Activist Funds, Leverage, and Procyclicality*

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Abstract

We provide a theoretical framework to study blockholder activism by funds who compete for investor flow. In our model, activists are intrinsically able to raise the value of target firms through monitoring. Competition for investor flow induces them to enhance the returns generated by monitoring by raising external funding at the level of the target firm. We adopt a microfounded approach to account for the lack of macro-state contingency in such financing contracts and show that debt is optimal for raising external funding. When good funds are sufficiently better than bad funds, competition for flow can generate excessive leverage which fosters debt overhang in low macroeconomic states and shuts down activist effort. As a result, investing in activist hedge funds is more desirable when macroeconomic prospects are good. Our model thus links the observed procyclicality of activism with documented increases in the leverage or payouts ratios of target firms. In addition, the model generates several new testable implications and reconciles seemingly contradictory evidence on the wealth effects of activism for shareholders and bondholders.

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1 Introduction

Activist blockholders play a key role in limiting the governance problem that affects publicly traded corporations with dispersed owners who have limited incentive to monitor managers. The potential benefits of blockholders has been widely recognized in the theoretical literature on corporate governance since Grossman and Hart (1980) and Shleifer and Vishny (1986). In recent decades, a specific type of blockholder – activist hedge funds – has taken centre stage in activism (e.g., Gillan and Starks (2007), Armour and Cheffins (2009)), generating gains to targets in terms of shareholder value and operating performance (Brav, Jiang, Partnoy, and Thomas (2008), Clifford (2008), Becht, Franks, Mayer, and Rossi (2009), Klein and Zur (2009), and Boyson and Mooradian (2011)).

It is important to recognize that – unlike the blockholders of classical corporate governance models – activist hedge funds are delegated portfolio managers: Their survival relies on the approval of the investors who finance them. It is well known that investor flows are positively related to fund performance, and that hedge funds are affected by such "flow-performance" relationships (Fung, Hsieh, Naik, and Ramadorai (2008), Agarwal, Daniel, and Naik (2009), Baquero and Verbeek (2009)). Indeed, flows can be four-times as important to hedge fund managers as incentive fees (Lim, Sensoy, and Weisbach (2013)). Thus, hedge funds compete for investor flow.

In this paper we present a theoretical framework to study blockholder monitoring by activist hedge funds who compete for investor flow. We show that competition for flow is a critical ingredient that links together the observed procyclicality of investment in activist funds with the documented effect of such funds on the leverage of their target companies. In addition to providing a lens through which to view hedge fund activism, our model also offers a potential resolution for some seemingly contradictory empirical evidence in this area, and outlines new testable predictions.

The key elements of our model can be summarized as follows. Activist hedge funds own blocks in target firms and aim to engage in a wide variety of potentially beneficial governance activities. Initially they aim to release excess cash from target firms. Subsequently they wish to engage in business improvements, restructuring, or merger of the target firm. All these activities are commonly declared goals of activist hedge funds (Brav, Jiang, and Kim (2010)). Hedge funds differ in their intrinsic ability to productively engage in all such activities: Good funds are able to generate higher cash flows than bad ones from each activity. In addition, the cash flows generated by the latter three types of activities are linked to macroeconomic conditions: Takeover premia may be higher in upturns, for example.

Funding for hedge funds is provided by their fee-paying investors to whom the funds must provide periodic returns. These investors make (rational) inferences about the ability of their hedge funds from these returns, and then decide whether to take their money elsewhere. Given the need to compete to keep investor capital, hedge funds are (rationally) tempted to enhance their intrinsically generated returns, as long as their investors are not perfectly able to distinguish whether returns are attributable to intrinsic ability or enhancement activity. A natural way in which to limit the ability of investors to distinguish between ability and enhancement is to engage in enhancement activities at the level of a target firm (which is, perhaps, one of many in the hedge fund's portfolio, or where enhancement activities may be hidden in complex corporate structures). There are two possible forms of enhancement: Funds can raise external finance at the level of target firms (increasing net leverage) or they can (destructively) divert internal resources from target firms.

Our key result is that such enhancement of payout induced by competition for investor flow can make hedge fund activism procyclical with respect to macroeconomic conditions. The intuition is as follows. Activism is costly for hedge fund managers who are equity holders in target firms. Yet, we show that the optimal form of enhancement activity involves issuing external claims that are senior to equity. The existence of such claims reduces the incentives to exert activist effort ex post, shutting down activism in economic downturns when overall cash flows from activism are lower. As a result, investment in activist hedge funds may be a bull-market phenomenon, a finding that resonates with the observed market-procycality of 13D filings discussed below.[1]

In our baseline model, we focus on external financing at the level of the target firm for payout enhancement. Needless to say, if such external financing is perfectly observable to hedge fund investors, it is useless for influencing their beliefs. Accordingly, we assume that the payout of hedge funds is opaque: Hedge fund investors cannot immediately and *directly* determine (though they *can* infer in equilibrium) the degree to which returns are generated by external financing activities. We believe that this assumption is not unreasonable for activist hedge funds, which may have multiple targets in their portfolio. These targets may be relatively small firms which are made more opaque by the strategic use of complex corporate structures. Finally, external financing activities include a wide range of activities including bank borrowing (e.g., Li and Xu (2010) document that a significant fraction of hedge fund target borrowing is bank based) and operational activities such as the lengthening of trade payment terms to suppliers. That said, the opacity of leverage is *not* essential for our results. We also present a variation of the model with fully observable leverage where the possibility of cash flow diversion preserves our qualitative results in full. In neither version of our model

[1] According to Section 12 of the Securities Exchange Act of 1934, any entity acquiring a stake of 5% or more of the voting shares of a publicly traded company must file a Schedule 13D with the SEC within ten days of the purchase. The schedule 13D provides information to the investing public about blockholders in public companies and their intentions with regard to the company.

is leverage intrinsically a signalling device, contrary to Ross (1977). Rather, funds signal via returns, and (optimally) use leverage to enhance returns.

We characterize hedge fund activism via a series of results. First, we show that competition for flow is an essential part of equilibrium, because pooling equilibria do not exist (Proposition 1). Second, we establish a minimum level of payout to hedge fund investors in any separating equilibrium (Proposition 2), which implies that external financing is essential for separation. Third, we adopt a microfounded approach (based on the non-verifiability of cash flows) to the lack of macro-state contingency in the financing activities of target firms and show that debt is the optimal way to raise external funds (Proposition 3). Finally, we characterize conditions under which – even in separating equilibria with the *minimal* amount of leverage – borrowing is high enough to generate debt overhang in low macro states leading to a shutdown in activist effort (Proposition 4). Knowing this, hedge fund investors will only fund activist blocks ex ante if macroeconomic prospects are sufficiently good. If – as is often claimed – broad equity markets are a leading predictor of macroeconomic prospects, then our results imply that activist block formation and resulting SEC 13D filings would be a bull-market phenomenon.

The conditions generating procyclicality are economically meaningful. Procyclicality with respect to macroeconomic states arises when ability differences between good and bad hedge funds are large enough. High ability differences induce investors to chase performance and it is the resulting competition for flow that fosters leverage and thus debt overhang. Indeed, we show that competition for flow is not only sufficient, but also *necessary* for procyclicality (Implication 1): Absent such competition, the desirability of investment in activist hedge funds is independent of macroeconomic prospects.

From an applied perspective, two key themes emerge from our analysis. First, since activist funds pay out free cash flow and enhance payouts via leverage, target firms should experience increases in payout and leverage. Second, as a result of the procyclicality discussed above, investment in activist funds should be higher in bull markets. Both implications resonate with the available empirical evidence.

The empirical literature suggests that activist hedge funds increase target firm leverage or payout or both (e.g. Brav, Jiang, Partnoy, and Thomas (2008), Klein and Zur (2009)). Further, there is evidence – consistent with our results – that the induced rise in leverage increases the credit risk of target firms: Target companies disproportionately experience credit downgrades (Byrd, Hambly, and Watson (2007), Aslan and Maraachlian (2009), and Klein and Zur (2011)).

There is also growing evidence that activist block formation is higher in bull markets. See, for example, Figures 1 and 2, which depict engagement disclosures (e.g., 13D filings) by activist hedge funds over time in the US and elsewhere. These findings are echoed in the financial press. According to *The Economist*, "In America investors began only two new activist campaigns in the fourth quarter of 2008, down from 32 in the preceding nine months and 61 in 2007."[2] It is only after a "strangely quiet" period during the two years following this steep decline in activism, during which "[m]any [activist investors] scaled back or even closed shop,"[3] that activist campaigns started to re-emerge.[4] Indeed, it is only another eighteen months later, in mid-2012, when the market had regained most of the value lost in the 2008 crisis, that – according to Peter Harkins of D.F. King, a proxy-advisor – shareholder activism is "getting back to normal after the financial crisis of 2008."[5]

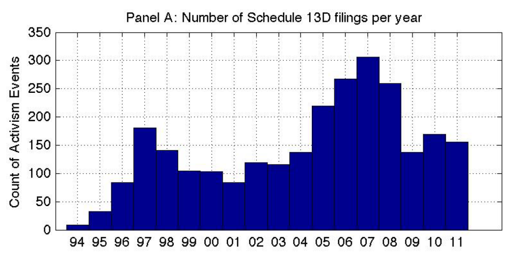

Figure 1: Reproduced with Alon Brav's permission. Based on an updated sample and the same data collection procedure and estimation methods as in Brav, Jiang, Partnoy, and Thomas (2008).

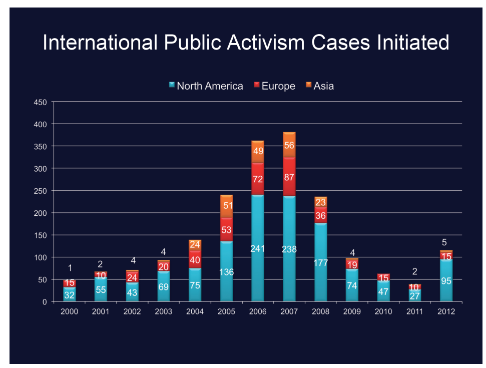

Figure 2: Reproduced with Marco Becht's permission. Based on Becht, Franks, Grant, and Wagner (2014).

It is sometimes suggested in the financial press that the procyclicality of returns from activist hedge funds is caused by the relative lack of diversification of activist portfolios.[6] Further, since one of the commonly declared objectives of activist hedge funds is the eventual merger of the target firm, it may also be tempting to attribute the procyclicality of hedge fund activism to the procyclicality of M&A markets. While these other potential channels may have a bearing on the procyclicality of activism, it is worth emphasizing that our analy-

[2] *The Economist*, "Activist Investors: Flight of the Locusts", April 8, 2009.

[3] *The Economist*, "Shareholder activism: Ready, set dough", December 2, 2010.

[4] Examples of activist campaigns launched in late 2010 include a successful joint attempt by Icahn and Seneca Capital to block the sale of Dynegy to Blackstone, a campaign by Trian Partners to induce Family Dollar to increase payouts, and a campaign by Jana partners to break up TNT (*The Economist*, "Shareholder activism: Ready, set dough", December 2, 2010.).

[5] *The Economist*, "Corporate Governance in America: Heating Up," April 7, 2012.

