterion. In this case, the boundary generated by the selection criterion substitutes for φ^* 's in the following analysis.

Before we derive the optimal size of the initial toehold for \mathcal{A} , we characterize \mathcal{A} 's choice of φ for a given level of period 1 prices.

Lemma 2. *\mathcal{A} 's choice of φ for a given level of period 1 prices is $\varphi_{\mathcal{A}} = \frac{1}{\phi}\nu(p_1 - p_{BM})$. \mathcal{A} 's expected profit is $V(\varphi_{\mathcal{A}}; p_1) = \frac{1}{2\phi}\nu^2(p_1 - p_{BM})^2$.*

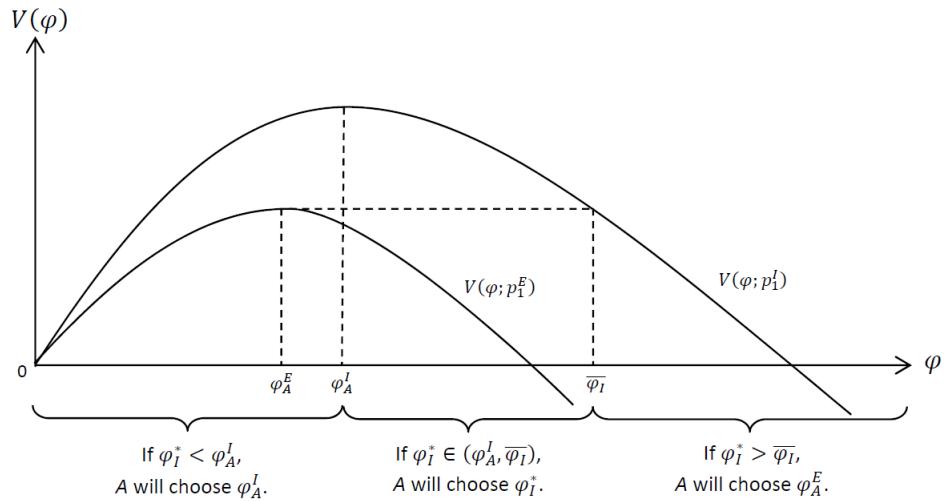
In other words, \mathcal{A} will choose higher φ when liquidity trading is large (ν), the cost of holding the block is small (ϕ), and \mathcal{A} 's impact on firm value is large ($p_1 - p_{BM}$). The following propositions characterize \mathcal{A} 's optimal φ , while taking into account the effect of φ on the form of governance that will prevail in equilibrium. First consider the case when intervention is the effective type of governance:

Proposition 5. *Suppose $p_1^I > p_1^E$. Let $\overline{\varphi}_I$ be such that $V(\overline{\varphi}_I; p_1^I) = V(\varphi_{\mathcal{A}}; p_1^E)$. \mathcal{A} will choose $\varphi_{\mathcal{A}} \geq \varphi_I^*$ as long as $\varphi_I^* \leq \overline{\varphi}_I$. In this case intervention will be the equilibrium type of governance. If $\varphi_I^* > \overline{\varphi}_I$, \mathcal{A} will choose $\varphi_{\mathcal{A}} = \varphi_{\mathcal{A}}^E$. In this case exit will be the equilibrium type of governance.*

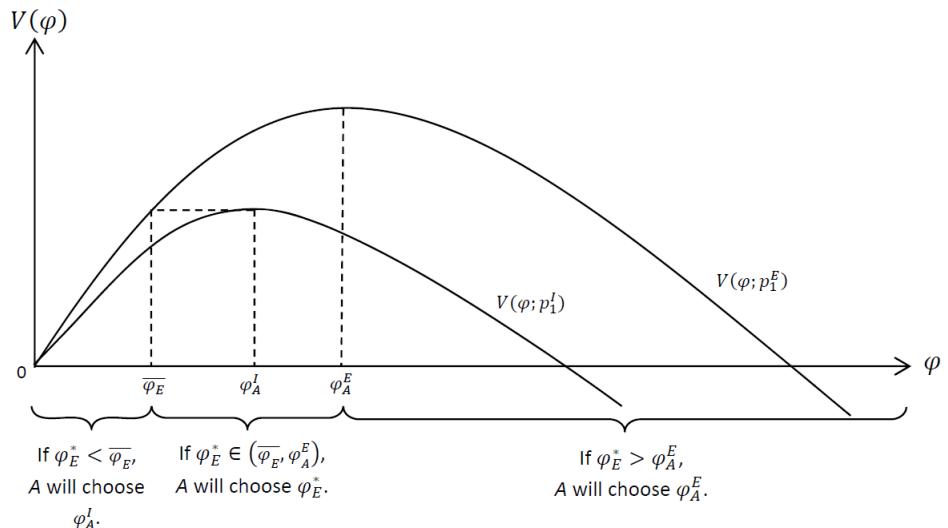
The intuition behind Proposition 5 is presented in Panel A of Figure 9.

