Government as Borrower of First Resort

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Abstract

We examine optimal supply of safe government bonds accounting for their effect on corporate debt markets. Government bonds are shown to influence leverage under asymmetric information regarding corporate cash flows and safe asset scarcity. Corporations have incentives to issue junk debt in response to safe asset scarcity since uninformed investors then migrate to junk debt markets. Uninformed demand stimulates informed speculation which drives junk debt prices closer to fundamentals, encouraging pooling at high leverage. Acting as borrower of first resort, the government can issue safe bonds which siphon off uninformed demand for risky corporate debt and reduce socially wasteful informed speculation. Thus, government bonds either eliminate pooling at high leverage or improve risk sharing in such equilibria. An optimal supply of government bonds is increasing in both marginal Q and the intrinsic demand for safe assets.

In recent years the set of safe stores of value has contracted. A number of factors are responsible. The credit crisis of 2007/8 revealed the exposure of senior tranches of securitizations to correlated defaults. The Eurozone crisis called into question the safety of some sovereign debts. Finally, fiscal weakness undermined confidence in deposit insurance in some jurisdictions. At the same time, it has been argued, these crises stimulated investor demand for safe stores of value in a flight-to-quality. The perceived combination of diminished supply and increased demand for safe assets has led some to argue that there is a scarcity of safe assets. In this vein, a recent Global Financial Stability

Report of the IMF (2012) states, "In the future, there will be rising demand for safe assets, but fewer of them will be available..."

Of course, safe asset scarcity must be an endogenous phenomenon. Moreover, one might reasonably expect the problem to be self-correcting. For example, Gourinchas and Jeanne (2012) argue "the economy as a system will strive to compensate for any shortage." In this paper, we take a corporate finance perspective on the self-correction mechanism, analyzing whether corporations capable of supplying safe (long-term) debt necessarily have the incentive to do so. We then assess the potential role to be played by government-supplied safe bonds in light of their effect on corporate debt markets.

In contrast to contemporaneous work by Gorton and Ordonez (2013), who focus on the collateral value of safe assets in repurchase agreements, our focus is on the distinct role of government and corporate bonds as long-term stores of value. In addition to its role as high-grade collateral for short-term borrowing, this investment role of safe debt is noted by the IMF (2012): "Safe assets are used as a reliable store of value and aid in capital preservation in portfolio construction." Here it is worth noting that even if one could readily identify riskless money-like short-term assets, such as money market mutual funds or demand deposits, a long-term investor still potentially faces a scarcity of financial investments generating a (near) riskless long-horizon return. For example, pension funds in the U.K. have long argued that they need access to government bonds with very long maturities. H.M. Treasury responded by offering ultra-long gilts.

To the extent that pension funds and insurers have a strong demand for ultra-long-duration safe stores of value, one might expect an issuing corporation with long-lived capital assets to adopt a low leverage ratio, supplying the market with long maturity debt with low default risk. However, there is little evidence that corporations are supplying low risk debt in response to investor demands for it. To the contrary, Stein (2013) points out that in the U.S. it is the junk bond market that has grown in recent years, with record volumes of high-yield debt issuance, leveraged loans, and dividend recapitalization transactions, as well as high debt-to-EBITDA multiples in leveraged buyouts. A

similar junk bond boom has taken place in European corporate debt markets.[1] This growth pattern is mirrored in the market for long-maturity project finance and infrastructure bonds.[2] In light of recent trends, policymakers such Stein (2013) have called for tools to identify and respond to debt market "overheating," which presumably means a debt market featuring high leverage and low social welfare.

Rather than punishing aggressive corporate financing decisions, the market has actually increased the relative reward to highly levered transactions in recent years, even as low risk government bond yields have fallen, consistent with the notion of a scarcity of safe assets. As shown in Figure 1, after the financial crisis of 2007/8, U.S. government bond yields fell. At the same time, the yield spread between corporate debt rated CCC and below and debt rated AAA actually narrowed, nearing historic lows.

In this paper we put forward a framework for understanding the conjunction of safe asset scarcity and an overheated corporate debt market. The model is predicated on a canonical corporate finance friction: asymmetric information between the corporation and investors regarding asset values or cash flows, as in Ross (1977). Our objective is to evaluate whether and how the offering of safe government bonds can be used as a policy tool to influence equilibrium in corporate debt markets and increase social welfare. The analysis leads to a novel theory regarding the optimal supply of safe government bonds.

We consider the following economy. A corporation chooses a debt face value and then uses the proceeds from the bond flotation to fund a scalable investment providing the shareholder with a private benefit Q per unit invested. Marginal Q is greater than one, so each unit invested has positive social NPV. The terminal period payoff on the asset-in-place backing the debt is either low (L) or high (H). This payoff is verifiable ex post, but is only privately observed by the corporation ex ante.

We depart from the extant debt signaling literature by allowing investors to purchase corporate

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[1] See "European junk bond volumes rise as banks retrench," Financial Times 22 August 2013.

[2] See "Infrastructure bonds grab investor attention," in Financial Times 2 December 2013.

debt in a securities market modeled à la Kyle (1985). There are perfectly competitive market-makers who clear the market and a speculator who can exert costly effort to acquire a noisy signal regarding the asset payoff. In addition, there is a continuum of uninformed investors who would prefer to carry funds across periods using a riskless store of value. There is safe asset scarcity, which the corporation can remedy by issuing riskless debt. A positive question addressed is whether the corporation will provide the uninformed investors with an information-insensitive store of value, facilitating efficient risk-sharing. The normative question addressed is whether the government can increase social welfare by acting as a borrower of first resort with an eye toward influencing the corporate debt market equilibrium.

One potential outcome is a separating equilibrium: If the corporation has positive private information, it signals this by issuing debt with face value L with the shareholder bearing all cash flow risk. From a social welfare perspective, this equilibrium is attractive in that uninformed investors are insulated from adverse selection, so they make efficient intertemporal transfers. Further, there is no socially wasteful speculator information production. However, corporate investment is low. Another possible outcome is pooling at riskless debt. This equilibrium has the same welfare properties as the separating equilibrium.

The final potential outcome is pooling at risky debt. This equilibrium has features that are typically treated as indicative of so-called overheating: leverage is high, bonds are mispriced, and yields are low given high corporate leverage. Relative to the separating equilibrium, the yield on a highly levered transaction is low, reflecting the averaging of default risk over the two potential types. Bonds are mispriced in that a low quality issuer is charged a yield that is too low and a high quality issuer is charged a yield that is too high. Despite being charged too high of a yield, a corporation with positive information is willing to pool at risky debt if new investment has high NPV and/or there is high intrinsic demand for safe bonds (captured by an uninformed investor endowment shock in the model).

We show that the problem of safe asset scarcity need not self-correct. To contrary, when the

demand for safe assets is strong, the private sector may very well supply risky rather than safe debt. That is, risky debt can be imposed on investors precisely when doing so generates a large negative externality. The argument is as follows. If there is safe asset scarcity, a portion of uninformed investor demand migrates to the risky corporate debt market. The prospect of trading against uninformed investors encourages speculator information production and informed trading. In turn, informed trading brings the risky debt price closer to fundamental value. And with less severe underpricing, a corporation with positive private information is more willing to issue risky debt even if investment NPV is quite low. This also benefits the corporation with negative private information as this firm then issues overvalued claims and invests more. Although this outcome leaves both issuer types better off, it entails low social welfare in the instance of low project NPV. After all, anticipation of adverse selection in the risky debt market will induce distortions in the portfolios of uninformed investors. For example, some uninformed investors will simply choose to consume today rather than save, resulting in utility/distress costs due to inadequate terminal period resources. In addition, there are direct costs of speculator information production. Against these costs, government must weigh the benefit of increased investment financed by the higher debt issuance.

We consider that the government acts as a Stackelberg leader and offers investors safe bonds anticipating their effect on the corporate debt market equilibrium. In the model, the government has insufficient debt capacity to meet all uninformed demand for safe assets. Despite this constraint, the government can increase social welfare, ensuring an adequate aggregate supply of safe assets by offering a limited amount of bonds that serve to influence corporate debt markets. The argument is as follows. Uninformed investors will substitute any available riskless government bonds for risky corporate debt in their portfolios. The anticipation of less uninformed trading in the corporate debt market deters speculator information production. This widens the gap between debt prices and fundamentals. With sufficient underpricing anticipated, high debt equilibria unravel, as a corporation with positive information then refuses to pool at risky debt. Thus, government debt

serves to *crowd-out* risky debt and *crowd-in* riskless corporate debt.

The preceding paragraph illustrates one potential rationale for the government to offer safe bonds to investors: eliminating the possibility of pooling at risky corporate debt. However, this policy involves a tradeoff. Risk-sharing becomes efficient but corporate investment is low. Crowding out junk debt (and concomitant marginal investment) in this way increases social welfare only if marginal Q is sufficiently low. This bring up the second, more subtle, rationale for some limited offering of safe government bonds. If marginal Q is high, social welfare will be higher if there is pooling at risky corporate debt. Here a limited supply of government bonds serves to increase social welfare given pooling at risky debt. After all, the siphoning argument still applies: Government bonds siphon uninformed demand, decrease speculator effort, and mitigate distortions in the portfolios of uninformed investors.

Regardless of whether the objective for offering safe government bonds is to prevent pooling at risky debt or to simply make such an equilibrium more efficient, we find that a simple government bond supply function suffices. This supply function is increasing in the intrinsic demand for safe storage. Intuitively, if the goal of the government is to deter pooling at risky debt, it must siphon off more uninformed demand if storage demand is higher. If instead the government's goal is to efficiently implement pooling at risky debt, higher storage demand allows it to better protect uninformed investors by offering more safe bonds while still preserving the corporation's willingness to implement this equilibrium. The government bond supply function is also increasing in marginal Q. If the goal of the government is to deter pooling at risky debt, it must siphon off more uninformed demand if Q is higher. If instead the government's goal is to efficiently implement pooling at risky debt, higher Q allows it to offer investors the protection of even more safe bonds while still preserving the corporation's willingness to implement this equilibrium.