[6] It is worth noting that an explanation based upon idiosyncratic shocks is hard to square with patterns related to the business cycle.

sis – apart from delivering a self-contained model with fully rational agents – delivers an endogenous link between the observed procyclicality of activism and the documented effect of activism on the net debt of target firms.

In addition to these core results, our model generates several new and potentially testable implications. First, we connect the leverage of hedge fund target firms with macroeconomic prospects. The better are these prospects, the higher is target fund leverage, because when good times are more likely, target firms have higher debt capacity, resulting in a higher level of borrowing necessary to separate good from bad activists. Second, we link macroeconomic prospects to the time-pattern of returns to target firm shareholders. In particular, the better are these prospects, the more front-loaded are these returns. This is because better prospects lead to greater leverage at the target level, moving payouts to shareholders forward in time. Third, we connect the nature of ability differences within the activist hedge fund industry to target leverage and the time-pattern of returns. We show that it is exactly when activist hedge funds are principally distinguished by their ability to restructure target firms (rather than pay out free cash) that target firm leverage will be highest and, correspondingly, the returns to target shareholders will be most front loaded.

Finally, our model also helps to resolve seemingly contradictory evidence on whether the documented gains to shareholders of firms targeted by hedge fund activists can be wholly or partly attributed to the expropriation of existing bondholders. At one end of the spectrum, Klein and Zur (2011) argue that hedge fund activism leads to an expropriation of existing bondholders. In contrast, Brav, Jiang, Partnoy, and Thomas (2008) argue that expropriation of existing bondholders is unlikely to be a source of significant shareholder value because they find that announcement returns to target shareholders are *higher* in companies which are previously *un*levered. While our core mechanism does not require us to take any stance on the wealth effects of hedge fund activism on existing long-term creditors, our model provides a framework for interpreting this seemingly conflicting evidence. In particular – as we show in section 5.3 – when the target firm has long-term pre-existing debt, existing creditors may be expropriated as a result of hedge fund activism while returns to equity holders are *reduced* by the presence of pre-existing leverage. Since leverage created by activist hedge funds is motivated by competition for investor flows, it may well end up reducing the cash available to pay existing creditors when economic conditions sour. However, target-level borrowing is carried out on rational credit markets and pre-existing leverage reduces the (residual) debt capacity of the firm. The reduced debt capacity, in turn, reduces the payout necessary for separation and lowers the cash flows received by target shareholders.

Following our baseline analysis, we extend the model to allow the leverage of target firms to be directly observable by hedge fund investors and introduce the possibility of resource diversion by the fund manager. We show that our results are qualitatively unchanged: Hedge fund activist effort is procyclical exactly when good and bad funds are sufficiently different.

While our model is motivated by activist hedge funds, the analysis and results may apply more generally. It is often argued, for example, that the buyout activity of private equity funds is procyclical.[7] Like hedge funds, private equity funds also receive more capital if their performance on existing projects is high (Chung, Sensoy, Stern, and Weisbach (2012)). In addition, the use of extensive leverage in private equity buyouts is well known. Thus, at a qualitative level, our debt overhang story provides an explanation for the cyclical features of private equity buyout activity as well. Indeed, consistent with our results in section 5.1, Axelson, Jenkinson, Stromberg, and Weisbach (2013) find that private equity buyout leverage is procyclical. Two recent papers that theoretically examine the procyclicality of private equity buyout activity are Martos-Vila, Rhodes-Kropf, and Harford (2013) and Malenko and Malenko (2014).

Our paper engages with a large literature, both theoretical and empirical. The empirical literature has already been discussed above. At the broadest level, our paper belongs to the rich theoretical tradition of modeling blockholder monitoring in publicly traded corporations (e.g. Grossman and Hart (1980), Shleifer and Vishny (1986), Admati, Pfleiderer, and Zechner (1994), Burkart, Gromb, and Panunzi (1997), Bolton and von Thadden (1998), Kahn and Winton (1998), Maug (1998), Tirole (2001), Noe (2002), Faure-Grimaud and Gromb (2004), Admati and Pfleiderer (2009), Edmans (2009), and Edmans and Manso (2011)). This literature does not account directly for the delegated nature of blockholding, a phenomenon particularly prominent in the US and the UK, but also relevant elsewhere. A handful of recent papers have started to consider the role of incentives in delegated portfolio management in affecting the nature of delegated blockholder monitoring. Goldman and Strobl (2011) examine how a given degree of fund managers' short-termism affects firm investment policy. Dasgupta and Piacentino (2011) model the effect of competition for investor flows on the ability of delegated blockholders to govern via the threat of exit. While these papers share, in the broadest of terms, our interest in modeling the effect of incentive conflicts arising from the delegation of portfolio management on blockholder monitoring, none of them consider the issue of the procyclicality of hedge fund activism. Finally, our paper has a family connection to the more established literature on how competition for investor flows affect the prices, returns, volume, and volatility of assets traded by money managers (Dasgupta and Prat (2008), Dasgupta, Prat, and Verardo (2011), or Guerrieri and Kondor (2011)).

[7] In their model of the optimal financing structure of private equity funds, Axelson, Stromberg, and Weisbach (2009) demonstrate how the procyclicality of funding implies overinvestment in booms and underinvestment in busts.

2 Model

We consider a setting with two periods and four sets of actors. There are *hedge funds* (HF) which acquire stakes in *target* firms to seek changes (increasing payouts, business restructuring, sale of assets, etc.). In other words, hedge funds are shareholder activists. Hedge funds are financed by *investors* who pay fees to them and monitor their performance in order to maximize private returns. Finally, there are *financiers* who may provide external finance to firms targeted by hedge funds. All actors are rational and risk-neutral.[8] For simplicity, we ignore discounting.

To be specific, there is a continuum of identical firms, a continuum of ex ante identical hedge funds, and a continuum of competitive deep pocketed financiers. Each continuum is of measure 1. Each hedge fund enters the first period having used their investors' capital to acquire a stake in a target firm, i.e. blocks in firms are formed at some unmodelled prior period.[9] The match between firms and funds is random. Hedge fund activism potentially occurs in each period. Each hedge fund is financed by a continuum of identical investors. Each target firm can raise funds from a deep pocketed financier.

Hedge funds come in two types $\theta \in \{G, B\}$, where $\Pr(\theta = G) = \gamma_\theta$. Type G are better activists: They are able to produce higher cash flows from target firms. Each hedge fund, regardless of type, can engage in two types of activism.

The first – short-term – form of activism occurs during the first period and involves mitigating a free cash flow problem in target firms. Each target firm has excess cash of $C > 0$ in the first period which, if left under the discretion of the firm's manager, will be wasted (e.g., invested in zero gross return projects or otherwise diverted). Hedge funds can identify a type-dependent amount of free cash x_1^θ. We assume that x_1^G is distributed according to a cumulative distribution function H on the domain $[\Delta x_1, C]$ and that $x_1^B = x_1^G - \Delta x_1$ where $\Delta x_1 > 0$. Thus, the good type is better able to salvage excess cash from managerial waste. Any identified free cash is disbursed to shareholders at the end of the first period. In addition, by expending an infinitesimal effort cost, hedge funds can increase the payout by raising external finance against the second period cash flows of the target firm. They may choose an amount $F \in R_+$ to raise from competitive financiers. As a result the payout at

[8] As a result of the assumption of universal risk-neutrality, we ignore issues related to block size. In particular, we write the payoffs to hedge funds and their investors "as if" funds owned the entire target firm. This is not true in practice, but – in our model – accounting for block size would amount to a simple scaling of all payoffs, leaving the qualitative results unchanged. Potential concerns about additional information that could be impounded in secondary market prices by the trade of direct owners of the target firms are mitigated by the fact (as shown below) that our equilibria are fully revealing.

[9] We ignore the investors' participation constraint at this stage. Such participation decisions are analyzed in section 4.

the end of the first period is $D_1 = x_1^\theta + F$.[10]

The second – long-term – form of activism occurs during the second period and involves business improvements, restructuring, or the merger of the target firm. This form of activism differs from the first in several aspects: First, it is – as noted already – long-term, and requires substantial time and effort from the hedge fund. Second, the cash flows generated by such activism depend on the aggregate (macro) state of the economy. There are two possible macro states, $s \in \{H, L\}$, with $\Pr(s = H) = \gamma_s$. The state is publicly revealed at the beginning of the second period. Following the revelation of the state, hedge funds can exert effort $e \in \{0, \bar{e}\}$ at private cost $c_e > 0$, giving rise to cash flows, $\bar{X}_s^\theta$ with probability $\bar{e}$ and $\underline{X}_s^\theta$ with probability $1 - \bar{e}$.

It is well known that contracting on aggregate states is difficult. For example, Shiller (1998) writes (p. 2): "These economic causes of changes in standards of living that should be insurable without moral hazard because they are beyond individual control are still not insurable today because they are not so objective or easy to verify as fires or disabilities." In line with Shiller's observation, we assume that project success or failure (within a given macro state) is verifiable, while the macro state itself is not. We then trace the implications of such non-verifiability throughout our analysis, which includes but is not restricted to the lack of macro-contingency in financing. Thus, we offer a microfounded approach for determining optimal external financing that is *un*contingent on macroeconomic states.[11] Further, we assume that:

$$\bar{X}_s^\theta > \underline{X}_s^\theta \text{ for all } \theta \text{ and } s. \tag{1}$$

$$\bar{X}_s^G > \bar{X}_s^B \text{ for all } s. \tag{2}$$

$$\bar{X}_H^\theta > \bar{X}_L^\theta \text{ for all } \theta. \tag{3}$$

$$\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right) > 0. \tag{4}$$

Assumptions (1), (2), and (3) are natural monotonicity conditions. The first implies that activist effort increases firm cash flow. The second implies that good activists are better than bad ones. The third implies that activist efforts generate higher cash flows in the high macro

[10] These cash flows do not literally have to paid out to investors, but can also be reinvested by hedge funds in other targets on behalf of investors.

[11] Other microfoundations for limited macro-state contingency can be found in Krishnamurthy (2003) and Korinek (2009).

state, which could be interpreted, for example, as better business opportunities (a greater choice of positive NPV projects) or higher takeover premia during booms. Assumption (4) implies that the marginal returns to activist effort by a good fund is higher in booms relative to busts. This assumption is consistent with Kadyrzhanova and Rhodes-Kropf (2013) who find that activism is most valuable during periods of high market valuation. Finally, to exclude the uninteresting case in which the good hedge fund does not exert effort in the low state *purely* due to the high cost of activism, we assume:

$$c_e \leq \alpha \bar{e} \left(\bar{X}_L^G - \underline{X}_L^G \right) \tag{5}$$

Cash flows produced in the second period, net of any payments to financiers, are paid out to shareholders at the end of the period (D_2).