When $\varphi_I^* < \varphi_{\mathcal{A}}^I$, \mathcal{A} finds it optimal to choose $\varphi_{\mathcal{A}}^I$ that maximizes \mathcal{A} 's profits in the intervention equilibrium. Thus, the Intervention is the equilibrium type of governance and because $\varphi_I^* < \varphi_{\mathcal{A}}^I$, \mathcal{A} 's private choice leads to the optimal type of governance.

When $\varphi_I^* \in (\varphi_{\mathcal{A}}^I, \overline{\varphi}_I)$, \mathcal{A} realizes that he needs to choose stake large enough to maintain prices from the intervention equilibrium because $\varphi_{\mathcal{A}}^I$ is not large enough to maintain the intervention equilibrium. Thus, \mathcal{A} will choose the lowest possible $\varphi = \varphi_I^*$ such that intervention is the equilibrium type of governance.



(a) $p_1^I > p_1^E$: Intervention is socially optimal type of governance



(b) $p_1^E > p_1^I$: Exit is socially optimal type of governance

Figure 9: \mathcal{A} 's choice of φ .

In the above two cases \mathcal{A} 's private choice leads to adoption of the effective type of governance. In contrast, when $\varphi_I^* > \overline{\varphi}_I$, \mathcal{A} does not find it optimal to build a toehold which is large enough to support the intervention equilibrium. As a result, \mathcal{A} will prefer to switch to the exit equilibrium and will select the reduced level of toehold $\varphi_{\mathcal{A}} = \varphi_{\mathcal{A}}^E$. In this case \mathcal{A} 's private choice leads to adoption of the less effective type of governance.

Next consider the case when exit is the effective type of governance:

Proposition 6. *Suppose $p_1^E > p_1^I$. Let $\overline{\varphi}_E$ be such that $V(\overline{\varphi}_E; p_1^E) = V(\varphi_{\mathcal{A}}; p_1^I)$. \mathcal{A} will choose $\varphi_{\mathcal{A}} \leq \varphi_E^*$ as long as $\varphi_E^* \geq \overline{\varphi}_E$. In this case exit will be the equilibrium type of governance. If $\varphi_E^* < \overline{\varphi}_E$, \mathcal{A} will choose $\varphi_{\mathcal{A}} = \varphi_{\mathcal{A}}^I$. In this case intervention will be the equilibrium type of governance.*

The intuition is presented in panel B. Proposition 6 shows that when $\varphi_E^* > \varphi_{\mathcal{A}}^E$, \mathcal{A} chooses a block size small enough to maintain the exit equilibrium. In this case \mathcal{A} 's private choice leads to the optimal type of governance. When $\varphi_E^* \in (\overline{\varphi}_E, \varphi_{\mathcal{A}}^E)$, \mathcal{A} realizes that $\varphi_{\mathcal{A}}^E$ is too large to maintain the exit equilibrium. Therefore, \mathcal{A} will choose the highest possible $\varphi = \varphi_E^*$ such that exit is the equilibrium type of governance.

In the above two cases \mathcal{A} 's private choice leads to adoption of the effective type of governance. In contrast, when $\varphi_E^* < \overline{\varphi}_E$, \mathcal{A} 's private choice leads to adoption of the less effective governance. Instead \mathcal{A} finds it optimal to build a toehold too large to support the exit equilibrium, choosing instead the profit maximizing intervention equilibrium $\varphi_{\mathcal{A}} = \varphi_{\mathcal{A}}^I$.