The separating equilibrium in our model is standard, although earlier papers ignore the beneficial effect of signaling in terms of improved risk sharing. In the model of Ross (1977), high debt face value is a positive signal under his assumption that the manager bears a personal cost in the event

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of default. In contrast, Stiglitz and Weiss (1981) argue that higher debt face values constitute a negative signal, since those with negative information do not intend to pay what they promise. We modify standard signaling frameworks by allowing for the possibility of informed trading driving prices closer to fundamentals. Our model of price formation extends the tractable models of Maug (1998) and Faure-Grimaud and Gromb (2004), for example. However, these papers assume pure noise-trading. Such setups preclude welfare analysis and rule out our central causal mechanism: endogenous changes in uninformed demand.

Gorton and Pennacchi (1990) also analyze the equilibrium supply of riskless debt in a setting where uninformed investors prefer safe storage. However, in their model the issuer does not have private information and it is the uninformed investors who exercise effective control over the intermediary's financial structure. In their setting, uninformed investors carve out a safe debt claim for themselves. In contrast, we show a privately informed issuer can have the opposite incentive, switching from riskless to risky debt when uninformed investors have high demand for safe storage. Although they do not focus on government bond provision, the analysis of Gorton and Pennacchi suggests it would be optimal for the government to serve all uninformed demand for safe bonds. We analyze how a government can meet this demand even if it cannot do so unilaterally. However, we also show that it is not generally optimal for government to fill all demand for safe bonds even if it can do so.

In the model of Woodford (1990), government bonds directly increase welfare by providing agents with an intertemporal store of value. Holmström and Tirole (1998) analyze the social welfare benefits of government bonds in a setting where limits on income verifiability constrain the private supply of stores of value. They consider a setting with hidden action and potential production inefficiencies, while we consider a setting with hidden information, speculative information production, and inefficient risk sharing across investors. In their model the role of government debt is a direct one, in that it increases aggregate storage dollar for dollar. In our model, government bonds can have a disproportionate multiplier effect on the aggregate supply of safe assets by crowding out corporate

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junk debt and crowding in riskless corporate debt.

Dang, Gorton and Holmström (2012) predict the privately optimal security is debt, which minimizes incentives for information acquisition. In contrast, we show a privately-informed owner may have an incentive to promote information acquisition by speculators since this drives prices closer to fundamentals in noisy rational expectations equilibria.

In the model of Gorton and Ordonez (2013), projects are positive NPV but loans must be backed by collateral. Some producers have low quality collateral and will be cut off from credit if investors acquire this information. The government can increase producer collateral by making them a gift of its bonds backed by future tax collections. Essentially, the government is allowing producers to credibly pledge future income via the tax system. The increase in collateral deters information production and increases investment. In their model, information production results in lower investment while in our model equilibria with information production feature higher investment. In their model, the indirect cost of information production takes the form of lower investment while in our model it takes the form of uninformed investor portfolio distortions. In their model, safe government debt serves to crowd-in risky borrowing while in our model government debt serves to crowd-out risky borrowing.

In our model, high debt is a negative signal. Eckbo (1986) finds that straight debt offerings have "non-positive valuation effects." Greenwood, Hanson and Stein (2010) present empirical evidence that corporate debt issuance fills gaps left by government debt. Greenwood and Hanson (2012) present evidence suggesting this gap filling may not be like-for-like. In particular, they find that the high-yield share of corporate debt flotations is inversely related to Treasury yields. This suggests corporations respond to scarcity of government debt by supplying riskier debt. In our model, safe asset scarcity can lead to high corporate leverage. Philippon (2010) documents a leading role for yield spreads in predicting corporate investment. In our model, an "overheated" debt market is associated with high investment and low yields given high leverage. Krishnamurthy and Vissing-Jorgensen (2012) find that episodes of safe asset scarcity are helpful in predicting subsequent crises,

although their focus is on short-term money-like assets. In our model, safe asset scarcity results in corporate debt markets with adverse selection, inducing some uninformed investors to save less, resulting in their potentially becoming distressed in the future.

The remainder of the paper is as follows. Section I describes the economic setting. Section II describes bond pricing. Section III describes the equilibrium corporate leverage decision taking as given the supply of government safe bonds. Section IV analyzes the optimal supply of safe government bonds.

I. The Economic Setting

There are two periods (1 and 2) and three categories of agents: government, corporation(s), and investors. The government and corporation offer bonds and investors buy them. All investors enter the model with sufficiently large endowments in period 1 to finance their desired portfolios. The period 2 endowments of investors are not verifiable so they cannot issue securities backed by them. Allen and Gale (1988) and Holmström and Tirole (1998) also rule out unsecured credit based on limited income verifiability.

Investors cannot privately store their period 1 endowments, e.g. privately stored goods will be stolen or decay. The absence of such private storage creates the possibility of a scarcity of stores of value, in the spirit of Holmström and Tirole (1998). In fact, each investor could be endowed with a limited amount of safe storage capacity without changing the results. The critical assumption is that private storage capacity, which has been normalized to zero, is smaller than intrinsic storage demand.

The government has the unique ability to store goods from period 1 to period 2 without any risk of theft or decay. To illustrate most clearly the ability of the government to raise social welfare via the borrower-of-first-resort channel, it is assumed to have no other capabilities. In particular, the government can neither verify endowments in order to collect taxes nor redistribute resources.[3]

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[3]Endowing government with ability to tax and redistribute creates a trivial rationale for government to transfer

In our parable economy, the government simply has the ability to collect goods from investors in period 1, place them in public storage, and return them in period 2. Essentially, government bond investors receive risk-free inflation-protected debt claims. The government's ability to provide such risk free stores of value is assumed to be limited, however. In particular, the maximum capacity of the public storage facility is $\overline{G} \in [0, \infty)$.

The objective of the government is to maximize a utilitarian social welfare function placing equal weight on each agent. The government acts as a Stackelberg leader in debt markets, specifying the amount of storage $G \in [0, \overline{G}]$ that it will make available to investors. In the event that requested storage exceeds G, it will be allocated on a pro rata basis.

The corporate sector acts as follower in a Stackelberg game. Specifically, just after the government specifies G, a private corporation chooses its own debt level. For simplicity, we initially focus on the leverage decision of a single corporation, with interdependence between corporate capital structure decisions analyzed as an extension. The corporation is controlled by a manager-owner ("the manager" below) who cannot raise outside equity funding due to his ability to costlessly divert discretionary cash flow.[4] The manager has vNM utility function over consumption $QC_1 + C_2$, where $Q > 1$. The corporation has an asset-in-place but no internal funds, and the manager has no other funds. The asset-in-place will generate an observable and verifiable cash flow in period 2. The cash flow is either L or H, where $H > L > 0$. The asset *type*, denoted $T \in \{L, H\}$, is equivalent to the cash flow that the asset-in-place will generate. Each asset type is equally likely, and investors do not know the true asset type. In contrast, the manager privately observes the asset type in period 1.

The privately informed manager chooses his corporation's leverage by specifying a debt face value D due in period 2. The proceeds raised by the debt flotation are used to finance a dividend in period 1. The manager enjoys limited liability so the period 2 payoff on the debt is equal to

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funds to positive NPV investments.

[4] With outside equity, the qualitative welfare tradeoffs remain. Low corporate leverage leads to efficient risk sharing and high leverage leads to high investment.

the minimum of T and D. As captured by the manager's utility function, each unit of funding the corporation receives in period 1 provides the manager with Q units of utility. There are two alternative interpretations for why the manager utility parameter Q is greater than one. First, one may think of the manager as being impatient. Second, one may think of the manager as using the funds received from investors to finance a new investment providing him with a private benefit. In this, our chosen interpretation, Q represents marginal Q.

There are three categories of investors who invest in government and corporate bonds. There is a measure one continuum of uninformed investors (UI). By construction, the UI are analogous to the liquidity traders in the model of Gorton and Pennacchi (1990). They are akin to pension funds and insurance companies in that they are risk-averse and have an intrinsic preference for safe storage. The UI have identical stochastic period 2 endowments $Y_2 \in \{Z - N, Z\}$ where $N \in (0, Z]$.[5] Each realization of Y_2 is equiprobable. It is assumed that the negative endowment shock satisfies the following two inequalities:

$$N > \overline{G}$$
$$N \leq \frac{L}{2}.$$

The first inequality implies that the government does not have the capacity to meet all intrinsic demand for safe assets. The second inequality implies the corporation has the ability to meet the intrinsic demand for safe assets by issuing debt with face value L. The specific form of the second inequality plays an additional technical role ensuring market-makers never face a call to take infeasible short positions in the corporate debt market.

Each UI has linear utility over period 1 consumption and concave utility over period 2 consumption. We follow the tractable specification of risk-aversion employed by Dow (1998) in that the period 2 utility of each UI is piecewise linear, with a concave kink at a critical consumption level which is just equal to Z.[6] The UI are heterogeneous in that they differ in the intensity of their

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[5] Assuming perfect correlation only serves to simplify the algebra.

[6] Other smooth utility functions could be assumed, with more complex aggregate UI demands.

aversion to consumption shortfalls. An uninformed investor with preference parameter θ has vNM utility function:

$$U(C_1, C_2; \theta) \equiv C_1 + \theta \min\{0, C_2 - Z\}. \tag{1}$$

By construction, each UI is averse to period 2 consumption falling below the critical level Z, creating an intrinsic demand for safe storage when confronted with a low terminal endowment. The intensity of aversion to low terminal consumption is captured by the idiosyncratic preference parameter θ. The θ parameters have support $\Theta \equiv [1, \infty)$ with density f and cumulative density F. The distribution is atomless, with f strictly positive and continuously differentiable. Given the preferences described in equation (1), it is apparent that when faced with the prospect of a low future endowment, each UI would like to invest in a riskless security delivering N units in period 2, bringing C_2 up to the critical level Z.