Our assumptions about activism above are well-supported by the data. The mitigation of free cash flow problems is a central goal of activist hedge funds. As Brav, Jiang, and Kim (2010) note in their survey, hedge fund targets can be characterised as "…"cash-cows" with low growth potentials that may sufffer from the agency problem of free cash flow." Longer-term forms of activism by hedge funds often include changes in business strategy and the merger of target companies. In the sample of Brav, Jiang, and Kim (2010) (which is an augmented version of the sample of Brav, Jiang, Partnoy, and Thomas (2008)) such changes, taken together, constitute 43% of 13D filings. Finally, our model requires that a given hedge fund potentially engages in more than one form of activism. There is also persuasive evidence for this. In the Brav, Jiang, and Kim (2010) sample, 48% of 13D filings between 2001 and 2007 do not declare a specific intent (i.e., state "general undervaluation" as the reason for intervention). The remaining 13D filings declare intent to (i) make changes to capital structure or (ii) business strategy, (iii) engage in a sale of the target company, or (iv) improve governance. While specific declarations of intent (13Ds that did not fall into the "general undervaluation" category) constituted only 52% of the sample, the percentages of 13D filings that declared goals (i)-(iv) above sum to nearly 85%. Thus, *on average*, hedge funds state around two distinct activist goals per 13D declaration.[12]

We now turn to our informational assumptions. Hedge funds are the most informed party in the model. Hedge fund investors and financiers have less information and their information sets are non-nested. At the beginning of the first period hedge funds learn the realized value of x_1^B and x_1^G and also discover their own type $\theta = G$ or B. In contrast, hedge funds investors only learn the realized value of x_1^B and x_1^G. They do not directly observe the types of their hedge fund.

[12] It is also reasonable to model payout policy changes as being a shorter-term form of activism as Brav, Jiang, and Kim (2010) present evidence that changes in payout policy happen more quickly than other changes (Table 5).

At the end of the first period, hedge fund investors see the payout D_1 but do not directly observe how much of this payout was due to additional funding (F). Since hedge funds have multiple methods for raising external finance at the level of the target firm (which may include, for example, drawing down of bank credit lines, additional borrowing from banks, lengthening of trade credit terms, private placements etc) it seems plausible that hedge fund investors may not observe the precise composition of their payout (which in general would involve knowing the external financing position of each target firm in their fund's portfolio) in real time. There is evidence, for example, that a significant amount of hedge fund target funding is bank-based (Li and Xu (2010)). At the time of the funding decision (when D_1 has not yet been paid), financiers do not know the realized values of x_1^G, x_1^B, but they observe F (since they are providing it). They form beliefs $\mu_F(F) = \Pr(\theta = G|F)$ and set the repayment terms $R\left(X_s^\theta\right)$ due at the end of the second period to break even, making all relevant equilibrium inferences.

It is worth noting that our assumptions that hedge fund investors do not directly observe external financing of the target and conversely that financiers do not directly know the potential abilities of hedge funds do *not* bind in equilibrium. As will be clear below, in equilibrium, hedge fund investors can infer the external financing of their fund's target and financiers correctly anticipate the type and ability of the hedge fund.

To conclude the description of the model, we now specify the actions of hedge fund investors and the payoffs of the hedge funds. After observing the payout D_1, hedge fund investors form their beliefs $\mu_I(D_1) = \Pr(\theta = G|D_1)$ about the type of the hedge fund and choose whether to retain ($a_I = R$) or to fire the fund ($a_I = W$). If $a_I = W$, the fund is shut down, and the target firm is sold to external markets (at fair prices) and continues to operate generating cash flows $\underline{X}_s^\theta$. Non-verifiability implies that of these cash flows, only $\underline{X}_L^\theta$ is available to be divided amongst financiers and the new equity holders according to the seniority specified in the contracts.

Motivated by observed compensation arrangements hedge fund fees in our model are made up of two parts. The first part is an assets-under-management fee, w, paid during each period of employment, at the beginning of the period. The second part is an incentive fee – a so-called "carry" – which is $\alpha \max(D_2, 0)$ for some $\alpha \in (0,1)$.[13] This implies that hedge funds that are retained by their investors for the second period get a share of the liquidating cash flows to equity holders.[14]

[13] Since we assume that $w > 0$ is paid at the beginning, even the bad type fund's participation constraint is trivially satisfied.

[14] Abstracting from the first-period carry is a simplification which – as will be clear later – *reduces* incentives for raising external finance under the optimal financing contract. Since our paper emphasizes the negative implications of excessive external financing induced by competition for investor flow, this simplification works

We end the model description with two assumptions about the bad hedge fund. First, we assume that:

$$\bar{X}_H^B - \underline{X}_H^B < \frac{c_e}{\bar{e}}. \tag{6}$$

It is straightforward to show (see the proof of Proposition 1) that (6) implies that bad hedge funds will not exert effort in the second period. Hence hedge fund investors would never knowingly retain a bad hedge fund into the second period. This is because by retaining a bad fund which does not exert effort, the investor expects a payoff of $-w+(1-\alpha)\underline{X}_L^B$ whereas by firing a fund at the end of the first period and liquidating the firm at fair prices, the investor receives $\underline{X}_L^B$. Second, we assume that

$$\Delta x_1 > \underline{X}_L^B. \tag{7}$$

This means that financiers would never knowingly provide funding of Δx_1 to a bad hedge fund: Since the bad hedge fund will not subsequently exert effort, the maximum amount that such a fund can in principle deliver to financiers in the second period is $\underline{X}_L^B$.

3 Equilibrium

Since all firms are identical, each firm is matched to one fund, and all funds of any type are identical, the discussion below is framed in terms of a representative firm ("the firm") and a representative fund ("the fund") that has invested in it. Similarly, since all investors and financiers are identical, the discussion is couched in terms of a representative investor ("the investor") and a representative financier ("the financier").

A perfect Bayesian equilibrium of this game is a n-tuple $(F^*, e^*(\cdot), a_I^*, R^*(\cdot), \mu_I^*, \mu_F^*)$ where (i) the retention decision a_I^* is optimal for the investor given beliefs μ_I^*; (ii) The repayment $R^*(\cdot)$ allows the financier to break even given beliefs μ_F^*; (iii) Funding F^* and state-contingent effort $e^*(\cdot)$ are best responses of the fund to (a_I^*, μ_I^*) and $(R^*(\cdot), \mu_F^*)$; and (iv) The posterior beliefs μ_I^*, μ_F^* are consistent with Bayes updating along the equilibrium path and are arbitrarily chosen otherwise. In this section, we characterize the perfect Bayesian equilibria of our model.

3.1 The impossibility of pooling

We first show that:

Proposition 1 *There exists no pooling equilibrium.*

against us.

The proof of this result, as well as that of all other results, is in the appendix. Intuitively, a pooling equilibrium can only exist if the first period payout D_1 does not reveal the hedge fund type to the investors. This requires that bad funds raise Δx_1 more than the good funds. Conditions (6) and (7) above imply that bad funds – if identified – cannot raise Δx_1 from financiers. Thus, bad funds can only raise Δx_1 or more from financiers if they raise the same amount as good funds. This, in turn, prevents them from offering the same payout. That is, mimicking good funds in the investor market forces bad funds to reveal their type in the funding market or vice versa.

Proposition 1 implies that we can focus on separating equilibria only for the remainder of the paper. We do not consider separating equilibria in which financiers can commit to enforcing arbitrary predetermined fundings amounts or firms can commit to raising arbitrary predetermined amounts F. Such equilibria with commitment do not seem realistic because they require perfect coordination across firms and/or financiers. Below, we focus on equilibria without such commitment.[15]

3.2 The need for external finance for separation

We begin the analysis by making a few straightforward observations about separating equilibria. The corresponding results are formally stated and proved in the appendix. Since investors never knowingly retain bad funds such funds are always closed down at the end of the first period in any separating equilibrium. This means that in any separating equilibrium, the bad fund will not raise external financing (Lemma 1). Since he will be discovered and closed down, it is not worth paying even the inifinitesimal cost of raising external financing in the first period. Now, since the bad fund does not raise any funds F in a separating equilibrium, the financier will rationally assume that any positive amount F is raised by a good type (Lemma 2) and therefore is willing to invest up to the pledgable income of the good type (PI^G).

We show that these observations sharply restrict the set of separating equilbria that can arise. Since the financier does not know x_1^B and x_1^G he cannot infer how much the good type would need to raise in equilibrium. Thus, the financier cannot detect potential deviations by the bad type which involve raising any amount up to the pledgable income of the good type. But this means that, to separate, the good type hedge fund must pay out an amount so high that, even by receiving the same financing terms as a good type, the bad type cannot imitate.

Proposition 2 *In separating equilibria,* $D_1^*(G) > x_1^B + PI^G$.

[15] In game theoretic terms, such equilibria with commitment could be ruled out, for example, by imposing the requirement that financiers' beliefs are always $\mu_F^*(\widehat{F}) = 1$ for all $\widehat{F} \neq F^*$.

Except in the uninteresting case in which future cash flows that can be generated by the activist hedge fund are so low that $x_1^B + PI^G \leq x_1^G$, i.e., that $PI^G < \Delta x_1$, separation requires the use of external finance. We therefore proceed to characterizing the optimal form of external financing, i.e., the contract that maximizes the incentives of the good fund to exert effort in the second period.

3.3 Optimal financing contract

We now solve for the optimal financing contract $R(\cdot)$ taking into account the fact that only the good type seeks external financing (by Lemma 1 above).

Proposition 3 *Debt is the optimal contract for raising external funding F.*

Since project success/failure is verifiable but the macro state is not, promised repayments can take on at most two possible values, say $\bar{R}$ and $\underline{R}$. Since, conditional on separation (which eliminates the bad fund in the first period) the future cash flows are increasing in the good hedge fund's effort, we look for $\bar{R}$ and $\underline{R}$ which maximize the good fund's incentives to exert effort. While effort is costly for the fund, it allows it to obtain an $\alpha-$share of a larger cash flow with probability $\bar{e}$. In addition, the fund can appropriate additional cash flows in state $s = H$ as a result of the non-verifiability of the macro state: If the project succeeds, then the additional appropriation amount is $\bar{X}_H^G - \bar{X}_L^G$ whereas if the project fails the amount is $\underline{X}_H^G - \underline{X}_L^G < \bar{X}_H^G - \bar{X}_L^G$ (the inequality follows from assumption 4).[16] Since effort increases the probability of success from 0 to $\bar{e}$, in the high state effort also generates an additional payoff of $\bar{e}\left(\left(\bar{X}_H^G - \bar{X}_L^G\right) - \left(\underline{X}_H^G - \underline{X}_L^G\right)\right)$ to the fund. Thus, as the proof in the appendix shows, the incentive compatibility constraints of the good fund are:

$$\alpha\bar{e}\left(\left(\bar{X}_L^G - \underline{X}_L^G\right) - \left(\bar{R} - \underline{R}\right)\right) \geq c_e \text{ in state } s = L, \text{ and}$$

$$\alpha\bar{e}\left(\left(\bar{X}_L^G - \underline{X}_L^G\right) - \left(\bar{R} - \underline{R}\right)\right) + \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right) \geq c_e \text{ in state } s = H.$$

For arbitrarily chosen parameters, these two constraints are clearly most slack if $\bar{R} - \underline{R}$ is minimized, an observation related to the key insight of Jensen and Meckling (1976). Imposing monotonicity, as is standard in this literature following Innes (1990), leads to two possible optimal financing arrangements: If the hedge fund raises less than $\underline{X}_L^G$, we have safe debt with repayment $\bar{R} = \underline{R} < \underline{X}_L^G$. Otherwise, optimal external financing is achieved via defaultable debt with $\bar{R} > \underline{R} = \underline{X}_L^G$.[17]

[16] The assumption that the hedge fund (rather than the target firm's management) can appropriate non-verifiable cash flows $\bar{X}_H^G - \bar{X}_L^G$ and $\underline{X}_H^G - \underline{X}_L^G$ amounts to stipulating that the hedge fund is able to directly observe all firm cash flows.