In general, the intervention equilibrium requires a large initial toehold, and the exit equilibrium requires a small initial toehold. Thus depending on

which of the two types of governance is more effective, factors that encourage larger initial toeholds (liquid markets in the initial period, lower costs to amassing the initial toehold) can increase *or* reduce the effectiveness of governance. For example, *smaller* liquidity trading and larger cost of holding the toehold are beneficial when the effective equilibrium is the exit equilibrium.

5. Conclusion

In this paper we have developed a model to compare two categories of disciplinary mechanisms used by activist shareholders: intervention and exit. We have derived predictions as to when one or the other is more likely to be available and more likely to be effective in disciplining the manager. In addition to the considerations which have been frequently alluded to in studies of the individual categories (size of share needed for a toehold and other costs of engaging in activism, relevant for intervention; sensitivity of managerial compensation to short run share price, relevant for exit), we have also examined factors which are of importance in both intervention and exit mechanisms.

When a manager engages in destructive behavior, intervention can have two effects: it can restore firm value, or it can reduce managerial benefit from the behavior. As we have seen, because of the interactions with the behavior of the manager, these two effects lead to different implications for effectiveness of intervention, and the likelihood that it is observed instead of exit activity. Empirically, these predictions could be tested using data on institutional ownership. Activist shareholders are likely to have skills in recovering firm value (e.g., Brav et al., 2008). Moreover, activist shareholders are able

to impose private costs on management when proxy contests are concerned (e.g., Fos and Tsoutsoura, 2014). Changes in beneficial ownership of activist shareholders could therefore be used to study what type of governance is effective.¹⁹

Finally, liquidity has a complicated effect on the forms of governance. Our model suggests that empiricists need to be more cautious when studying implications of stock liquidity on corporate governance—first, because the empirical measures of stock liquidity, such as bid-ask spreads are endogenous to the equilibrium trading decisions made by activist shareholders. But even if the underlying liquidity shocks faced by activists are themselves exogenous, we show that type of the liquidity shock faced by the activist can have different effects in the model. While the effect of liquidity shocks on the existence of exit equilibria is ambiguous, we have provided numerical examples to show that when the likelihood of liquidity shocks is small, increases in the frequency of unexpected need to buy reduces the likelihood of an exit equilibrium and increases in the unexpected need to sell increases the likelihood of an exit equilibrium. On the other hand, liquidity shocks that force the activist to sell have no impact on the existence of the intervention equilibrium, while liquidity shocks that force the activist to buy have a positive effect on the existence of intervention equilibrium. When both exit and intervention equilibria exist, frequency of liquidity shocks of either type reduces the effectiveness of intervention relative to that of exit.

¹⁹For example, one could argue that an increase in activist hedge funds ownership enhances the role of the intervention mechanism. However, endogeneity of activist ownership suggests that an extraordinary caution is required when such empirical analysis is performed.

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Supplemental Internal Materials for the paper
“Governance through Threats of Intervention and Exit”

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Proof of Lemma 1.

\mathcal{A} 's profits, as measured by $\pi - \varphi v$, are as follows:

	$a = 0$ (no damage) (1)	$a = 1$ (damage) (2)
Buy	$(\alpha - \varphi)\kappa\Lambda \frac{\Phi\sigma_1^B + \bar{\zeta}\Phi}{\Phi\sigma_1^B + (1-\Phi)\sigma_0^B + \zeta}$	$-\alpha\kappa\Lambda + (\alpha - \varphi)\kappa\Lambda \frac{\Phi\sigma_1^B + \bar{\zeta}\Phi}{\Phi\sigma_1^B + (1-\Phi)\sigma_0^B + \zeta}$
Hold	0	$-\varphi\Lambda$
Sell	$-\varphi\Lambda \frac{\Phi\sigma_1^S + \bar{\theta}\Phi}{\Phi\sigma_1^S + (1-\Phi)\sigma_0^S + \bar{\theta}}$	$-\varphi\Lambda \frac{\Phi\sigma_1^S + \bar{\theta}\Phi}{\Phi\sigma_1^S + (1-\Phi)\sigma_0^S + \bar{\theta}}$

$\pi_0^B - \varphi v > \pi_0^H - \varphi v > \pi_0^S - \varphi v$ implies $\sigma_0^B = 1$, $\sigma_0^H = 0$, and $\sigma_0^S = 0$.