There is a risk-neutral speculator S with vNM utility function $C_1 + C_2$. Her period 1 endowment is Y_1^s and her period 2 endowment is normalized at zero without loss of generality. The speculator is unique amongst investors in that she observes a private signal $s \in \{s_L, s_H\}$ of the true asset type. The speculator chooses the precision of her signal, σ, from a feasible set $\Sigma \equiv [\frac{1}{2}, 1]$. Signal precision is defined as follows:

$$\sigma \equiv \Pr[T = H | s = s_H] = \Pr[T = L | s = s_L].$$

The speculator must exert costly effort in order to generate an informative signal. The speculator's effort cost function e is twice continuously differentiable, strictly increasing and convex, with

$$\lim_{\sigma \downarrow \frac{1}{2}} e(\sigma) = 0$$
$$\lim_{\sigma \downarrow \frac{1}{2}} e'(\sigma) = 0$$
$$\lim_{\sigma \uparrow 1} e'(\sigma) = \infty.$$

Since e is strictly increasing, it has a well-defined inverse

$$\Psi \equiv e^{-1}.$$

In addition to the uninformed investors and speculator, there are a large number of risk-neutral market-makers (MM below). Each MM has vNM utility function $C_1 + C_2$. Their aggregate period 1 endowment is Y_1^{mm} and their period 2 endowment is normalized at zero without loss of generality.

Investors form beliefs regarding the true asset type based upon the manager's choice of D. They anonymously submit simultaneous orders for government safe storage and the corporate debt. Prior to submitting orders, the speculator pays the effort cost $e(\sigma)$ and observes the signal s regarding the asset type T. Prior to placing orders, the UI privately observe their period 2 endowment. The corporate debt price is set as in Kyle (1985): the MM observe aggregate order flows and bid up the corporate debt price until it reaches its conditional expected payoff.

We solve for pure strategy perfect Bayesian equilibria (PBE). For each $D \in \mathcal{D}$ that may be chosen by the manager, each investor must have an assessment $a : \mathcal{D} \to [0,1]$ regarding the probability that the true asset type is H. In response to debt face values chosen on the equilibrium path, investor beliefs regarding the type must be consistent with Bayes' rule. Actions of all agents must be sequentially optimal given their beliefs regarding the asset type and the actions of the other agents. Our primary interest is in pinning down the socially optimal amount of bonds G for the government to offer investors in light of the fact that corporate leverage will vary in response.

Anticipating, the central mechanism in the model is the interplay between government borrowing and asymmetric information in the corporate debt market. To this, note that if the true asset type were common knowledge, the manager would sell debt with face value equal to the true cash flow ($D = T$). The MM would then set the debt price $P = T$. Since the corporate debt would be priced at its true payoff, the speculator would have no incentive to exert costly effort. On the other hand, the UI with low terminal endowments would purchase N/T units of debt, ensuring they achieve the critical consumption level Z. That is, under symmetric information, the manager would raise T units of outside funding and first-best sharing of risks would be achieved across investors. Thus, safe government bonds would be superfluous under common knowledge of the asset type.

II. The Corporate Debt Market

We solve via backward induction. Consider first the equilibrium price (P) of the corporate debt. Order flow is irrelevant for debt pricing if the debt face value is sufficiently low. In particular

$$D \leq L \Rightarrow P = D. \tag{2}$$

Of course, if the corporation issues riskless debt, the speculator has no incentive to acquire a costly signal regarding the true asset type. In the special case where $D = L$, UI hit with a negative endowment shock can submit orders for N/L units of the debt, just enough to achieve the target terminal period consumption level Z. This corresponds to a perfect sharing of risks across investors.

Consider next the pricing of debt for higher face values. Here we must distinguish between two types of equilibria. In a separating equilibrium the choice of debt face value varies with the true type T fully revealing the manager's private information. In such cases, the MM will set the debt price equal to its true type-contingent payoff. We have:

$$\text{Separating Equilibrium} \Rightarrow P = \min\{D, T\}. \tag{3}$$

Note that when the manager's choice of face value reveals the true asset type, the speculator has no incentive to exert costly effort. Further, if the true asset type is revealed to investors, each UI hit with a negative income shock can submit an order for $N/\min\{D, T\}$ units of the debt, just enough to achieve the target consumption level Z. This corresponds to perfect risk sharing across investors.

Consider next price determination in the event of a pooling equilibrium in which the debt face value $D \in (L, H]$ is invariant to the true asset type. Here the debt price set by the MM will depend upon order flow. Consider then the aggregate demand of the UI. In a pooling equilibrium, UI enter the debt market holding their prior belief that the true asset type is H with probability one-half. If the UI have a high period 2 endowment, they have no motive to buy any debt. Conversely, if the UI anticipate the low period 2 endowment, they may be willing to buy debt depending on the intensity of their aversion to a consumption shortfall, as well as their expectation of the equilibrium

debt price. Let $x^*(\theta, D, G, \sigma)$ denote the optimal θ-contingent demand for an UI in the event of a low period 2 endowment. Aggregate UI demand in the event of a negative period 2 endowment shock is:

$$X_U(D, G, \sigma) \equiv \int_1^\infty x^*(\theta, D, G, \sigma) f(\theta) d\theta. \tag{4}$$

We will return to the determination of the UI demand function x^* below. Before doing so, it is necessary to consider how the speculator will trade.

The speculator relies on the trading of the UI in the corporate debt market to provide camouflage. In fact, as shown below, her trading gain is increasing in UI demand for corporate debt. Since the UI prefer safe stores of value, they would put all their savings in the government storage if this were feasible. To limit their access to such safe assets, the speculator will submit an infinite order for government bonds, causing them to be allocated on a pro rata basis.[7] It follows that in the event of a low period 2 endowment, the UI will have a residual demand for safe storage equal to $N - G$. The critical role played by the government bond offering is to alter the amount of residual UI storage demand since it is this residual demand that migrates, in part, to the corporate debt market.

Consider next the speculator's optimal order in the corporate debt market. Since she cannot short-sell, her optimal strategy is to place a buy order for the debt if and only if she receives the positive signal s_H. As in Maug (1998), the size of the speculator's buy order is constrained by her need to mask her trades. If the speculator is to make positive expected trading gains, she must choose her order size such that the MM cannot infer her signal. This can only be achieved by choosing an order size such that MM cannot distinguish between no UI endowment shock combined with speculator buying versus UI endowment shock combined with speculator not buying. Thus, the speculator will submit an order for X_U units of corporate debt upon observing a positive signal and place zero order otherwise. Critically, the size of the speculator's order size, and hence her effort incentive, is constrained by the equilibrium volume of uninformed demand the corporate debt attracts.

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[7]Alternatively, the speculator could submit a random order no less than G. Both suffice to mask the UI endowment state as required to confound the MM.

Table 1 depicts the order flow possibilities.

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Table 1: Aggregate Demand Outcomes

Type	Speculator Signal	UI Period 2 Endowment	Speculator Order	UI Order	Aggregate Order	Probability
H	s_H	$Z - N$	X_U	X_U	$2X_U$	$\frac{\sigma}{4}$
H	s_H	Z	X_U	0	X_U	$\frac{\sigma}{4}$
H	s_L	$Z - N$	0	X_U	X_U	$\frac{1-\sigma}{4}$
H	s_L	Z	0	0	0	$\frac{1-\sigma}{4}$
L	s_L	$Z - N$	0	X_U	X_U	$\frac{\sigma}{4}$
L	s_L	Z	0	0	0	$\frac{\sigma}{4}$
L	s_H	$Z - N$	X_U	X_U	$2X_U$	$\frac{1-\sigma}{4}$
L	s_H	Z	X_U	0	X_U	$\frac{1-\sigma}{4}$

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The MM set the debt price based upon aggregate demand (X_A) as follows:

$$P(X_A) = D \Pr(T = H|X_A) + L \Pr(T = L|X_A) \quad \forall \quad X_A \in \{0, X_U, 2X_U\}. \tag{5}$$

As shown in Table 1, the MM will face one of three order flows. The highest and lowest order flows fully reveal the speculator signal, while the intermediate order flow leaves the MM confounded as to the signal and the true asset type. Using Bayes' rule the MM form beliefs as follows:

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\begin{aligned}
\Pr[T &= H|X_A = 2X_U] = \sigma \\
\Pr[T &= H|X_A = X_U] = \frac{1}{2} \\
\Pr[T &= H|X_A = 0] = 1 - \sigma.
\end{aligned}
\tag{6}
$$

It follows from equations (5) and (6) that the debt price is increasing in aggregate demand. Further, the responsiveness of the debt price to aggregate demand is increasing in the speculator's signal precision. Intuitively, the MM revise beliefs more aggressively in response to order flow if the speculator has more precise information.

Continuing the backward induction, we must pin down Nash configurations for the pair (σ, X_U), the speculator signal precision and UI demand. Consider first the speculator's signal precision. The speculator chooses some $\widetilde{\sigma} \in \Sigma$ taking as given the UI demand factor X_U and the pricing rule used by the MM, with the pricing rule itself being predicated upon the speculator signal precision postulated by the MM, call it σ. Using Table 1, the speculator's expected trading gain is:

$$\Pi(\widetilde{\sigma}, \sigma, X_U) = \left[\begin{array}{c} \frac{\widetilde{\sigma}}{4}[D - P(2X_U)] + \frac{\widetilde{\sigma}}{4}[D - P(X_U)] \\ +\frac{1-\widetilde{\sigma}}{4}[L - P(2X_U)] + \frac{1-\widetilde{\sigma}}{4}[L - P(X_U)] \end{array} \right] \times X_U. \tag{7}$$

An incentive compatible speculator signal precision, call it σ_{ic}, equates the marginal change in expected trading gains resulting from a change in $\widetilde{\sigma}$ with the marginal effort cost. This implies:

$$e'(\sigma_{ic}) = \Pi_1(\widetilde{\sigma}_{ic}, \sigma, X_U) = \frac{1}{2}(D - L)X_U. \tag{8}$$

Thus:

$$\sigma_{ic}(X_U) = \Psi\left[(D - L)X_U/2\right]. \tag{9}$$

Since Ψ is increasing, it follows from the preceding equation that, holding all else constant, speculator effort is increasing in the uninformed corporate debt demand X_U. Intuitively, higher UI demand allows the speculator to place larger orders and to make higher trading gains, strengthening her effort incentive.

We now return to determining the debt demands of the individual UI in the event of a low future endowment. In order to formulate their optimal debt demand, the UI must form an expectation of the equilibrium debt price, conditional on being hit with a negative endowment shock. From Table 1 it follows:

$$E[P|Y_2 = Z - N] = \frac{1}{2}\left[D + L + (D - L)\left(\sigma - \frac{1}{2}\right)\right]. \tag{10}$$

Equation (10) shows UI perceive themselves as facing adverse selection in that they expect to pay a price in excess of the unconditional expected debt payoff, which is just $(D + L)/2$. The intuition for this effect is as follows. A negative endowment shock implies higher expected order flow. And in the presence of an informed speculator, the MM will respond to high order flow by setting a

higher debt price. In fact, the price set by the MM is more sensitive to order flow the higher the speculator's signal precision. Thus, the intensity of the adverse selection problem, as perceived by the UI, is increasing in the speculator's signal precision.