[17] Needless to say, in the absence of any contracting frictions state contingent debt is the optimal contract. Such financial contracts would, however, be at odds with the prevalence of straight debt in the real world.

3.4 The consequences of borrowing to separate

We have shown to date that competition for investor flow implies that good hedge funds always separate in equilibrium, and that such separation implies raising external finance, which is best achieved by borrowing. In this section, we explore the consequences of borrowing to separate. The subsequent analysis needs to be split, for technical reasons, into two cases:

$$\text{Case A: } \left(\bar{X}_H^G - \underline{X}_H^G\right) \geq (1+\alpha)\left(\bar{X}_L^G - \underline{X}_L^G\right) \tag{8}$$

and

$$\text{Case B: } \left(\bar{X}_L^G - \underline{X}_L^G\right) < \left(\bar{X}_H^G - \underline{X}_H^G\right) < (1+\alpha)\left(\bar{X}_L^G - \underline{X}_L^G\right). \tag{9}$$

Since α is typically on the order of 0.2 for hedge funds (and lower of late for new hedge funds, e.g., *The Economist*, February 8, 2014) Case B is quite restrictive. Accordingly, we focus on Case A in the body of the paper and relegate Case B to the appendix, where we show that the economic content of our results is essentially identical across the cases.

Before stating our formal result, it is useful to introduce some suggestive terminology. To motivate this terminology, note that since the hedge fund receives only the second-period carry, he does not wish to borrow too much: The more he borrows, the less is this carry (by definition). So, it is reasonable to focus on the separating equilibrium which delivers separation with as little leverage as possible. In addition, since – as will be clear from our result below – borrowing to separate may (under certain conditions) shut down hedge fund activism in low macro states, focussing on separating equilibria with *minimal* leverage establishes the conditions under which such reduced activism is an *essential* element of equilibrium. In the remainder of the paper, we shall refer to the equilibrium which delivers separation with as little leverage as possible as the *separating equilibrium with minimal leverage* (SEML).

Proposition 4 *As long as* $\bar{X}_L^G > \underline{X}_L^G + \frac{\Delta x_1}{\gamma_s(1-\gamma_s)\bar{e}}$ *and* $\Delta x_1 > \frac{w}{1-\alpha}$*, the separating equilibrium with minimal leverage involves:*

i. *For* $c_e \in \left(0, (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$*,* $e^*(s) = \bar{e}$ *for all* s*.*

ii. *For* $c_e \in \left[(1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right], \alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$*,* $e^*(H) = \bar{e}$ *and* $e^*(L) = 0$*.*

When effort costs are relatively low, the fund exerts effort in both macro states, but when effort costs are relatively high it does so only in the high state. This reduction of activist effort is, however, *not* down to high effort cost *alone*: Given condition (5), if the fund were the sole residual claimant to the incremental expected cash flows generated by effort in the low state, he *would* have exerted effort in that state. He does not do so because, in equilibrium, he can*not* claim a sufficient fraction of the incremental cash flow due to leverage taken on to separate from the bad type. Thus, leverage induced by competition amongst funds generates debt overhang in the low state and shuts down activist effort. Since this arises in the separating equilibrium with minimal leverage, for the relevant range of effort cost, such a state-contingent reduction of activist effort is an *essential* part of equilibrium.

The proof of this result involves four steps which are detailed in the appendix and heuristically summarized here. First, we compute the minimum face value $\underline{K}$ which triggers debt overhang in state $s = L$. This is determined using the incentive compatibility condition in the low state and is equal to $\bar{X}_L^G - \frac{c_e}{\alpha \bar{e}}$.

Next, we compute the maximum face value $\bar{K}$ which ensures effort exertion in state $s = H$. There are two natural bounds on $\bar{K}$. First, conditional on paying $\bar{K}$ the hedge fund must retain enough expected payoffs to have incentives to exert effort. At the same time, $\bar{K}$ cannot be larger than $\bar{X}_L^G$, because – since macro states are non-verifiable – the fund can always claim that total cash flow is $\bar{X}_L^G$ in case of success. It turns out that in Case A (i.e., if condition 8 holds), the relevant upper bound on $\bar{K}$ is always $\bar{X}_L^G$.

Thus, a debt contract which promises $\bar{X}_L^G - \frac{c_e}{\alpha \bar{e}}$ induces the fund to make an effort in both states. A debt contract that promises $\bar{X}_L^G$ induces the fund to make an effort in the high state only. The pledgeable income associated with each of these contracts determines which one will be relevant in equilibrium. In the SEML the good fund pays out just enough to separate even if the bad fund were to borrow the full pledgeable income of the good. Hence, good fund must use the contract with the higher pledgeable income. Otherwise, the bad type could mimic the good type's SEML payout, contradicting separation.

The choice between the contract that promises $\bar{X}_L^G - \frac{c_e}{\alpha \bar{e}}$ and one that promises $\bar{X}_L^G$ involves the following trade-off. On the one hand, the former contract pays less conditional on success than the latter and the difference is increasing in the effort cost. On the other hand, creditors are paid in full more often under the former contract (with probability $\bar{e}$) than under the latter (with probability $\gamma_s \bar{e}$). Therefore, the pledgeable income associated with the former contract will be higher precisely when the effort cost is low. In that case, separation involves the use of a lower face-value contract which maintains incentives to exert effort in both states. In contrast, when effort costs are relatively high, separation involves the use of a higher face-value contract which destroys incentives to exert effort in the low state. This is the dichotomy captured in the result above.

Finally, note that Proposition 4 requires that $\bar{X}_L^G$ and Δx_1 be large enough. The parameter Δx_1 measures the difference in skills between the good and bad hedge fund in payout activism. Since $\bar{X}_L^B < \bar{X}_H^B$, and $\bar{X}_H^B$ is bounded above by condition (6), a high $\bar{X}_L^G$ translates into a large difference $\bar{X}_L^G - \bar{X}_L^B$. But this, in turn, is a measure of the difference in restructuring ability across good and bad funds. Taking these two observations together, competition for flow generates a tournament amongst hedge funds that induces sufficient leverage to prevent activist effort in low states precisely when ability differences across funds are not small.

There are two interpretations of the cost variation captured in Proposition 4. First, cost variations may be seen as representative of different activist styles. If, for example, restructuring is more costly than the merger of the target, then one may expect to see hedge funds aiming for restructuring to be more prone to reduce effort in downturns. Second, and perhaps more intriguingly, one could view the cost variation as a time-series phenomenon, related to target selection. The evidence discussed in the introduction suggests that hedge fund activism occurs in waves. It has been observed that early in a wave activist funds select target firms where it is realistic to achieve value improvements, whereas late in a wave – when easy targets are scarce – they aim for targets where value improvement may be more difficult to attain.[18] Viewed through the lens of our model, this variation can be interpreted in terms of costs of activist effort: Early in waves hedge funds engage in targets where activism is less costly and robust i.e., immune, to an economic downturns. Late in waves hedge funds engage in targets where activism is more difficult which makes activism itself more fragile and sensitive to macroeconomic conditions.

We conclude this section with two observations about when activist effort is more or less likely to be sensitive to macroeconomic conditions:

Corollary 1 *The effect of macroeconomic prospects:*

a. *Better macroeconomic prospects (higher γ_s) make hedge fund activism more prone to procyclicality.*

b. *When macroeconomic prospects are good ($\gamma_s > \frac{1}{2}$) hedge fund activism is more prone to procyclicality when it creates more value ($\bar{X}_L^G$ is larger).*

Statement (a) follows from the fact that the interval $\left(0, (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$ is decreasing in γ_s while the interval $\left[(1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right], \alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$ is increasing in γ_s. Thus better macroeconomic prospects increase the range over which there is debt overhang. Statement (b) follows from the fact that, for $\gamma_s > \frac{1}{2}$, increasing $\bar{X}_L^G$ lengthens the interval $\left[(1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right], \alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$ more than it lengthens the interval $\left(0, (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$. Thus, when macroeconomic prospects are good, higher potential

[18] In *The Wall Street Journal* (online) 13 August 2013, referring to Ackman's stake in J. C. Penney, Justin Lahart writes, quoting Alon Brav: "Activists did well in 2009, but by late 2010... the easiest pickings may have been taken. To create value under those circumstances, says Mr. Brav, "you will have to do something that is not so simple." One example: entering the cutthroat world of department-store retail and pushing through a huge reconfiguration of the business."

cash flows from activism increases the relative range of activities over which such cash flows are not produced in economic downturns.

4 Procyclicality

Proposition 4 identifies a range of effort costs over which hedge fund activism becomes sensitive to macroeconomic conditions. In this section we show that a consequence of such sensitivity is that investment in activist funds becomes more attractive when macroeconomic prospects are better, and that this provides a basis for interpreting the available evidence on the procyclicality of 13D filings. We also pin down the role of competition for flow in delivering our results: It is both necessary and sufficient in fostering procyclicality.

Since our focus is on procyclicality, we consider investment incentives in the case where $c_e \in \left[(1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right], \alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$. The analysis for $c_e \leq (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]$ is in the appendix. It is easy to see that, if there were ex ante uncertainty about the cost parameter c_e, then our characterization of investment incentives would hold qualitatively for *any* c_e.

4.1 When are activist blocks formed?

To characterize the attractiveness of investment in activist hedge funds, we begin by analysing the investors' ex ante participation decision. We normalize the block price in period 0 to be 1. The precise block price depends on the nature of the trading game between the hedge fund and the prior owners of the block, a topic beyond the scope of this paper. Our qualitative results only require that the block price does not fully reflect all information about the future cash flows generated by activist funds. This would arise naturally if, for example, the fund acquired the block from investors who were forced to sell due to idiosyncratic liquidity shocks. Then the price would simply reflect the reservation value of the seller. Gantchev and Jotikasthira (2013) provide evidence suggesting that activist hedge funds do indeed exploit liquidity sales by other institutions in forming blocks. Suppose that the investor has initial wealth $1 + w$, and can either invest it in a storage asset (with zero net return), or give 1 to an activist hedge fund to form a block and pay him a fee of w for the first period. If the investor employs a hedge fund, then (since all hedge funds of either type participate) with probability γ_θ he is matched with a good fund. In the SEML, the good fund pays out $x_1^G + \gamma_s\bar{e}\bar{X}_L^G + (1-\gamma_s\bar{e})\,\underline{X}_L^G - \Delta x_1$ in the first period, and then in the second period the investor always pays w but the hedge fund exerts effort only in the high state. Hence, conditional on being matched with a good fund (with probability γ_θ), the investor receives in expectation

$$(1-\alpha)\left[\gamma_s\left(\bar{e}\left(\bar{X}_L^G-K^*\right)+(1-\bar{e})\max\left(\underline{X}_L^G-K^*,0\right)\right)+(1-\gamma_s)\max\left(\underline{X}_L^G-K^*,0\right)\right]-w.$$

Given that (as shown in the proof of Proposition 4) $K^*=\bar{X}_L^G-\frac{\Delta x_1}{\gamma_s\bar{e}}>\underline{X}_L^G$, the investor's expected payoff in the second period in this case is $(1-\alpha)\Delta x_1-w$. Instead, with probability $1-\gamma_\theta$ he is matched with a bad fund. The bad fund pays out $E\left(x_1^B\right)$ in the first period and is closed down, and the investor sells the firm for a price $\underline{X}_L^B$. Thus, the investor's expected total cash flows are:

$$\gamma_\theta\left[E\left(x_1^G\right)+\gamma_s\bar{e}\bar{X}_L^G+(1-\gamma_s\bar{e})\underline{X}_L^G-\Delta x_1-w+(1-\alpha)\Delta x_1\right]+(1-\gamma_\theta)\left[E\left(x_1^B\right)+\underline{X}_L^B\right] \tag{10}$$

This is to be compared with the net return on the outside option which is zero. Thus, the investor participates if and only if the value of the expression in (10) exceeds the initial investment cost $1+w$. It is clear that as long as the non-divertible return from hedge fund activism ($\bar{X}_L^G$) is high enough the participation constraint is satisfied, without violating any of the equilibrium conditions.