Similarly, $\pi_1^S - \varphi v > \pi_1^H - \varphi v$ implies $\sigma_1^H = 0$ if $\Phi < 1$. A sufficient condition for $\Phi < 1$ is that the support of the distribution include sufficiently high values of δ such that the manager is uninterested in taking the action. For example, it is sufficient to assume $F(\beta/\omega_2) < 1$.

Proof of Proposition 1.

Condition 1 follows from comparing π_1^B and π_1^S when $\sigma_1^B = 1$.

Given the beliefs, \mathcal{M} expects $P_1(a = 0) = P_1(a = 1) = (1-\theta)p_B + \theta p_S$.

Moreover, in equilibrium \mathcal{M} consumes private benefits $\beta(\theta + (1-\theta)\gamma)$ if $a = 1$. Thus, if \mathcal{M} does not take the action, his expected utility is $\omega_1((1-\theta)p_B + \theta p_S) + \omega_2 v$. If \mathcal{M} takes the action, his expected utility is $\omega_1((1-\theta)p_B + \theta p_S) + \omega_2(v - \tilde{\delta}(\theta + (1-\theta)\kappa) + \beta(\theta + (1-\theta)\gamma))$. The cutoff point is therefore $\delta_I = \beta/\omega_2 \frac{\theta + (1-\theta)\gamma}{\theta + (1-\theta)\kappa}$.

Proof of Proposition 2.

Condition 2 follows from comparing π_1^B and π_1^S when $\sigma_1^B = 0$. Given the beliefs, \mathcal{M} expects $P_1(a = 0) = (1-\theta-\zeta)p_B + \zeta p_B + \theta p_S$ and $P_1(a = 1) = (1-$

$\theta - \zeta)p_S + \zeta p_B + \theta p_S$. Moreover, in equilibrium \mathcal{M} consumes private benefits $\beta(\gamma\zeta + (1 - \zeta))$ if $a = 1$. Thus, if \mathcal{M} does not take the action, his expected utility is $\omega_1(p_B - \theta(p_B - p_S)) + \omega_2 v$. If \mathcal{M} takes the action, his expected utility is $\omega_1(p_S + \zeta(p_B - p_S)) + \omega_2(v - \tilde{\delta}(\kappa\zeta + (1 - \zeta))) + \beta(\gamma\zeta + (1 - \zeta))$. The cutoff point is therefore $\delta_E = \frac{\beta}{\omega_2} \frac{\gamma\zeta + (1 - \zeta)}{\kappa\zeta + (1 - \zeta)} - (1 - \theta - \zeta) \frac{\omega_1}{\omega_2} \frac{p_B - p_S}{\kappa\zeta + (1 - \zeta)}$, where $p_B - p_S = \Lambda \Phi \left[\frac{1 + \bar{\theta}}{\Phi + \bar{\theta}} - \frac{\kappa \bar{\zeta}}{(1 - \Phi) + \bar{\zeta}} \right]$.

Proof of Proposition 3.

Consider $G(\sigma_1^B) \equiv \pi_1^B - \pi_1^S$:

$$G(\sigma_1^B) = -\alpha\kappa + (\alpha - \varphi)\kappa\Phi \frac{\sigma_1^B + \bar{\zeta}}{\Phi\sigma_1^B + (1 - \Phi) + \bar{\zeta}} + \varphi\Phi \frac{(1 - \sigma_1^B) + \bar{\theta}}{\Phi(1 - \sigma_1^B) + \bar{\theta}}. \quad (1)$$

For there to be an equilibrium with $\sigma_1^B \in (0, 1)$ for a given Φ , it is necessary and sufficient that $G(\sigma_1^B) = 0$ at some σ_1^B . Eliminating denominators yields a quadratic function:

$$\begin{aligned} & -\alpha\kappa(\Phi\sigma_1^B + (1 - \Phi) + \bar{\zeta})(\Phi(1 - \sigma_1^B) + \bar{\theta}) \\ & + (\alpha - \varphi)\kappa\Phi(\sigma_1^B + \bar{\zeta})(\Phi(1 - \sigma_1^B) + \bar{\theta}) \\ & + \varphi\Phi((1 - \sigma_1^B) + \bar{\theta})(\Phi\sigma_1^B + (1 - \Phi) + \bar{\zeta}), \end{aligned}$$

for which the quadratic coefficient is

$$(\alpha\kappa - (\alpha - \varphi)\kappa - \varphi)\Phi^2 = -(1 - \kappa)\varphi\Phi^2 < 0,$$