With their conditional expectation of the debt price determined, we can now pin down the optimal debt demand for those UI with low period 2 endowments. The optimal corporate debt demand maximizes expected period 2 utility less the expected debt price. The program is:

$$\max_{x \geq 0} \quad \frac{1}{2}\theta \min\{0, -N + G + xL\} + \frac{1}{2}\theta \min\{0, -N + G + xD\} - xE[P|Y_2 = Z - N]. \tag{11}$$

Solving the preceding program, we obtain the following characterization of the optimal UI portfolios:

$$
\begin{aligned}
\theta &\in [1, \theta_1) \Rightarrow x^*(\theta) = 0 \\
\theta &\in [\theta_1, \theta_2) \Rightarrow x^*(\theta) = \frac{N - G}{D} \\
\theta &\geq \theta_2 \Rightarrow x^*(\theta) = \frac{N - G}{L}
\end{aligned}
\tag{12}
$$

where

$$
\begin{aligned}
\theta_1(\sigma, D) &\equiv 1 + \left(\sigma - \frac{1}{2}\right)\left(\frac{D - L}{D + L}\right) \\
\theta_2(\sigma, D) &\equiv 1 + \frac{D}{L} + \left(\sigma - \frac{1}{2}\right)\left(\frac{D - L}{L}\right).
\end{aligned}
\tag{13}
$$

The intuition behind the UI demand function is straightforward. If θ is sufficiently low, adverse selection dominates the storage motive and so the investor boycotts the corporate debt market. For intermediate values of θ, the UI partially insures in the sense of buying just enough units of corporate debt to ensure he will achieve his target consumption Z if $T = H$, which implies his consumption falls short of Z if $T = L$. Finally, if θ is sufficiently high, the investor completely insures in the sense of purchasing enough units of corporate debt to ensure he achieves his target consumption level even if $T = L$, implying his consumption actually overshoots Z if $T = H$.

Integrating over the individual debt demands, we obtain the following expression for aggregate

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UI demand:

$$X_U(D, G, \sigma) = (N - G) \left(\frac{1}{L} [1 - F(\theta_2(\sigma, D))] + \frac{1}{D} [F(\theta_2(\sigma, D)) - F(\theta_1(\sigma, D))] \right). \qquad (14)$$

There are two points worth noting regarding the aggregate UI demand schedule. First, UI demand is predicated upon a conjecture by these investors regarding the signal precision that will be chosen by the speculator. To see this, note that the UI demand cutoffs θ_1 and θ_2 are both increasing in σ, implying aggregate UI demand is decreasing in the speculator signal precision posited by the UI. The second point worth noting is that aggregate UI demand is linear in the size of the residual safe storage demand $N - G$. Thus, we see that government bonds act as a demand shifter in the corporate debt market.

For each given debt face value $D \in (L, H]$ at which we wish to consider the possibility of a pooling equilibrium of the full game, we can now determine the *continuation equilibrium* values for uninformed demand and speculator signal precision. Such continuation equilibrium pairs will be denoted (X_U^{eq}, σ^{eq}), and are found as D-contingent solutions to equations (9) and (14). Substituting the uninformed demand equation (14) into the speculator's incentive compatibility condition (9), equilibrium is defined implicitly by the following equation:

$$\Psi \left[\frac{1}{2}(D - L) X_U(\sigma^{eq}) \right] - \sigma^{eq} = 0. \qquad (15)$$

The appendix shows that for each $D \in (L, H]$, the continuation equilibrium defined by the equation (15) is unique.

Figure 2 depicts the UI demand schedule and speculator signal precision in the event of pooling at risky debt. The continuation equilibrium is found at the intersection of the two curves. The upward sloping line depicts the schedule σ_{ic}. From equation (9) it follows that this schedule is increasing in X_U. Intuitively, the speculator's effort incentive is higher when there is a larger volume of uninformed trading providing camouflage. The downward sloping line depicts the schedule X_U. From equation (14) it follows the UI demand schedule is strictly decreasing in σ. Intuitively, uninformed investors face a more severe adverse selection problem when the speculator has more precise information. They respond by cutting their debt demands.

How then does the government offering of risk free bonds affect equilibrium in the event of pooling at risky debt? Applying the Implicit Function Theorem to equation (15) we find:

$$
\begin{aligned}
\frac{\partial \sigma^{eq}}{\partial G} &= \frac{\frac{1}{2}\Psi'(\cdot)(D-L)\frac{\partial X_U}{\partial G}}{1-\frac{1}{2}\Psi'(\cdot)(D-L)\frac{\partial X_U}{\partial \sigma}} < 0 \qquad\qquad (16) \\
\frac{\partial X_U}{\partial G} &= -\frac{1}{L}\left[1-F(\theta_2(\sigma,D))\right] - \frac{1}{D}[F(\theta_2(\sigma,D))-F(\theta_1(\sigma,D))] < 0.
\end{aligned}
$$

The preceding equations show that the equilibrium level of speculator effort is decreasing in the amount of safe government bonds offered to investors. Intuitively, an increase in the availability of government bonds reduces UI demand for corporate debt. And it is this demand that provides the camouflage and subsidy to informed speculation. As shown in Figure 2, an increase in G manifests itself as a parallel shift downward of the schedule X_U, implying lower equilibrium speculator signal precision.

Again applying the Implicit Function Theorem to equation (15) it can be verified that an increase in speculator effort would arise from an increase in the size of the negative endowment shock N. Similarly, a first-order stochastic dominant shift in the θ parameters would also increase speculator effort, since this too increases uninformed demand at each given level of speculator effort. Intuitively, an increase in θ implies the investor is more willing to incur trading losses in the corporate debt market given he is more averse to a consumption shortfall in period 2. In turn, an increase in uninformed demand for risky debt provides the speculator with more camouflage, allowing her to buy a larger block. This increases the speculator's marginal benefit from acquiring better information.

At this point it is worth emphasizing that the continuation game captured in Figure 2 is only reached in the event that the issuer pools, choosing a T-invariant debt face value $D > L$. The next section turns to the issue of whether and when such a continuation game will be reached in equilibrium.

III. The Corporate Leverage Choice

The previous section described the pricing and trading of debt, taking as given the firm's leverage. This section analyzes the choice of leverage. Recall, in a PBE investors must have a belief for each debt face that can be chosen, and beliefs must be based on Bayes' rule where possible. In turn, agents' actions must be sequentially optimal given beliefs and the actions of the other agents.

A. Equilibrium Set

The following lemma is useful in characterizing potential equilibria.

Lemma 1 *The set of equilibria includes all debt face value configurations such that a manager owning a low value asset-in-place attains at least $V_L^{\min} \equiv QL$, while a manager owning a high value asset-in-place attains at least $V_H^{\min} \equiv QL + H - L$.*

Based on the preceding lemma, we obtain the following proposition.

Proposition 1 *There is no equilibrium in which the manager chooses debt with face value less than L. There is a pooling equilibrium in which, regardless of the true asset type, the manager chooses riskless debt with face value L. The set of separating equilibria are those in which the owner of a high value asset issues debt with face value L while the owner of a low value asset issues debt with face value in $(L, H]$.*

There are a number of points worth noting from the preceding proposition. The first statement in the proposition allows us to confine attention to debt face values no less than L for the remainder of the analysis. The second statement indicates that there is always a pooling equilibrium in which the manager issues riskless debt with face value L, regardless of the true value of his asset-in-place. Pooling at riskless debt is socially attractive since the firm has supplied the riskless debt preferred by uninformed investors. In such an equilibrium, each UI facing a low terminal period endowment will buy $(N-G)/L$ units of corporate debt, ensuring they achieve their target consumption level. At the same time, the speculator does not exert costly effort. The drawback of this equilibrium from a

social welfare perspective is that the owner of a high value asset only receives L units of investment funding whereas he would obtain H units of investment funding if T were common knowledge.

The last statement in Proposition 1 describes separating equilibria. In a separating equilibrium, the high type signals positive information by issuing debt with a low face value and bearing all cash flow risk through his levered equity stake. In contrast, a low type is willing to issue debt with a high face value. That is, the willingness to promise the payment of a high debt face value is a negative signal in this setting. To understand this, note that the manager's utility, expressed in units of current consumption (C_1) and debt face values (D) is:

$$V_T = QC_1 + \max\{T - D, 0\}.$$

Figure 3 depicts indifference curves for the high (solid line) and low (dashed kinked lines) types. As shown, a standard single-crossing condition is satisfied for $D > L$. Relative to the high type, the low type is more willing to increase debt face value beyond L. Intuitively, once D exceeds L, the low type has no qualms raising his debt face value at the margin, since he views it as costless given that he will default.

An important, and generally neglected, feature of separating equilibria is that they have good risk-sharing properties. In each separating equilibrium the private information of the issuer is revealed so that investors can choose the appropriate portfolio without fearing mispricing of debt claims. Regardless of the issuer's private information, investors with a negative endowment shock will buy just enough units of debt to achieve the target terminal period consumption level. The revelation of private information allows for first-best saving.

Consider finally potential pooling equilibria. It was mentioned above that there is always a pooling equilibrium in which the manager chooses face value L regardless of the true type. Consider now whether there exist pooling equilibria in which the issuer, regardless of the true type, chooses some $D > L$. From Lemma 1 it follows that any viable pooling equilibrium has the property that the issuer attains at least his type-contingent minimum utility $V_T^{\min}$. With this in mind, we use Table 1 to compute the type-contingent expected utility of the issuer in the event of pooling at some

face value $D \in (L, H]$. The expected utility (V_T) of the issuer in the event of pooling is equal to Q times the type-conditional expectation of the debt price plus the terminal period dividend. We have:

$$
\begin{aligned}
V_H(D) &= Q\left[I(\sigma^{eq})D + (1 - I(\sigma^{eq}))L\right] + H - D &\quad (17) \\
V_L(D) &= Q\left[I(\sigma^{eq})L + (1 - I(\sigma^{eq}))D\right] \\
I(\sigma) &= \frac{3}{4} + \sigma^2 - \sigma.
\end{aligned}
$$

The endogenous variable I plays an important role in the model, capturing the informational efficiency of prices. For example, if $I = 1/2$ the debt price is completely uninformative, as would be the case in a standard signaling model sans informed trading. In fact, the function I is increasing in σ with

$$
\begin{aligned}
I\left(\frac{1}{2}\right) &= \frac{1}{2} \\
I(1) &= \frac{3}{4}.
\end{aligned}
$$

As shown in equation (17), the high (low) type benefits (suffers) from an increase in I.