4.2 Competition for flow and macroeconomic prospects

Our analysis of the investors' participation decision reveals a salient property. Since $\bar{X}_L^G>\underline{X}_L^G$, for any given $\left(E\left(x_1^G\right),E\left(x_1^B\right),\Delta x_1,\bar{X}_L^G,\underline{X}_L^G,\gamma_\theta,\bar{e},w,\alpha\right)$ the expected payoff to investors from investing in an activist fund (given by (10)) is increasing in γ_s: Investors will find it more attractive to invest in activist funds if macroeconomic prospects are better. To understand how competition for flow is relevant for such macroeconomic sensitivity, imagine an alternative where investors (counterfactually) do not chase flows, and instead retain the hedge fund with some arbitrary exogenous probability $\xi\in(0,1)$. Now, funds cannot influence their retention probability by their first period return, and thus do not compete to influence investors. In particular, since funds receive only a second-period carry, which is reduced by borrowing in the first period, they choose not to leverage at all. Instead, they pay out x_1^θ and then (if retained exogenously into the second period) the good fund exerts effort in the second period regardless of the macroeconomic state (assumption 5) while the bad fund does not (assumption 6). Due to the non-verifiability of macro states the cash flows available to investors is $\bar{X}_L^\theta$ in case of success and $\underline{X}_L^\theta$ in case of failure. Thus, investors' payoffs are:

$$\left(\begin{array}{c}\gamma_\theta\left[E\left(x_1^G\right)+\xi(1-\alpha)\left(\bar{e}\bar{X}_L^G+(1-\bar{e})\underline{X}_L^G\right)+(1-\xi)\underline{X}_L^G\right]\\+(1-\gamma_\theta)\left[E\left(x_1^B\right)+\xi(1-\alpha)\underline{X}_L^B+(1-\xi)\underline{X}_L^B\right]\end{array}\right)-\xi w, \tag{11}$$

which is independent of γ_s. We can thus pinpoint the critical role of competition for flow in rendering macroeconomic prospects relevant for investment in activist funds:

Implication 1 *Competition for flow is necessary and sufficient to ensure that the attractiveness of investment in activist funds is increasing in macroeconomic prospects.*

4.3 Interpreting the evidence on procyclicality

The evidence on procyclicality discussed in the introduction suggests that there is a positive association between market valuations and the number of 13D filings by activist hedge funds: If an econometrician regresses the number of 13D filings on market valuations, they would find a positive coefficient. In this section, we show that competition for flow implies a positive predicted coefficient on market valuations in a regression of the number of 13D filings on market valuations. The coefficient would be zero in the absence of competition for flow.

13D filings are usually associated with block formation which, in turn, requires that activist hedge funds are financed, i.e. that investors agree to participate. Building on the analysis above, we first demonstrate that investors may not agree to participate unless macroeconomic prospects are sufficiently good. To see this in the starkest possible manner, restrict attention to cases in which (i) $\bar{X}_L^G$ is large, (ii) $\underline{X}_s^\theta = 0$ for all θ, s (for simplicity) and (iii) The return from free cash flow mitigation by bad types is not very high: $E\left(x_1^B\right) < 1 + (1+\gamma_\theta)\, w - \gamma_\theta\,(1-\alpha)\,\Delta x_1$.

When $\gamma_s \to 0$ in expression (10), the expected return from investing in hedge funds is $\gamma_\theta\left[E\left(x_1^G\right) - \Delta x_1 - w + (1-\alpha)\,\Delta x_1\right] + (1-\gamma_\theta)\,E\left(x_1^B\right)$ which is smaller than $1+w$ given (iii) above. Thus, in the presence of competition for flow there exists a positive threshold level of γ_s, say γ_s^{DO}, such that the investor participates, and thus a block is formed, only if $\gamma_s \geq \gamma_s^{DO}$. In contrast, since $\bar{X}_L^G$ is large, inspection of (11) implies that without flow competition, participation occurs for all $\gamma_s \geq 0$.

Now, we can formalize the sense in which competition for flow affects the predicted coefficient on market valuations in a regression of the number of 13D filings on market valuations. In our model γ_s is the appropriate proxy for broad market valuations since it is a sufficient statistic in the first period for future macroeconomic conditions.

Suppose that γ_s is distributed ex ante according to some CDF $J(\cdot)$ which is defined on $[0,1]$ and with a density that is strictly positive everywhere. Consider a hypothetical regression of the number of 13Ds on our proxy for market valuations, γ_s. Recall that the measure of firms is 1, so that the maximum "number" (measure) of 13Ds that can be filed is 1. Denote by γ_s^* the relevant threshold for investor participation, depending on whether there is competition for flow ($\gamma_s^* = \gamma_s^{DO} > 0$) or not ($\gamma_s^* = 0$). Let $N13D$ denote the random variable for the number of filings at $t=0$. Formally our model predicts that

$$N13D = \begin{cases} 1 & \text{if } \gamma_s > \gamma_s^* \\ 0 & \text{otherwise} \end{cases}.$$

This is because, whenever $\gamma_s > \gamma_s^*$ the investor's participation constraint will be satisfied, and thus blocks will be formed and 13Ds filed. Thus,

$$E(N13D) = \int_{\gamma_s^*}^{1} dJ(\gamma_s).$$

If $N13D$ were regressed on γ_s the regression coefficient would be:

$$\frac{cov(\gamma_s, N13D)}{Var(\gamma_s)}.$$

Comparing the regression coeffiecient with and without competition for flow amounts to comparing the covariance term:

$$cov(\gamma_s, N13D) = \int_{\gamma_s^*}^{1} (\gamma_s - E(\gamma_s))\, dJ(\gamma_s).$$

It is immediate that if there is no competition for flow, so that $\gamma_s^* = 0$, then $cov(\gamma_s, N13D) = 0$, while if there is competition for flow, so that $\gamma_s^* = \gamma_s^{DO} > 0$, then $cov(\gamma_s, N13D) > 0$.

5 Further Empirical Implications

In this section, we outline further empirical implications of our model. Some of these are new testable implications (sections 5.1 and 5.2), while others reconcile existing empirical evidence (section 5.3). As before we focus throughout on the case where effort costs are high enough that activist efforts cease in the low state and comment in passing on the low costs case.

5.1 Economic prospects, target leverage, and returns to target shares

The amount of borrowing in the SEML is $PI_K^G - \Delta x_1 = \gamma_s \bar{e} \bar{X}_L^G + (1 - \gamma_s \bar{e}) \underline{X}_L^G - \Delta x_1$, while the face value of the debt is $\bar{X}_L^G - \frac{\Delta x_1}{\gamma_s \bar{e}}$. Both quantities are increasing in γ_s. Thus, when γ_s is higher, hedge fund activists will impose greater leverage on their target firms in equilibrium. The reason is that better economic prospects implies a higher debt capacity for the target, which in turn implies that more borrowing is necessary for good type funds to separate.

Implication 2 *When economic prospects are better, hedge funds target firms are more highly leveraged.*

While we are not aware of any systematic empirical investigation of this question, there is anecdotal evidence that activist hedge funds changed their tactics when they resurfaced after the financial crisis. According to *The Economist*, "Activists are toning down their attempts to get companies to take on more debt. Many were burned before, and are reluctant to put their hands back in the fire."[19] Interpreted through the lens of our model, this may simply be a case of lower market confidence about future prospects for the economy in 2010 than in the heady days of optimism prior to the financial crisis.

It is also worth mentioning that target debt has a higher face value in times of better economic prospects. So, if investment were of variable scale, there would be more debt overhang if economic conditions soured (i.e., more projects would be shut down).

Finally, economic prospects also have implications for the time pattern of expected returns to target shareholders. The expected equilibrium payoff to target shareholders is $\gamma_\theta \left(\gamma_s \bar{e} \bar{X}_L^G + (1-\gamma_s \bar{e}) \underline{X}_L^G - \Delta x_1 + E\left(x_1^G\right)\right) + (1-\gamma_\theta)\left(E\left(x_1^B\right) + \underline{X}_L^B\right)$ in the first period and $\gamma_\theta \Delta x_1$ in the second period. Better economic prospects enhance first period payoffs without affecting second period payoffs, because they lead to higher leverage for separation, moving payouts to target shareholders forward in time.

Implication 3 *When economic prospects are better, the returns to target firms' shareholders from hedge fund activism are more front-loaded.*

The evidence in Brav, Jiang, and Kim (2010) (see Table 4) suggests that in the 2001-2006 period – a time of significant optimism about economic prospects – the abnormal returns to target shareholders accrued in the early months of activist campaigns. This is consistent with Implication 3. In addition, Implication 3 may also suggest that activist hedge funds would be particularly attractive to impatient investors during periods of significant optimism about future prospects.[20]

5.2 Payout vs Restructuring

Our model also relates the nature of ability differences within activist hedge funds to the leverage of their targets, providing another set of potentially testable implications. Keeping Δx_1 large enough to satisfy the SEML conditions, it is clear that lower Δx_1 implies higher leverage $\gamma_s \bar{e} \bar{X}_L^G + (1-\gamma_s \bar{e}) \underline{X}_L^G - \Delta x_1$. Δx_1 is a measure of managerial talent differences in combating the free cash flow problem. Thus, the less managerial talent matters in the short-run payout enhancement form of activism, the higher is leverage and the higher is the potential for debt overhang.

Implication 4 *When talent differences across activists matter little for mitigating free cash flow problems, target leverage is higher.*

[19] *The Economist*, "Shareholder activism: Ready, set dough", December 2, 2010.

[20] For $c_e \in \left(0, (1-\gamma_s)\alpha \bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$, the pledgable income and thus leverage is independent of γ_s since activist effort is independent of macro states. Thus, the two implications considered here are moot for that case.

Excessive target leverage is what gives rise to procyclicality and thus shuts down restructuring in economic downturns. In turn, as ability differences in mitigating free cash flow problems become less important, a higher utilization of the target's debt capacity is required for separation. Thus, it is precisely when activist hedge funds are principally differentiated by restructuring ability that restructuring becomes less likely in downturns.

Ability differences in tackling free cash flow problems also affect the time pattern of expected returns to target shareholders.