implying that the quadratic coefficient is negative thus the function has a single maximum. Thus, the function is continuous, has a single maxi-

mum, and by hypothesis $G(0) > 0$ and $G(1) < 0$. Therefore the function equals zero for some $\sigma_1^B \in (0, 1)$. The values of p_B and p_S are immediate from their definitions. Given the beliefs, \mathcal{M} expects $P_1(a = 0) = (1 - \theta)p_B + \theta p_S$ and $P_1(a = 1) = (1 - \theta - \zeta)[\sigma_1^B p_B + (1 - \sigma_1^B)p_S] + \zeta p_B + \theta p_S$. Moreover, in equilibrium \mathcal{M} consumes private benefits $B(\theta, \zeta, \sigma_1^B, \gamma) = \gamma\beta[(1 - \theta - \zeta)\sigma_1^B + \zeta] + \beta[(1 - \theta - \zeta)(1 - \sigma_1^B) + \theta]$. Thus, if \mathcal{M} does not take the action, his expected utility is $\omega_1 P_1(a = 0) + \omega_2 v$. If \mathcal{M} takes the action, his expected utility is $\omega_1 P_1(a = 1) + \omega_2(v - \tilde{\delta}[(1 - \theta - \zeta)\sigma_1^B + \zeta]\kappa + ((1 - \theta - \zeta)(1 - \sigma_1^B) + \theta)] + B(\theta, \zeta, \sigma_1^B, \gamma)$. The cutoff point is therefore

$$\delta_{Mixed} = \frac{B(\theta, \zeta, \sigma_1^B, \gamma)/\omega_2 - (\omega_1/\omega_2)(1 - \theta - \zeta)(1 - \sigma_1^B)(p_B - p_S)}{((1 - \theta - \zeta)\sigma_1^B + \zeta)\kappa + ((1 - \theta - \zeta)(1 - \sigma_1^B) + \theta)}.$$

Proof of Proposition 4

Suppose we have a sequence of parameter values satisfying the conditions of the proposition. Then those sequences yield a sequence of exit equilibria. However they do not necessarily yield a sequence of intervention equilibria, since the value Φ that applies for the exit equilibrium will not in general be the same value for the intervention equilibrium. Nonetheless, examination of the condition (1) demonstrates that if it is satisfied for one value of Φ , it is also satisfied for all higher values of Φ . Thus if the conditions defining the cutoff in the case of intervention are less stringent than in the case of the corresponding exit equilibrium, we in fact have an intervention

equilibrium as well. By the earlier propositions, the intervention cutoff is

$$\delta_I = \beta/\omega_2 \frac{\theta + (1 - \theta)\gamma}{\theta + (1 - \theta)\kappa}$$

and the exit cutoff is

$$\delta_E = \frac{\beta}{\omega_2} \frac{\gamma\zeta + (1 - \zeta)}{\kappa\zeta + (1 - \zeta)} - (1 - \theta - \zeta) \frac{\omega_1}{\omega_2} \frac{p_B - p_S}{\kappa\zeta + (1 - \zeta)},$$

where

$$p_B - p_S = \Lambda\Phi \left[\frac{1 + \bar{\theta}}{\Phi + \bar{\theta}} - \frac{\kappa\bar{\zeta}}{(1 - \Phi) + \bar{\zeta}} \right].$$

Thus $\delta_I > \delta_E$ is equivalent to

$$\beta/\omega_2 \frac{\theta + (1 - \theta)\gamma}{\theta + (1 - \theta)\kappa} > \frac{\beta}{\omega_2} \frac{\gamma\zeta + (1 - \zeta)}{\kappa\zeta + (1 - \zeta)} - (1 - \theta - \zeta) \frac{\omega_1}{\omega_2} \frac{p_B - p_S}{\kappa\zeta + (1 - \zeta)}.$$

In the limit as $\gamma \rightarrow 1$ and $\zeta \rightarrow 0$ the expression is:

$$(1 - \theta)(p_B - p_S) > \frac{\beta}{\omega_1} \left[1 - \frac{1}{\theta + (1 - \theta)\kappa} \right].$$

The inequality holds in the limit because $p_B > p_S$ and $\theta + (1 - \theta)\kappa < 1$.