From Lemma 1 and equation (17) we have the following proposition characterizing pooling equilibria with risky debt.

Proposition 2 *If $Q \leq 4/3$, there is no pooling equilibrium in which the manager, regardless of the true asset type, chooses a debt face value greater than L. For $Q \in (4/3, 2)$, pooling at $D > L$ can be sustained if and only if it results in a continuation equilibrium in which the speculator signal precision is sufficiently high to satisfy:*

$$
\sigma^{eq} \geq \frac{1}{2} + \frac{1}{2}\sqrt{4/Q - 2}.
$$

If $Q \geq 2$, pooling can be sustained at any face value in $(L, H]$.

The intuition for Proposition 2 is as follows. From Lemma 1 we know a pooling equilibrium can be sustained if and only if, regardless of the true asset type, the issuer is better off pooling than he

would be under the issuance of riskless debt with face value L. The owner of a low quality asset is better off in the event of pooling with $D > L$, since he benefits from overpricing of his debt. Whether the owner of a high value asset is better or worse off depends on the magnitude of competing effects. On one hand, by raising the debt face value, the issuer raises more funding, which is valuable given $Q > 1$. On the other hand, the owner of the high value asset knows the market will underprice his debt. This latter effect is attenuated by high speculator effort, which serves to drive price closer to fundamentals. This explains why the critical σ threshold for sustaining a pooling equilibrium is decreasing in Q.

It is worthwhile to contrast the separating equilibria described in Proposition 1 with the pooling equilibria described in Proposition 2. A separating equilibrium is better for the uninformed investors since the revelation of the issuer's private information insulates them from adverse selection, allowing them to achieve their target consumption in a manner consistent with symmetric information. However, in a separating equilibrium, the manager only raises funding equal to L. In contrast, in the pooling equilibria described in Proposition 2, the manager raises $(D + L)/2$ units of funding in expectation. Relative to any separating equilibrium, the manager has higher utility when there is pooling at risky debt, regardless of the true asset type. Conversely, such pooling equilibria are unattractive from the perspective of uninformed investors. After all, under pooling at risky debt uninformed investors are exposed to adverse selection, leading to distortions in their portfolios and savings as described in the preceding section. This results in deadweight losses.

It is worth stressing that when it exists the pooling equilibrium featuring risky debt is arguably "focal" relative to the separating equilibria or pooling at safe debt. After all, it follows from Lemma 1 that when such an equilibrium exists, it leaves the issuer better off regardless of the true asset type.

Before proceeding, we consider which of the perfect Bayesian equilibria satisfy the Intuitive Criterion of Cho and Kreps (1987). In the present context, a posited equilibrium fails to satisfy the Intuitive Criterion if one of the issuer types would benefit from choosing a different face value,

provided this were sufficient to convince investors of his true type, while the other type would be strictly worse off choosing that same face value regardless of the beliefs formed in response. We have the following proposition.

Proposition 3 *All separating equilibria satisfy the Intuitive Criterion, as does pooling at riskless debt with face value L. If $Q \leq 3/2$, pooling at risky debt does not satisfy the Intuitive Criterion. For $Q > 3/2$, pooling at risky debt satisfies the Intuitive Criterion if and only if it results in a continuation equilibrium in which the speculator signal precision is sufficiently high to satisfy*

$$\sigma^{eq} \geq \frac{1}{2} \left[1 + \sqrt{4Q/(2Q-1) - 2} \right]. \tag{18}$$

The preceding proposition shows that only a subset of perfect Bayesian equilibria cum pooling satisfy the Intuitive Criterion. In particular, satisfaction of the Intuitive Criterion demands an even higher level of speculator effort and informational efficiency than in a PBE. And returning to the role of safe government bonds, we recall the key comparative static result from equation (16) that speculator effort in a posited post-pooling trading game is decreasing in the quantity of government-provided safe assets. Together these results imply it will be easier for the government to weed out pooling equilibria featuring high debt, should that be desired by the planner, if investors exhibit the higher degree of sophistication demanded by the Intuitive Criterion.

A fundamental question to be addressed is whether, as a general proposition, one should expect corporations to respond to demand for safe assets by supplying them. Propositions 2 and 3 show that it is possible for there to be a pooling equilibrium in which the issuer chooses risky debt with a face value $D > L$ provided speculator signal precision is sufficiently high. But recall, as shown in Section II, an increase in the intrinsic demand for safe storage (as captured by N) has the effect of stimulating the demand for risky debt when safe government bonds are scarce. And with greater uninformed demand for risky debt, an informed speculator has more camouflage for her trades in the high-yield debt market. Anticipating this ability to place larger orders, she will exert more effort. Prices will be driven closer to fundamentals (higher I) and pooling is more readily sustained

$(QI \geq 1)$. Thus, paradoxically, the corporation may be induced to issue risky rather than riskless debt precisely when investor demand for safe storage is strongest.

B. Multiple Corporate Debt Issuers

This subsection considers how equilibrium is affected if there are other corporate debt issuers. To limit the number of equilibrium permutations that are possible, attention is confined to symmetric equilibria with two corporations issuing debt.

Suppose now there are two managers, each valuing immediate funding at Q and each having private information about the true payoff on his respective asset-in-place. The value of the asset-in-place owned by each manager is an i.i.d. random variable with equal probability of generating the cash flows L and H.

We begin by noting that once again it is possible to sustain a given debt configuration as an equilibrium provided that regardless of type, a manager gets at least his payoff from issuing riskless debt with face value L (Lemma 1). Further, the existence of another issuer has no effect on a manager's equilibrium payoff in any of the separating equilibria described in Proposition 1. So one symmetric equilibrium entails both managers signaling their private information as described in Proposition 1.

Similarly, the existence of another issuer has no effect on a manager's equilibrium payoff in the event that he issues riskless debt with face value L regardless of his type (pooling). Thus, there is another symmetric equilibrium in which both managers issue riskless debt with face value L regardless of their true asset type.

Consider the final class of equilibria discussed in the preceding subsection, those in which the manager chooses some $D > L$ regardless of T (Proposition 2). In this class of equilibria, the presence of another issuer does indeed have an effect on the manager's payoff. To see this, note that, as shown in the appendix, the presence of a rival debt issuer causes an inward shift of the uninformed demand curve X_U facing each issuer. And we know an inward shift in the uninformed demand curve results in a reduction of speculator effort (σ^{eq}) in the continuation equilibrium. And

with lower speculator effort, pooling at risky debt becomes less viable since the necessary condition (18) is less likely to be satisfied. Intuitively, the presence of another corporate debt issuer siphons off some of the debt demand, resulting in lower informational efficiency in the event of pooling. And with lower informational efficiency, a high type is less willing to pool given that he will face more severe underpricing of his debt. Of course, with sufficiently large endowment shocks, this equilibrium may still be implemented by the private sector even if there are multiple corporate issuers.

IV. The Optimal Quantity of Government Bonds

This section analyses the role that safe government debt can play in increasing social welfare. As described below, the central mechanism at work is that the quantity of government debt influences equilibrium in corporate debt markets. As described in the previous section, there are multiple potential corporate debt market equilibria: separating equilibria, pooling at riskless debt, and pooling at risky debt. In the interest of brevity, we consider here that if pooling at risky debt occurs, it occurs at the highest possible face value H, which yields maximal expected corporate investment. In fact, all pooling equilibria with risky debt entail a similar social tradeoff in that they feature relatively high expected investment but distorted investor-level risk-sharing. Relative to lower face values, pooling at the highest possible face value H is on the Pareto frontier from an issuer perspective. And finally, we recall that such a pooling equilibrium only exists if it Pareto-dominates the separating equilibria and pooling at riskless debt. In this sense, such an equilibrium might be perceived as "focal" provided it can be sustained.

Looking across equilibria in which informed speculation does and does not occur, Pareto improvements are impossible since uninformed investors suffer when informed speculation occurs while the speculator benefits. Therefore, we take the perspective of a utilitarian social planner placing equal weight on all agents. We begin by calculating deadweight losses in the different equilibria. To set a benchmark, consider social welfare if the type of the asset-in-place was common knowledge. Since investment has positive NPV, the manager would raise the maximum funding possible by marketing

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debt with face value T, converting each unit of funds raised into Q units of private benefits. The speculator and market-makers would have total consumption equal to their endowments. Each UI facing a low period 2 endowment would save N in period 1 in order to receive N in period 2, insuring against any consumption shortfall. Thus, with symmetric information, social welfare is:

$$W^* = \frac{1}{2}(H + L)Q + Y_1^s + Y_1^{mm} + Y_1^{ui} - \frac{1}{2}N. \tag{19}$$

Consider next social welfare in any of the separating equilibria described in Proposition 1. In a separating equilibrium there is no socially wasteful speculator effort. And with the manager's private information revealed, the uninformed investors fully insure against endowment shocks, with debt claims priced at their true payoff. So investors achieve efficient risk-sharing. However, the high type receives only L units of investment funding, less than he would under symmetric information. Thus, relative to social welfare under common knowledge of the type (W^*), a separating equilibrium generates the following deadweight loss attributable to foregone positive NPV investments:

$$DWL^{SEP} = \frac{1}{2}(Q - 1)(H - L). \tag{20}$$

Notice, the amount of government bonds offered would have no effect on social welfare in the event of a separating equilibrium. However, as argued below, a separating equilibrium is more likely to be implemented if the quantity of government bonds is high.