Implication 5 *When talent differences across activists matter little for mitigating free cash flow problems, the returns to target firms' shareholders from hedge fund activism is more front loaded.*

Again, the effect works through the amount of leverage. Lower talent differences in tackling free cash flow problems translate into higher leverage, which moves payoffs to target shareholders forward in time.[21]

5.3 Do activists expropriate bondholders?

There is general agreement in the literature on the fact that – as in our model – hedge fund activism produces significant positive returns to target shareholders. However, the empirical literature is not unanimous on whether (some of) these gains derive from the expropriation of existing bondholders. At one end of the spectrum, Klein and Zur (2011) argue that hedge fund activism leads to an expropriation of existing bondholders, a conclusion shared – with caveats and qualifications – by Li and Xu (2010) and Sunder, Sunder, and Wongsunwai (2010). However, Brav, Jiang, Partnoy, and Thomas (2008) argue that expropriation of existing bondholders is unlikely to be a source of significant shareholder value because they find that returns to target shareholders are *higher* in companies which are previously *un*levered.

Our core mechanism does not turn on the interaction between existing bondholders and shareholders: Since the representative target firm is unlevered in our model, our baseline results are silent on the issue of bondholder expropriation. Nevertheless, our framework can be used to interpret the seemingly conflicting evidence in Brav, Jiang, Partnoy, and Thomas (2008) and Klein and Zur (2011). Reconsider the baseline model with the following modifications. Assume that the representative firm has some liquid assets of $Y_0 > 0$ in the first period. Unlike the pre-existing excess cash C, which is subject to a free cash flow problem, these liquid assets Y_0 cannot be diverted by company management. Thus, absent hedge fund activists, this Y_0 would be retained until the second period and available to pay pre-existing

[21] For $c_e \in \left(0, (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$ the amount of borrowing is also decreasing in Δx_1, so the two implications stated in this subsection hold for this range of costs as well.

creditor claims, if any. Hedge fund activists may pay out part or all of these liquid assets in the first period to enhance early returns to their investors, in addition to leveraging the target as in the baseline model. As before, investors cannot directly observe the composition of the payout (though they can infer it in equilibrium). We compare two capital structures for the target firm: Either the target firm has no pre-existing debt (as in the baseline model) or it has pre-existing debt maturing in the second period with a face value of $K_0 \in (\Delta x_1, Y_0)$. For simplicity, assume that $\underline{X}_s^\theta = 0$ for all θ, s and that $\bar{X}_H^G \geq (1+\alpha)\,\bar{X}_L^G$ (corresponding to baseline Case A). We can now state:

Proposition 5 *For $c_e \in \left[(1-\gamma_s)\alpha\bar{e}\bar{X}_L^G, \alpha\bar{e}\bar{X}_L^G\right]$, as long as $\bar{X}_L^G$ and Δx_1 are large enough, pre-existing target leverage may reduce shareholder returns from activism even when activism expropriates existing bondholders.*

Using arguments that parallel those of Proposition 4, we show that when effort costs and ability differences between good and bad funds are sufficiently high, competition for flow induces the good fund to pay out all available liquid assets in the first period and also to leverage the target sufficiently to generate debt overhang in the low macro state. This implies that activist funds reduce the cash available for existing creditors: In the absence of hedge funds, pre-existing debt is safe and creditors are paid in both states. In the presence of hedge funds, the pre-existing debt becomes risky and creditors are only paid with probability $\bar{e}$ in the high state, consistent with the findings of Klein and Zur (2011). However, comparing target firms with and without pre-existing leverage in the presence of activist funds, Proposition 5 shows that returns to shareholders are higher when the target firm is unlevered. This is because pre-existing target debt reduces the (residual) debt capacity of the target, which in turn reduces the payout necessary for separation and hence the equilibrium first period payout to target firm shareholders. The second period payout is unaffected because – as in the baseline model – activist funds borrow all but Δx_1 of the target's debt capacity. Thus, in the presence of activist funds, returns are lower to the target firm shareholders when there is pre-existing leverage, consistent with the findings of Brav, Jiang, Partnoy, and Thomas (2008). Thus, our model provides a simple, stylized, framework that helps to resolve some of the contradictory empirical evidence in Brav, Jiang, Partnoy, and Thomas (2008) and Klein and Zur (2011).

5.4 Excessive payout

The enriched framework introduced in section 5.3 delivers a further benefit: It enables us to examine whether our results hold if we restrict hedge funds to changing payout policy *only*, i.e., preclude them from issuing *new* target debt. If that is so, then our results can be interpreted in terms of increases in *net* debt – i.e., debt minus cash – extending our model's links to the empirical literature.

We show that our results are indeed robust to payout policy changes only as long as the target has both pre-existing debt and liquid assets: For target firms with pre-existing debt, a reduction in liquid assets increases net debt. Competition for flow can deliver sufficiently high net debt to foster debt overhang in the low macro state. We consider the same variation of the model as in section 5.3 except that new borrowing is prohibited. Activist hedge funds salvage excess cash of x_1^θ and pay it out at the end of the first period. They may augment the payment by tapping into liquid assets Y_0. As in the baseline model investors only directly observe total cash paid out but not its components. In the absence of a hedge fund activists, the liquid assets Y_0 would be retained until the second period and available to pay pre-existing creditor claims.

Proposition 6 *High payout to compete for investor flow may induce debt overhang even without new target firm borrowing.*

The intuition is that – as before – good funds must pay a high enough dividend at the end of the first period to prevent mimicking by bad funds. Since either fund can tap into the liquid assets, the good fund must pay out at least $x_1^B + Y_0$ to separate. But, then, for target firms with a sufficient amount of pre-existing leverage, debt overhang arises in the low state.

6 Observable leverage

In the baseline model target firm leverage is not (directly) observable to hedge fund investors. As discussed earlier, we believe that this assumption is not unrealistic in the context of hedge fund activism. However, to illustrate that our key economic results are not driven by this assumption, in this section we analyze a model in which target leverage is immediately and publicly observable.

In this variant of the model, as in the baseline, each hedge fund pays out x_1^θ in the first period from free cash flow mitigation where $x_1^G - x_1^B = \Delta x_1 > 0$ is constant and common knowledge. However, each hedge fund can enhance these cash flows in two ways: First, the fund can (secretly) monetize (liquidate/divert) assets from the firm of some amount $k \in [0, \bar{k}]$ where $\bar{k} > \Delta x_1$. Such monetization is costly in terms of future cash flow from restructuring in a way described below. Second, each hedge fund can leverage the target firm as before. However, now, we assume that the amount borrowed is publicly observed and creditors can directly infer the type of the fund as a result of due diligence. As in the baseline model, enhancement activity – now leverage and monetization – requires an infinitesimal cost.

Following the revelation of the macro state in the second period, hedge funds can exert effort $e \in \{0, \bar{e}\}$ at private cost $c_e > 0$, giving rise to cash flows, $\bar{X}_s^\theta$ with probability $\bar{e}$ and $\underline{X}_s^\theta$ with probability $1-\bar{e}$. Further, we retain the monotonicity assumptions (2) and (3) from the baseline model. As in Section 5.3, we simplify the analysis by assuming that failure payoffs are zero ($\underline{X}_s^\theta = 0$ for all θ, s). This implies that assumptions (1) and (4) of the baseline model become redundant and assumption (5) reduces to $c_e \leq \alpha\bar{e}\bar{X}_L^G$. We model the loss from monetization as follows: Monetizing assets $k \in [0, \bar{k}]$ during the first period reduces the second period cash flow $\bar{X}_s^\theta$ to $\left(1 - \frac{k}{\tau}\right)\bar{X}_s^\theta$ where $\tau > \bar{k}$.

Finally, as in the baseline, we make two further assumptions about the the bad fund. First, for simplicity, we assume that effort costs are such that the bad fund never exerts effort in the low state: $c_e > \alpha\bar{e}\bar{X}_L^B$. Note that this weakens our baseline assumption (6) which implied that bad funds did not exert effort in *either* state. In this version we always ensure that it is *feasible* (out of equilibrium) for the bad fund to exert effort in the *high* state. This is because when leverage is observable and creditors directly infer types, the bad fund can *attempt* to imitate the good only if he has a positive debt capacity. However, to allow for flow competition, we bound the bad fund's ability. We assume that:

$$\bar{X}_L^B < \frac{w}{\gamma_s\bar{e}\,(1-\alpha)}, \tag{12}$$

which implies that the bad fund will be fired if identified. We first provide a parallel to Proposition 2:

Proposition 7 *In separating equilibria,* $D_1^*(G) > x_1^G + PI^B\,(k = \Delta x_1)$

Since $\bar{k} > \Delta x_1$ the bad fund has a way of offsetting the good fund's advantage at free cash flow mitigation, i.e., if the good fund chose to pay out only x_1^G at $t = 1$ then the bad fund could imitate, destroying separation. One option for the good fund is to enhance payout by borrowing $F^G > 0$. Of course, so can the bad fund as long as $F^B = F^G$. In particular, if the good fund borrows F^G to pay out $x_1^G + F^G$ then the bad fund can set $k = \Delta x_1$, borrow $F^B = F^G$, and pay out $x_1^B + \Delta x_1 + F^B = x_1^G + F^G$. The only way to prevent this is that the good fund borrows enough that the bad fund cannot imitate. Such a level of borrowing exists only because credit markets can directly infer the type of the borrower by doing due diligence conditional on a loan application. Thus, now the good fund can borrow $\hat{F}^G = PI^B\,(k = \Delta x_1) + \epsilon$ for some $\epsilon > 0$ and pay out $x_1^G + \hat{F}^G$. Clearly, the bad fund cannot imitate this because raising $F^B > PI^B\,(k = \Delta x_1)$ is impossible. Recall that it is not possible for the bad fund to borrow $F^B < F^G$ and divert $k > \Delta x_1$, since the raised amount is publicly observed and this would immediately reveal the type. As before, we focus on separating equilibria with minimal leverage.

Proposition 8 *As long as* $\bar{X}_H^G$, $\bar{X}_L^G$ *and* $\bar{X}_H^B$ *are high enough*[22] *the good fund does not monetize.*

i. *For* $c_e \in \left(\alpha\bar{e}\bar{X}_L^B, \alpha\bar{e}\left(\bar{X}_L^G - \gamma_s\bar{X}_L^B\left(1-\frac{\Delta x_1}{\tau}\right)\right)\right]$ *there exist a SEML* $e^*(s) = \bar{e}$ *for all* s*.*

ii. *For* $c_e \in \left(\alpha\bar{e}\left(\bar{X}_L^G - \gamma_s\bar{X}_L^B\left(1-\frac{\Delta x_1}{\tau}\right)\right), \alpha\bar{e}\bar{X}_L^G\right]$*, the SEML involves* $e^*(H) = \bar{e}$ *and* $e^*(L) = 0$*.*

In equilibrium, the good funds leverage the target but do not monetize, whereas bad funds do not leverage or monetize. Thus, despite the fact that leverage and monetization are both available enhancement options for the good fund, in equilibrium he chooses not to monetize. The superiority of leverage over monetization as an enhancement method arises because, when $\bar{X}_H^G$ and $\bar{X}_L^G$ are high, monetization is very costly for the fund.

While this result is qualitatively identical to Proposition 4 there are two caveats: Unlike the baseline model, we require $\bar{X}_H^B$ to be high and impose a positive lower bound on c_e. These differences are for tractability: In the observable debt model, the required borrowing of the *good* fund is driven by the debt capacity of the *bad* fund. The debt capacity of the bad fund, in turn, depends on whether he exerts effort in both states or only in the high state. For simplicity, we examine only the case where the bad type does *not* exert effort in the low state. This imposes a strictly positive lower bound on c_e. For comparability with the main analysis of the baseline model we analyze the case where non-verifiability – rather than incentive compatibility – imposes the binding constraint on the pledgable income of the bad fund in the high state. This requires that $\bar{X}_H^B$ is high enough.

It is worth noting that our assumptions that $\bar{X}_H^G$ and $\bar{X}_H^B$ are high enough are *not* payoff relevant for hedge fund investors or target shareholders: Non-verifiability implies that the payoffs to all parties other than the hedge fund manager are determined by $\bar{X}_L^\theta$ only. Thus, qualitatively, the condition that is directly payoff relevant for hedge fund investors and target shareholders is that $\bar{X}_L^G$ is high enough, in particular (using condition (22) from the proof) that

$$\bar{X}_L^G \geq \bar{X}_L^B\left(1-\frac{\Delta x_1}{\tau}\right) + \frac{w}{\gamma_s\bar{e}\left(1-\alpha\right)},$$

i.e., $\bar{X}_L^G$ is high relative to $\bar{X}_L^B$. That is, it is exactly when good funds are able to produce sufficiently higher returns for investors that investors chase flow and the induced flow competition may result in hedge fund activist efforts becoming sensitive to macroeconomic conditions.

[22] To be specific, the bounds on $\bar{X}_H^G$, $\bar{X}_L^G$ and $\bar{X}_H^B$ are given as follows: $\bar{X}_H^G$ must satisfy (21) and (23), $\bar{X}_L^G$ must satisfy (22), and $\bar{X}_H^B$ must satisfy (20).

The comparative statics of this variant of our model are also qualitatively identical to that of the baseline model. As in Corollary 1, increasing γ_s increases the range over which hedge fund activism is procyclical. Thus, better macroeconomic prospects decreases the range over which there is no debt overhang and increases the range over which there is. Implications 2-5 in the baseline model follow from the fact that the leverage necessary to separate is given by the difference in the debt capacity of the firm under the good activist (which is increasing in γ_s) and Δx_1. In this variant of the model, the leverage necessary for separation is given by the debt capacity of the firm under the bad activist conditional on the (off equilibrium) monetization of Δx_1, which is $\gamma_s \bar{e} \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right)$ (see the proof of Proposition 8). This expression is also increasing in γ_s and decreasing in Δx_1. So, implications 2-5 carry over qualitatively to this model.[23]

7 Conclusions

We propose a simple benchmark model of hedge fund activism in the presence of competition for flows. Our self-contained story helps to explain the observed procyclicality of hedge fund activism and reconciles it with the documented effect of activist hedge funds on the net leverage of their target firms. In addition, we generate some testable implications and help to resolve some ostensibly contradictory empirical evidence on the wealth effects of hedge fund activism on different stakeholders in target firms. Our paper highlights how the agency frictions arising out of the delegation of portfolio management can affect the nature of blockholder monitoring and, more broadly, may help to enrich our understanding of corporate governance issues.

8 Appendix

Proof of Proposition 1: The bad fund's incentive compatibility constraint in state $s = H$ is:

$$\alpha \left(\bar{X}_L^B - \underline{X}_L^B\right) + \left(\left(\bar{X}_H^B - \underline{X}_H^B\right) - \left(\bar{X}_L^B - \underline{X}_L^B\right)\right) > \frac{c_e}{\bar{e}}.$$

This is never satisfied given $\bar{X}_H^B - \underline{X}_H^B < c_e/\bar{e}$ (constraint 6). Due to the non-verifiability of cash flows financiers are willing to knowingly lend at most $\underline{X}_L^B$ to a bad fund. Since $\Delta x_1 > \underline{X}_L^B$ (constraint 7), a bad fund can raise enough funding to imitate the good fund only

[23] We have focussed only on separating equilibria, but there always exist regions of parameters (in particular, those where $\gamma_\theta \to 0$) that the investor's participation constraint cannot be satisfied in a pooling equilibrium, and hence such equilibria cannot exist.

if he is not being identified as being bad. In a pooling equilibrium, the break-even constraint of the financiers implies that the support of $D \in \left[D_*^{\min}, D_*^{\max}\right]$ is bounded, with $D_*^{\min} \geq 0$ and $D_*^{\max} = x_1^B + PI^P < \infty$ denotes the equilibrium funding capacity. Consider any conjecture $\left(x_1^B, x_1^G\right)$ by financiers, giving rise to an conjectured payout support of $\left[D^{\min}, D^{\max}\right]$. Even if $D_{pool}^* = D^{\max}$, $F(G) \notin (D^{\max} - x_1^G, D^{\max}]$ because otherwise the payout of the good fund is $x_1^G + F(G) > D^{\max}$ which cannot be matched by a bad fund. Thus, to avoid identification while obtaining external financing, $F(B) \notin (D^{\max} - x_1^G, D^{\max}]$. This means in turn that the maximum possible pooling equilibrium payout is $\widehat{D}^{\max} = x_1^B + F(B) \leq x_1^B + D^{\max} - x_1^G = D^{\max} - \Delta x_1$. Now, even if $D_{pool}^* = \widehat{D}^{\max}$, $F(G) \notin (\widehat{D}^{\max} - x_1^G, \widehat{D}^{\max}]$. Further iterations of the argument, carried out over all feasible conjectures by financiers, rule out the possibility of external funding in any pooling equilibrium. But without external funding a pooling equilibrium cannot exist, because the bad fund cannot mimic the good fund's minimum payout of x_1^G.■

Lemma 1 *If $D_1^*(G) \neq D_1^*(B)$, then $F^*(B) = 0$.*

Proof: If $D_1^*(G) \neq D_1^*(B)$, then $\mu_I^*\left(D_1^*(B)\right) = 0$. The constraint (6) implies that bad funds will not exert effort. Therefore, investors would never knowingly retain a bad fund. By firing a bad fund and liquidating the firm at fair prices the investor receives $\underline{X}_L^B$, whereas retaining him results in a payoff of $-w + (1-\alpha)\underline{X}_L^B$. Thus, $a_I^*\left(D_1^*(B)\right) = W$, and $F^*(B) = 0$ since choosing $F > 0$ creates an infinitesimal cost for the fund.■

Lemma 2 *If $D_1^*(G) \neq D_1^*(B)$, then $\mu_F^*(F) = 1$ for $F \in \left(0, PI^G\right]$.*

Proof: The equilibrium payout $D_1^*(G)$ can be represented as a map $f : \left(x_1^G, x_1^B\right) \rightarrow \mathbb{R}_+$. The required borrowing is therefore $F^*(G) = f\left(x_1^G, x_1^B\right) - x_1^G$. Except in the special case in which $f\left(x_1^G, x_1^B\right) - x_1^G = k$ for some $k \in \mathbb{R}$ – which by definition can only arise in equilibria in which financiers commit/coordinate to lend only specific amounts and are thus ruled out in our analysis – financiers cannot compute $F^*(G)$ before the funding request is made because they do not know x_1^G . However, since $F^*(B) = 0$ (Lemma 1), any requested amount $F \in \left(0, PI^G\right]$ is consistent with $\mu_F^*(F) = 1$.■

Proof of Proposition 2: From Lemma 2 we know that in an equilibrium with $D_1^*(G) \neq D_1^*(B)$, $\mu_F^*(F) = 1$ for $F \in \left(0, PI^G\right]$. Thus financiers are happy to invest up to PI^G. Suppose that $D_1^*(G) < x_1^B + PI^G$. Then, type B can deviate and raise $D_1^*(G) - x_1^B < PI^G$ and successfully imitate type G violating $D_1^*(G) \neq D_1^*(B)$.■

Proof of Proposition 3: Since there are four possible cash flows generated by the good type (two aggregate states crossed with project success or failure) the repayment function $R(\cdot)$ takes four possible values: $R\left(\bar{X}_L^G\right)$, $R\left(\bar{X}_H^G\right)$, $R\left(\underline{X}_L^G\right)$, and $R\left(\underline{X}_H^G\right)$ respectively. The verifiability of project success coupled with the non-verifiability of realized cash flows implies that

$$R\left(\bar{X}_L^G\right) = R\left(\bar{X}_H^G\right) := \bar{R} \text{ and } R\left(\underline{X}_L^G\right) = R\left(\underline{X}_H^G\right) := \underline{R}.$$

It also implies that in state H the hedge fund captures the incremental cash flows $\bar{X}_H^G - \bar{X}_L^G$ and $\underline{X}_H^G - \underline{X}_L^G$ conditional on success and failure respectively, since hedge fund investors cannot verify whether $s = H$ or L.

Effort exertion in state $s = L$ requires that

$$\begin{aligned} \alpha\left(\bar{e}\left(\bar{X}_L^G - \bar{R}\right) + (1-\bar{e})\left(\underline{X}_L^G - \underline{R}\right)\right) - c_e &\geq \alpha\left(\underline{X}_L^G - \underline{R}\right), \\ \text{i.e., } \alpha\bar{e}\left(\left(\bar{X}_L^G - \underline{X}_L^G\right) - \left(\bar{R} - \underline{R}\right)\right) &\geq c_e. \end{aligned} \tag{13}$$

Effort exertion in state $s = H$ requires that

$$\left(\begin{array}{c} \alpha\bar{e}\left(\bar{X}_L^G - \bar{R}\right) + \bar{e}\left(\bar{X}_H^G - \bar{X}_L^G\right) + \\ \alpha(1-\bar{e})\left(\underline{X}_L^G - \underline{R}\right) + (1-\bar{e})\left(\underline{X}_H^G - \underline{X}_L^G\right) \end{array}\right) - c_e \geq \alpha\left(\underline{X}_L^G - \underline{R}\right) + \left(\underline{X}_H^G - \underline{X}_L^G\right),$$

$$\begin{aligned} \text{i.e., } \alpha\bar{e}\left(\left(\bar{X}_L^G - \underline{X}_L^G\right) - \left(\bar{R} - \underline{R}\right)\right) + \bar{e}\left(\left(\bar{X}_H^G - \bar{X}_L^G\right) - \left(\underline{X}_H^G - \underline{X}_L^G\right)\right) &\geq c_e, \\ \text{i.e., } \alpha\bar{e}\left(\left(\bar{X}_L^G - \underline{X}_L^G\right) - \left(\bar{R} - \underline{R}\right)\right) + \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right) &\geq c_e. \end{aligned} \tag{14}$$

For arbitrarily chosen parameters, (13) and (14) are clearly most slack if $\bar{R} - \underline{R}$ is minimized. With monotonicity $\bar{R} \geq \underline{R}$. This implies that the two possible optimal financing arrangements are: If the hedge fund raises less than $\underline{X}_L^G$, we have safe debt with repayment $\bar{R} = \underline{R} < \underline{X}_L^G$. Otherwise, optimal external financing is achieved via defaultable debt with $\bar{R} > \underline{R} = \underline{X}_L^G$, i.e., the face value of debt must be $K \geq \underline{X}_L^G$.The maximum (fulfillable) face value of debt is given by $K \leq \bar{X}_L^G$.■

Proof of Proposition 4: The derivation proceeds in four steps.