As described in Proposition 1, there is a pooling equilibrium in which, regardless of T, the issuer chooses face value L. We call this equilibrium LPOOL for short. If the corporation issues such riskless debt, the speculator will not exert socially wasteful effort. And with riskless debt issued, uninformed investors will fully insure against endowment shocks at the competitive price. However, the high type only receives L units of investment funding, less than what he would receive if T were observable. Relative to social welfare under observable types, LPOOL generates the following deadweight loss:

$$DWL^{LPOOL} = \frac{1}{2}(Q - 1)(H - L). \tag{21}$$

The amount of government bonds offered to investors has no effect on social welfare in the event of

LPOOL being implemented. However, as argued below, pooling at riskless debt is more likely to occur if the quantity of government bonds is high.

As described in Proposition 2, the final class of potential equilibria are those in which the manager pools in the sense of choosing a type-invariant debt face value $D > L$. As stated above, we focus on the case where pooling occurs at $D = H$. This outcome is denoted HPOOL below. Further, we assume the private sector will implement this potentially focal outcome whenever it is in the equilibrium set.

An attractive feature of HPOOL is that expected corporate investment is equal to $(H + L)/2$, which is equal to expected investment under observable types. However, this equilibrium entails socially wasteful speculator effort and inefficient risk sharing as the UI distort their portfolios in response to adverse selection in the corporate debt market. In particular, we obtain the following expression for the relative deadweight loss in the event of pooling at risky debt with face value H:

$$DWL^{HPOOL} = e(\sigma^{eq}) + \frac{1}{2}(N - G) \left[\begin{array}{c} \int_{1}^{\theta_1} (\theta - 1)f(d\theta) + \frac{1}{2}\left(\frac{H-L}{L}\right)[1 - F(\theta_2)] \\ + \frac{1}{2}\left(\frac{H-L}{H}\right)\int_{\theta_1}^{\theta_2}(\theta - 1)f(d\theta) \end{array} \right]. \tag{22}$$

Equation (22) has the following intuition. The first term reflects the fact that speculator effort is socially wasteful. The first term in the large brackets captures the fact that those UI with $\theta \in [1, \theta_1]$ forego the purchase of corporate debt altogether, despite the fact that there would be a social gain of $\theta - 1$ per unit of incremental safe assets held by these investors. The second term in the large brackets represents the social cost associated with overinsurance by extremely risk-averse UI with $\theta \geq \theta_2$. These investors buy $(N - G)/L$ units of corporate debt, implying accrual of excess resources in period 2 in the event that $T = H$. The final term in the large brackets reflects the fact that adverse selection induces those UI with $\theta \in (\theta_1, \theta_2)$ to only partially insure. These investors buy only $(N - G)/H$ units of corporate debt, implying a costly consumption shortfall in period 2 if $T = L$.

In order to understand the potential merits of public provision of safe bonds, it is useful to first

consider outcomes in the absence of government bonds. We therefore analyze this case first. In this setting, we can compare social welfare under HPOOL with that under LPOOL (each separating equilibrium generates the same social welfare as LPOOL). The key model parameters determining social welfare are marginal investment Q and the period 2 endowment shock N, the model's proxy for the intrinsic demand for safe assets.

Note that the deadweight loss in LPOOL is independent of N, but increasing in Q, reflecting social costs of underinvestment by the high type. In contrast, the deadweight loss in HPOOL is independent of Q but increasing in N, reflecting the fact that larger endowment shocks induce more socially wasteful speculator effort, as well as amplifying deadweight losses due to distortions in uninformed investor portfolios. In particular, from equation (22) we obtain the following comparative static:

$$
\frac{dDWL^{HPOOL}}{dN} = e'(\sigma)\frac{\partial\sigma}{\partial N} + \frac{1}{2}\left[\begin{array}{c} \int\limits_{1}^{\theta_1}(\theta-1)f(d\theta) + \frac{1}{2}\left(\frac{H-L}{L}\right)[1-F(\theta_2)] \\ +\frac{1}{2}\left(\frac{H-L}{H}\right)\int\limits_{\theta_1}^{\theta_2}(\theta-1)f(d\theta) \end{array}\right]
$$
$$
+\frac{1}{2}\frac{\partial\sigma}{\partial N}(N-G)\left(\sigma-\frac{1}{2}\right)\left[\left(\frac{H-L}{2H}\right)f(\theta_1)\frac{\partial\theta_1}{\partial\sigma} + \frac{(H-L)^2}{LH}f(\theta_2)\frac{\partial\theta_2}{\partial\sigma}\right]. \tag{23}
$$

Since speculator effort is increasing in N, as shown in Section 2, it is readily verified that the deadweight loss in HPOOL is indeed increasing in N.

By equating the deadweight losses across the two equilibria we pin down a critical value of Q, call it Q_{pub}, at which the social planner would be just indifferent between HPOOL and LPOOL. Specifically:

$$
Q_{pub} - 1 = \frac{e(\sigma) + \frac{1}{2}(N-G)\left[\int\limits_{1}^{\theta_1}(\theta-1)f(d\theta) + \frac{1}{2}\left(\frac{H-L}{L}\right)[1-F(\theta_2)] + \frac{1}{2}\left(\frac{H-L}{H}\right)\int\limits_{\theta_1}^{\theta_2}(\theta-1)f(d\theta)\right]}{\frac{1}{2}(H-L)}. \tag{24}
$$

It is readily verified that Q_{pub} is increasing in N. Intuitively, an increase in N raises the risk sharing cost arising from pooling at risky debt. Maintaining social planner indifference across the

equilibria then necessitates a compensating increase in Q, which raises the deadweight cost of the underinvestment associated with LPOOL.

Keeping in mind the social planner's preferences over HPOOL and LPOOL, consider that HPOOL can be sustained as a private sector equilibrium if the high type is better off than under riskless debt. Section III showed this requires $QI \geq 1$. Thus, there is a critical value of Q, call it Q_{priv} at which the high type would be just indifferent between HPOOL and LPOOL. We have:

$$Q_{priv} = [I(\sigma)]^{-1} \Rightarrow \frac{dQ_{priv}}{dN} = -[I(\sigma)]^{-2}\left[\frac{\partial \sigma}{\partial N}\right] < 0. \tag{25}$$

As illustrated by the preceding equation, the high type is more attracted to HPOOL for higher values of N since large endowment shocks stimulate uninformed demand and speculator effort, resulting in less underpricing if he issues risky debt. Hence, to maintain indifference between HPOOL and LPOOL, a compensating decrease in the investment value parameter Q is required.

Figure 4 pulls this analysis together, depicting the private sector equilibrium and the planner's preference between pooling at face value L versus face value H under the presently maintained assumption that no government bonds are offered. On Region 1, the social planner prefers pooling at the face value H given that the social cost of risk-sharing distortions are low given the low value of N. In contrast, $D = H$ cannot be sustained as a private sector equilibrium on this region since low values of N imply low uninformed trading, poor information quality, and high underpricing costs faced by the high type if he pools at risky debt. On Region 2, the social planner prefers $D = L$, and this is the outcome that the private sector will implement. On Regions 3 and 5 the social planner prefers pooling at face value H, as does the corporation. Here the high type is willing to issue risky debt given that the high value he places on funding (high Q) more than offsets any underpricing. The planner prefers pooling at risky debt on these same regions because efficient risk sharing is less socially important than maintaining high investment.

Consider finally Region 4. Here the planner prefers pooling at riskless debt. Safe assets are particularly socially valuable on Region 4 given the large storage demands of the uninformed investors. However, on this same region the private sector would pool at risky debt. Intuitively, the high type

recognizes that high N values serve to stimulate speculator effort and mitigate the extent of under-pricing of risky debt. Here the corporation prefers issuing risky debt despite the fact that safe debt and efficient risk sharing have high social value. But note, this is not simply a matter of the high type failing to account for the negative externality he imposes on uninformed investors. Rather, the private sector is more likely to impose the negative externality associated with the choice of risky over riskless debt precisely when the negative externality is large (high N).

What then is the utility of the government offering safe bonds to the investors? First, the issuance of safe government bonds can potentially be used to eliminate HPOOL as an equilibrium altogether, which may be optimal if Q is sufficiently low. Second, safe government bonds can be issued in order to reduce the deadweight losses generated in the event of HPOOL being implemented. To see this latter effect, totally differentiate equation (22) with respect to G to obtain:

$$
\frac{dDWL^{HPOOL}}{dG} = e'(\sigma)\frac{\partial\sigma}{\partial G} - \frac{1}{2}\left[\begin{array}{c} \int_{1}^{\theta_1}(\theta-1)f(d\theta) + \frac{1}{2}\left(\frac{H-L}{L}\right)[1-F(\theta_2)] \\ +\frac{1}{2}\left(\frac{H-L}{H}\right)\int_{\theta_1}^{\theta_2}(\theta-1)f(d\theta) \end{array}\right] \tag{26}
$$
$$
+\frac{1}{2}\frac{\partial\sigma}{\partial G}(N-G)\left(\sigma-\frac{1}{2}\right)\left[\left(\frac{H-L}{2H}\right)f(\theta_1)\frac{\partial\theta_1}{\partial\sigma} + \frac{(H-L)^2}{LH}f(\theta_2)\frac{\partial\theta_2}{\partial\sigma}\right].
$$

From the fact that speculator effort is decreasing in G (equation (16)) and the fact that both of the UI portfolio cutoffs are increasing in σ (equation (13)) it follows that the right side of the preceding equation is negative. The intuition for the preceding comparative static is as follows. Within the set of HPOOL equilibria, the provision of safe public debt serves to reduce speculator effort costs. Further, with additional safe assets, inframarginal uninformed investors can substitute out of corporate bonds which represent an imperfect savings vehicle. This is captured by the second term in large square brackets in the preceding equation. Finally, the induced reduction in speculator effort resulting from additional safe government bonds reduces the portfolio distortions of marginal uninformed investors. This effect is captured by the last term in the large brackets.

In order to express the optimal government bond policy most compactly, let $\underline{N}(Q)$ denote the

size of the negative liquidity shock such that in the absence of any government bond offering ($G = 0$), the manager owning a high quality asset would be just indifferent between HPOOL and LPOOL:

$$QI[\sigma(\underline{N}(Q))] = 1 \Rightarrow \underline{N}(Q) = \sigma^{-1}[I^{-1}(1/Q)].$$

It is readily verified that the function $\underline{N}$ is decreasing. Intuitively, the high type is more willing to pool for higher values of Q, so a reduction in N would be necessary to restore indifference between HPOOL and LPOOL. We have the following lemma.

Lemma 2 *If the corporation pools at risky debt with face value H, social welfare is increasing in G. In such an equilibrium, the optimal level of government bonds is*

$$G^*(Q) = \min\{\overline{G}, N - \underline{N}(Q)\}.$$

Consider then the optimal level of government bond provision when we consider the additional possibility that the private sector may pool at riskless debt. Returning to Figure 4 increases in government bonds have the same effect on social welfare as a reduction in N. Effectively, government bonds reduce the amount of residual uninformed storage demand, which is simply $N - G$. With this in mind, note that government bond provision serves no purpose on Regions 1 and 2, as the private sector would implement LPOOL regardless of the level of G. Here the optimal level of government bonds is zero. On Region 5 the government optimally offers the maximum possible amount of safe bonds ($\overline{G}$) in order to minimize the effort and risk sharing costs incurred at the equilibrium HPOOL. On Region 3, marginal Q is high relative to N and so the government prefers HPOOL to LPOOL. Here the government would offer the maximum G consistent with the private sector implementing HPOOL, subject to the upper bound $\overline{G}$.

Consider finally Region 4. On the top half of Region 4 the government would offer the maximum G consistent with the private sector implementing HPOOL, subject to the upper bound $\overline{G}$. Here the government prefers to implement HPOOL given that investment has high NPV. On the bottom half of Region 4 the government would place priority on efficient risk sharing given that Q is low relative to N. Therefore, its objective is to induce the private sector to implement LPOOL. There are two

cases to consider. If $\overline{G} < N - \underline{N}(Q)$, it is not possible for the government to induce the private sector to implement LPOOL and so it must content itself with minimizing the deadweight costs associated with HPOOL by offering $\overline{G}$. If $\overline{G} \geq N - \underline{N}(Q)$, the government will optimally induce the private sector to implement LPOOL. The minimum government bond offering that achieves this objective is $N - \underline{N}(Q)$. Alternatively, the government could implement the same outcome by offering $\overline{G}$. That is, the optimal government bond policy is not unique on the lower half of Region 4.

Before summarizing the optimal government bond policy, it is worth noting an important potential feature of the government's optimal policy illustrated by our analysis of Region 4. On the lower half of Region 4 we saw that the government would like to ensure all uninformed investors have access to safe debt given that the high magnitude of N places primacy on efficient risk sharing. But note, the government does not have to achieve this unilaterally. In fact, given $\overline{G} < N$, the government does not have the capacity to meet the intrinsic demand for safe assets. Nevertheless, provided that $\overline{G} \geq N - \underline{N}(Q)$ the government can offer a sufficient amount of government bonds to ensure the private sector provides the safe assets demanded by investors. That is, there is a multiplier effect by which each unit of safe government bonds has the effect of crowding in additional units of safe corporate debt.

Based on the preceding discussion we have the following characterization of the optimal government bond offering.

Proposition 4 *Suppose the corporation will either pool at face value H, pool at face value L, or implement one of the separating equilibria. Then it is optimal for the government to offer investors*

$$G^*(Q, N) = \max\{0, \min\{\overline{G}, N - \underline{N}(Q)\}\}.$$

The optimal government bond supply function described in the proposition is increasing in intrinsic demand for safe assets (N). The intuition is as follows. In the case where the government would like to prune HPOOL, it must increase its bond offering if N increases, since larger endowment shocks translate into higher uninformed demand for risky corporate debt, making pooling at risky

debt more attractive to the high type. In the case where the government would like to implement HPOOL with minimal risk sharing distortions, it can increase its bond offering if N increases while keeping the issuer willing to implement HPOOL. It follows from the proposition that an optimal level of government bond provision is increasing in the value of investment (Q). Intuitively, in the case where the government would like to prune HPOOL, it must increase its bond offering if Q increases in order to overcome the increased attraction of pooling at the high face value. And in the case where the government would like to implement HPOOL with minimal risk sharing distortions, it can increase its bond offering if Q increases while keeping the issuer willing to implement HPOOL.

Figure 5 plots the optimal supply of government bonds as described in the preceding proposition, with each line assuming a different value for the endowment shock parameter N. The plot labeled High N2 captures our analysis of the lower half of Region 4. In particular, it is assumed that $\overline{G} \geq N - \underline{N}(Q)$ for Q less than 1.4. Here the government will optimally induce the private sector to implement LPOOL and can actually do so by offering any $G \in [N - \underline{N}(Q), \overline{G}]$. The monotonic government debt function (solid black line) is as described in the preceding proposition. However, for low values of Q higher debt levels are also optimal.

Conclusions

In recent years there has been increasing concern over a potential scarcity of safe assets. Seemingly paradoxically, corporations have responded by increasing the supply of junk debt, consistent with a more general historical negative correlation between government bond yields and the high-yield share in total corporate debt. Further, the market seems to be rewarding such aggressive corporate finance decisions, charging low yields in highly-levered-transactions.

In this paper we present a positive framework for understanding the conjunction of safe asset scarcity and "overheated" debt markets. This provides the foundation for a normative framework for thinking about the welfare consequences of government-supplied safe bonds. We start from a canonical debt signaling framework, adding one additional element: endogenous trading by uninformed investors and an informed speculator. We argue that an overheated debt market, with low

social welfare, emerges when safe asset shortages support a speculative high yield debt market. If there is a safe asset shortage, uninformed demand migrates to the junk debt market. The increase in uninformed demand spurs speculator information production. This drives prices of junk debt closer to fundamentals, encouraging firms with positive information to pool at high face values. The social benefit of such an outcome is high corporate investment. One social cost of this outcome is the cost of speculator information production. A resulting social cost of asymmetric information across investors is distorted portfolios. And to the extent that some uninformed investors are biased away from saving adequately, distress costs may result. Paradoxically, we show a private issuer is more willing to impose this negative externality associated with risky debt when it is large.

In this economy, the government can increase social welfare by offering to investors even a limited amount of safe bonds. For example, the government can offer safe bonds with an eye toward deterring pooling at risky corporate debt. Safe government bonds siphon off uninformed demand from junk debt markets. This lowers speculative information production, driving prices away from fundamentals. If this effect is sufficiently strong, corporations will opt to issue riskless debt instead of junk debt. That is, riskless government bonds serve to crowd-in safe debt, while crowding out investment financed by risky debt. This increases social welfare if marginal Q is sufficiently low. Alternatively, the government can supply safe government bonds with an eye toward increasing the efficiency with which the private market implements pooling at risky debt. Here the siphoning effect of government bonds serves to induce marginal reductions in socially wasteful speculator effort and mitigates the extent of investor-level portfolio distortions.

References

[1] Allen, F., and D. Gale, 1988, "Optimal Security Design," *Review of Financial Studies* 89, 229-63.

[2] Barro, R., 1974, "Are Government Bonds Net Worth," *Journal of Political Economy* 82, 1095-1117.

[3] Boot, A., and A. Thakor, 1993, "Security Design," *Journal of Finance* 48 , 1349-78.

[4] Cho, I. and D. Kreps, 1987, "Signaling Games and Stable Equilibria," *Quarterly Journal of Economics*, 179-221.

[5] Dang,T.V., Gorton, G., and B. Holmström, 2012, "Ignorance, Debt and Financial Crises," working paper, Yale School of Management.

[6] Dow, J., 1998, "Arbitrage, Hedging, and Financial Innovation," *Review of Financial Studies* 11, 739-755.

[7] Eckbo, Espen, 1986, "Valuation Effects of Corporate Debt Offerings," *Journal of Financial Economics*, 119-151.

[8] Faure-Grimaud, A. and D. Gromb, 2004, "Public Trading and Private Incentives," *Review of Financial Studies* 17, 985-1014.

[9] Glosten, L., and P. Milgrom, 1985, "Bid, Ask and Transactions Prices in a Specialist Market with Heterogeneously Informed Traders," *Journal of Financial Economics* 14, 71-100.

[10] Gourinchas, P.O., and O. Jeanne, 2012, "Global Safe Assets," working paper, University of California, Berkeley.

[11] Gorton, G., and G. Ordonez, 2013, "The Supply and Demand for Safe Assets," working paper, University of Pennsylvania.

[12] Gorton, G., and G. Pennacchi, 1990, "Financial Intermediaries and Liquidity Creation," *Journal of Finance* 45, 49-71.

[13] Greenwood, R., and S. Hanson, 2013, "Issuer Quality and Corporate Bond Returns," *Review of Financial Studies*, 1483-1525.

[14] Greenwood, R., S. Hanson, and J. Stein, 2010, "A Gap Filling Theory of Corporate Debt Maturity Choices," *Journal of Finance* 65, 993-1028.

[15] Holmström, B., and J. Tirole, 1998, "Private and Public Supply of Liquidity," *Journal of Political Economy* 106, 1-40.

[16] Krishnamurthy, A., and A. Vissing-Jorgensen, 2012, "Short Term Debt and Financial Crises: What Can We Learn From U.S. Treasury Supply," working paper, Northwestern University.

[17] Kyle, A. S., 1985, "Continuous Auctions with Insider Trading," *Econometrica*, 1315-1335.

[18] Maug, E., 1998, "Large Shareholders as Monitors: Is There a Trade-off Between Liquidity and Control?," *Journal of Finance* 53, 65-98.

[19] Philippon, T., 2009, "The Bond Market's Q," *Quarterly Journal of Economics*, 1011-1056.

[20] Ross, S., 1977, "The Determination of Financial Structure: The Incentive Signalling Approach," *Bell Journal of Economics* 8, 23-40.

[21] Stein, J., "Overheating in Credit Markets: Origins, Measurement, and Policy Responses," Remarks at "Restoring Household Financial Stability after the Great Recession: Why Household Balance Sheets Matter." Symposium sponsored by Federal Reserve Bank of St. Louis.

[22] Stiglitz, J. and A. Weiss, 1981, "Credit Rationing in Markets with Imperfect Information," *American Economic Review* 71, 393-410.

[23] Woodford, M., 1990, "Public Debt as Private Liquidity," *American Economic Review Papers and Proceedings*, 382-388.
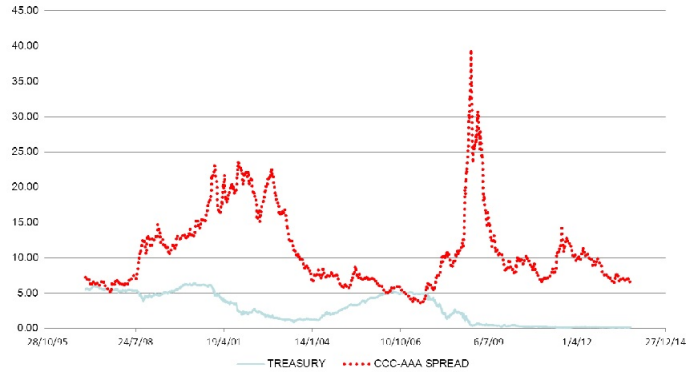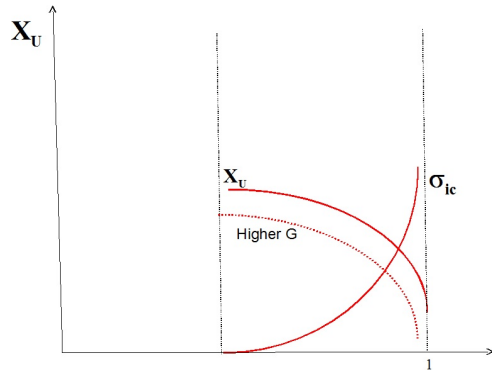
Figure 1: Treasury Yields and Spreads

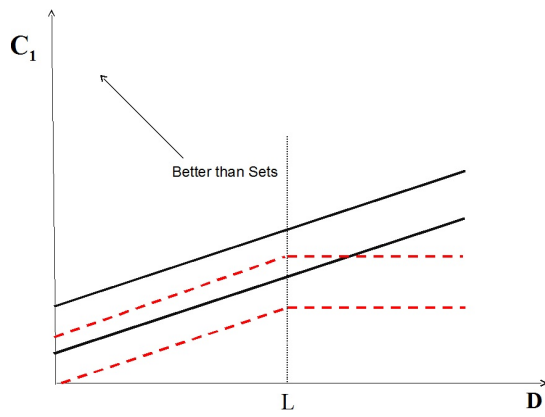Figure 2: Uninformed Demand and Speculator Signal with Risky Debt
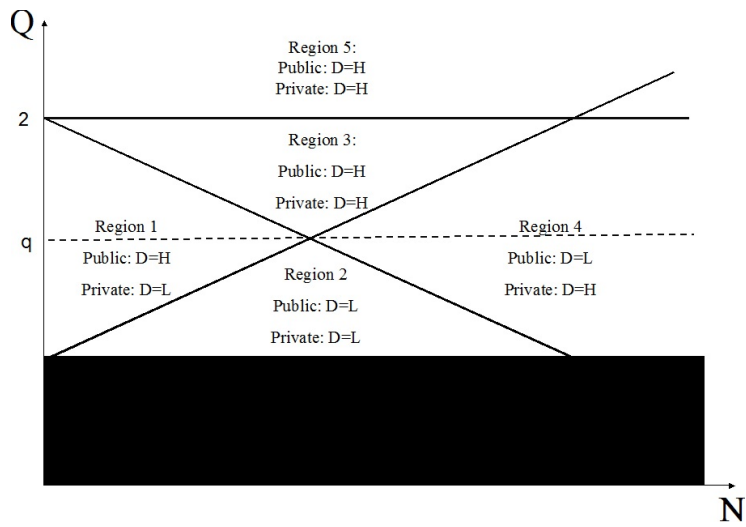

Figure 3: Debt Issuer Preferences
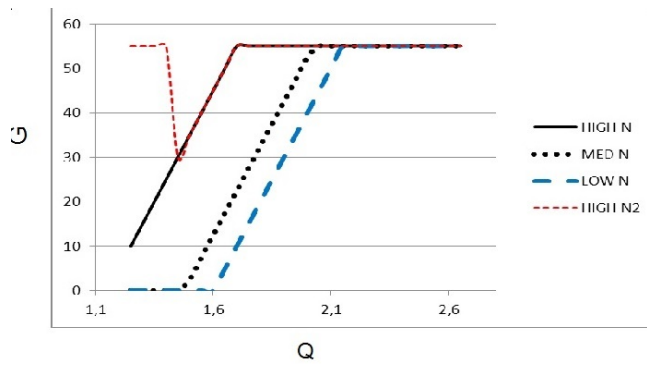
Figure 4: Private versus Public Preferences

Figure 5: Tobins Q and Optimal G

Lemma A1: Existence of Unique Continuation Equilibrium Define the function Γ with domain $[1/2, 1]$ based upon the speculator's incentive condition as follows:

$$\Gamma(\sigma) \equiv \Psi\left[\frac{1}{2}(D - L)(N - G)\left(\frac{1}{L}\left[1 - F(\theta_2(\sigma, D))\right] + \frac{1}{D}\left[F(\theta_2(\sigma, D)) - F(\theta_1(\sigma, D))\right]\right)\right]$$

The function Γ is continuous and strictly decreasing with $\Gamma(1/2) > 1/2$. It follows there exists a unique solution to the equation $\Gamma(\sigma) = \sigma$ in $(1/2, 1)$. ∎

Lemma A2: Reduction in Uninformed Demand with Two Corporate Debt Issuers For brevity, let $\phi \equiv N - G$. Consider then the portfolio problem of an individual UI. Let k denote the number of defaults against which the agent wants to insure, with $k \in \{0, 1, 2\}$. In this connection, let x_k denote the number of units of debt of each issuer the investor must hold in order to achieve a payoff of ϕ given that there are k defaults. We have:

$$x_k = [kL + (2 - k)D]^{-1}\phi$$

We can pin down the optimum portfolio here using perturbation arguments. Consider first an investor anticipating the low future endowment who holds zero units of debt. His gain from increasing his holdings of each issuer's debt infinitesimally is equal to

$$\theta\left[\frac{1}{4}(2L) + \frac{1}{4}(2D) + \frac{1}{2}(L + D)\right] - 2E(P).$$

Consider next an investor with initial portfolio holding of x_0 contemplating an increase in his holdings. His gain from increasing his holdings of each issuer's debt infinitesimally is equal to

$$\theta\left[\frac{1}{4}(2L) + \frac{1}{2}(L + D)\right] - 2E(P).$$

Finally, consider an investor with initial portfolio holding of x_1 contemplating an increase in his holdings. His gain from increasing his holdings of each issuer's debt infinitesimally is equal to

$$\theta\left[\frac{1}{4}(2L)\right] - 2E(P).$$

From the preceding perturbation gain equations we obtain the following critical cutoffs for a net gain to increasing the portfolio:

$$\widehat{\theta}_1 \equiv \frac{2E(P)}{L+D}; \widehat{\theta}_2 \equiv \frac{4E(P)}{2L+D}; \widehat{\theta}_3 \equiv \frac{4E(P)}{L}.$$

And we have the following portfolio rule:

$$\theta \leq \widehat{\theta}_1 \Rightarrow x^*(\theta) = 0$$
$$\theta \in (\widehat{\theta}_1, \widehat{\theta}_2) \Rightarrow x^*(\theta) = \frac{\phi}{2D}$$
$$\theta \in (\widehat{\theta}_2, \widehat{\theta}_3) \Rightarrow x^*(\theta) = \frac{\phi}{L+D}$$
$$\theta \geq \widehat{\theta}_3 \Rightarrow x^*(\theta) = \frac{\phi}{2L}.$$

In contrast, with one issuer we had the following thresholds:

$$\theta_1 = \frac{2E[P]}{L+D}; \theta_2 = \frac{2E[P]}{L}.$$

And the following portfolio rule.

$$\theta \leq \theta_1 \Rightarrow x^*(\theta) = 0$$
$$\theta \in (\theta_1, \theta_2) \Rightarrow x^*(\theta) = \frac{\phi}{D}$$
$$\theta \geq \theta_2 \Rightarrow x^*(\theta) = \frac{\phi}{L}.$$

And we verify that for all θ demand is lower with two issuers than with one issuer:

$$\theta \in (\theta_1, \widehat{\theta}_2) : \frac{\phi}{2D} < \frac{\phi}{D}$$
$$\theta \in (\widehat{\theta}_2, \theta_2) : \frac{\phi}{L+D} < \frac{\phi}{D}$$
$$\theta \in (\theta_2, \widehat{\theta}_3) : \frac{\phi}{L+D} < \frac{\phi}{L}$$
$$\theta > \widehat{\theta}_3 : \frac{\phi}{2L} < \frac{\phi}{L}. \blacksquare$$

Proof of Lemma 1.   To see the necessity of each type attaining the posited minimum utility, note that regardless of what beliefs investors might form in response, the manager can always attain the

stated minimum by issuing debt with face value L. For sufficiency, consider a posited equilibrium in which each type makes at least the posited minimum. This equilibrium can be sustained if investors impute a deviation to the manager holding a low value asset. Given such beliefs, any deviation will yield the deviating manager a payoff no greater than $V_T^{\min}$.∎

Proof of Proposition 1.   The first statement in the proposition follows from Lemma 1 and the fact that debt with face value less than L provides the issuer with less than $V_T^{\min}$. The second statement in the proposition follows from Lemma 1 and the fact that debt with face value L generates issuer utility equal to $V_T^{\min}$. The last statement in the proposition follows from the fact that the low type cannot make more than QL in a separating equilibrium. So he must make QL in any separating equilibrium. For this reason, the high type cannot sell debt for more than L in a separating equilibrium. So the high type must market debt with face value L in any separating equilibrium. The low type can then issue debt with face value in $(L, H]$ in any separating equilibrium. Each type then attains his respective minimum utility.∎

Proof of Proposition 3.   Given any PBE, the low type never gains from deviating if doing so identifies him, so attention can be confined to the high type's incentive to deviate. Consider then any separating equilibrium or pooling at $D = L$. Only a deviation to a face value greater than L can make the high type strictly better off, but then the low type would also gain from such a deviation under some beliefs. So the Intuitive Criterion admits imputing such a deviation to the low type. And given such beliefs, there is no incentive for either type to deviate. Next, consider that pruning a pooling PBE featuring face value D via the Intuitive Criterion demands finding a deviation D_0 such that:

$$QD_0 + H - D_0 \; > \; V_H(D)$$
$$QD_0 \; < \; V_L(D).$$

The final inequality stated in the proposition implies no such D_0 exists.∎