Step 1: Debt Overhang in $s = L$

For a given face value of debt K debt overhang arises in state $s = L$ only if

$$\alpha\left[\bar{e}\left(\bar{X}_L^G - K\right) - \bar{e}\left(\underline{X}_L^G - \min(K, \underline{X}_L^G)\right)\right] < c_e.$$

For $K < \underline{X}_L^G$ the above reduces to $\alpha\bar{e}\left(\bar{X}_L^G - \underline{X}_L^G\right) \leq c_e$, which violates assumption (5). Thus, $K > \underline{X}_L^G$, and the maximum face value of debt associated with effort exertion in state $s = L$ is

$$\underline{K} = \bar{X}_L^G - \frac{c_e}{\alpha\bar{e}}.$$

Step 2: No Debt Overhang in $s = H$

For a given face value K, there is no debt overhang in state $s = H$ if

$$\begin{pmatrix} \alpha \left[\bar{e}\left(\bar{X}_L^G - K\right) + (1-\bar{e})\left(\underline{X}_L^G - \min(\underline{X}_L^G, K)\right)\right] \\ +\bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right) \end{pmatrix} - c_e \geq \alpha\left(\underline{X}_L^G - \min(\underline{X}_L^G, K)\right)$$

Since we look for debt levels that induce debt overhang in state $s = L$, $K > \underline{K} > \underline{X}_L^G$ so that the expression above simplifies to:

$$\alpha\bar{e}\left(\bar{X}_L^G - K\right) + \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right) - c_e \geq 0,$$

which gives us

$$K \leq \bar{X}_L^G - \frac{c_e}{\alpha\bar{e}} + \frac{1}{\alpha}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right).$$

If

$$c_e \leq \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right)$$

then the relevant constraint for K is

$$K \leq \bar{X}_L^G,$$

because of the non-verifiability of macro states. Assumption (5) guarantees that

$$c_e \leq \alpha\bar{e}\left(\bar{X}_L^G - \underline{X}_L^G\right).$$

Thus, if

$$\begin{aligned} \alpha\bar{e}\left(\bar{X}_L^G - \underline{X}_L^G\right) &< \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right), \\ \text{i.e., } \left(\bar{X}_H^G - \underline{X}_H^G\right) &\geq (1+\alpha)\left(\bar{X}_L^G - \underline{X}_L^G\right), \end{aligned}$$

then, under Assumption (5) the relevant constraint for K is always

$$K \leq \bar{X}_L^G.$$

and

$$\bar{K} = \bar{X}_L^G.$$

Step 3: Pledgeable Income PI^G

To derive the conditions under which pledgable income is higher, we compare the maximum pledgable income with debt $\underline{K}$ and the one with debt $\bar{K}$. Without debt overhang in state $s = L$ pledgeable income is equal to

$$\bar{e}\underline{K} + (1-\bar{e})\,\underline{X}_L^G.$$

Inserting $\underline{K} = \bar{X}_L^G - c_e/\alpha\bar{e}$ yields the maximum pledgeable income $PI_{\underline{K}}^G$:

$$PI_{\underline{K}}^G = \bar{e}\left(\bar{X}_L^G - \frac{c_e}{\alpha\bar{e}}\right) + (1-\bar{e})\,\underline{X}_L^G.$$

With debt overhang in state $s = L$ pledgable income is equal to

$$\gamma_s\bar{e}\bar{K} + (1-\gamma_s\bar{e})\,\underline{X}_L^G.$$

Inserting the expression for $\bar{K} = \bar{X}_L^G$ yields the maximum pledgeable income $PI_{\bar{K}}^G$:

$$PI_{\bar{K}}^G = \gamma_s\bar{e}\bar{X}_L^G + (1-\gamma_s\bar{e})\,\underline{X}_L^G.$$

Then $PI_{\bar{K}}^G > PI_{\underline{K}}^G$ is equivalent to

$$c_e \geq (1-\gamma_s)\,\alpha\bar{e}\left(\bar{X}_L^G - \underline{X}_L^G\right).$$

Thus, for $c_e \in \left(0, (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$ the maximum pledgeable income is $PI_{\underline{K}}^G$ (Case A.1), while for $c_e \in \left[(1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right], \alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right]$, the maximum pledgeable income is $PI_{\bar{K}}^G$ (Case A.2).

Case A.1: $c_e \in \left(0, (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$

Step 4 for A.1: Funding amount for $PI_{\bar{K}}^G < PI_{\underline{K}}^G$

Proposition 2 implies that separation requires borrowing of

$$PI_{\underline{K}}^G - \Delta x_1 = \bar{e}\left(\bar{X}_L^G - \frac{c_e}{\alpha\bar{e}}\right) + (1-\bar{e})\,\underline{X}_L^G - \Delta x_1,$$

and the corresponding face value K^{**} solves

$$\bar{e}\left(\bar{X}_L^G - \frac{c_e}{\alpha\bar{e}}\right) + (1-\bar{e})\,\underline{X}_L^G - \Delta x_1 = \bar{e}K^{**} + (1-\bar{e})\min(K^{**}, \underline{X}_L^G). \tag{15}$$

Suppose $K^{**} > \underline{X}_L^G$, then $\min(K^{**}, \underline{X}_L^G) = \underline{X}_L^G$, in which case (15) gives:

$$K^{**} = \bar{X}_L^G - \frac{c_e}{\alpha\bar{e}} - \frac{\Delta x_1}{\bar{e}},$$

which is clearly smaller than $\underline{K} = \bar{X}_L^G - \frac{c_e}{\alpha\bar{e}}$ so that there is indeed no debt overhang in state $s = L$. Furthermore, the condition $\bar{X}_L^G > \underline{X}_L^G + \frac{\Delta x_1}{\gamma_s(1-\gamma_s)\bar{e}}$ in Proposition 4 ensures that $K^{**} > \underline{X}_L^G$. Indeed, a sufficient condition for $K^{**} > \underline{X}_L^G$ for all $c_e \in \left(0, (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$ is that

$$\bar{X}_L^G - \frac{c_e}{\alpha\bar{e}} - \frac{\Delta x_1}{\bar{e}} > \underline{X}_L^G$$

for $c_e = (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]$. This in turn, is equivalent to:

$$\bar{X}_L^G - \underline{X}_L^G > \frac{\Delta x_1}{\gamma_s\bar{e}} \tag{16}$$

which always holds since $\bar{X}_L^G - \underline{X}_L^G > \frac{\Delta x_1}{\gamma_s(1-\gamma_s)\bar{e}} > \frac{\Delta x_1}{\gamma_s \bar{e}}$.

It remains to check that it is in the investor's interest to retain a good hedge fund. Retaining the good fund generates a continuation payoff equal to

$$(1-\alpha)\,\bar{e}\left(\bar{X}_L^G - K^{**}\right) - w,$$

which does not depend on the aggregate state due to a combination of (i) no debt overhang and (ii) non verifiability of the macro state. Liquidating the fund/firm results in a payoff of $\max\left(\underline{X}_L^G - K^{**}, 0\right) = 0$. Thus retention requires:

$$(1-\alpha)\left(\frac{c_e}{\alpha} + \Delta x_1\right) - w \geq 0 \tag{17}$$

which is clearly always satisfied given $\Delta x_1 > \frac{w}{1-\alpha}$. This concludes the proof of the proposition for constellation A.1.

Case A.2: $c_e \in \left[(1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right], \alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right]$

Step 4 for A.2: Funding amount given that $PI_{\bar{K}}^G > PI_{\underline{K}}^G$

Separation requires borrowing of

$$PI_{\bar{K}}^G - \Delta x_1 = \gamma_s\bar{e}\bar{X}_L^G + (1-\gamma_s\bar{e})\,\underline{X}_L^G - \Delta x_1,$$

and the corresponding face value K^* is obtained by setting

$$\gamma_s\bar{e}\bar{X}_L^G + (1-\gamma_s\bar{e})\,\underline{X}_L^G - \Delta x_1 = \gamma_s\bar{e}K^* + (1-\gamma_s\bar{e})\,\underline{X}_L^G,$$

giving

$$K^* = \frac{\gamma_s\bar{e}\bar{X}_L^G - \Delta x_1}{\gamma_s\bar{e}} = \bar{X}_L^G - \frac{\Delta x_1}{\gamma_s\bar{e}}.$$

For consistency we need $K^* > \underline{K}$, i.e.,

$$\bar{X}_L^G - \frac{\Delta x_1}{\gamma_s\bar{e}} > \bar{X}_L^G - \frac{c_e}{\alpha\bar{e}},$$

i.e.,

$$\Delta x_1 < \frac{\gamma_s}{\alpha}c_e$$

Since $c_e \geq (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]$, the constraint above is always satisfied given

$$\bar{X}_L^G - \underline{X}_L^G > \frac{\Delta x_1}{\gamma_s(1-\gamma_s)\bar{e}}. \tag{18}$$

It remains to check that it is in the investor's interest to retain a good hedge fund. Liquidating the fund/firm results in a payoff equal of

$$(1-\alpha)\left(\gamma_s\left(\bar{e}\left(\bar{X}_L^G - K^*\right) + (1-\bar{e})\max\left(\underline{X}_L^G - K^*, 0\right)\right) + (1-\gamma_s)\max\left(\underline{X}_L^G - K^*, 0\right)\right) - w,$$

Liquidating the fund/firm results in a payoff of

$$\max\left(\underline{X}_L^G - K^*, 0\right).$$

Since $K^* = \bar{X}_L^G - \frac{\Delta x_1}{\gamma_s \bar{e}} > \underline{K} > \underline{X}_L^G$, the investor retains the good fund if:

$$(1-\alpha)\,\gamma_s\bar{e}\left(\bar{X}_L^G - \bar{X}_L^G + \frac{\Delta x_1}{\gamma_s \bar{e}}\right) - w \geq 0 \tag{19}$$

which is clearly satisfied given $\Delta x_1 > \frac{w}{(1-\alpha)}$. This concludes the proof of the proposition for case A.2.■

The consequences of borrowing to separate for Case B

When $\left(\bar{X}_L^G - \underline{X}_L^G\right) < \left(\bar{X}_H^G - \underline{X}_H^G\right) < (1+\alpha)\left(\bar{X}_L^G - \underline{X}_L^G\right)$, there are two possibilities: For $c_e \leq \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right)$, $\bar{K} = \bar{X}_L^G$, while for $c_e > \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right)$, $\bar{K} = \bar{X}_L^G - \frac{c_e}{\alpha\bar{e}} + \frac{1}{\alpha}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right)$.

For $c_e \leq \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]$, $\bar{K} = \bar{X}_L^G$ while $\underline{K} = \bar{X}_L^G - \frac{c_e}{\alpha\bar{e}}$ as before. Consequently,

$$PI_{\underline{K}}^G = \bar{e}\left(\bar{X}_L^G - \frac{c_e}{\alpha\bar{e}}\right) + (1-\bar{e})\,\underline{X}_L^G$$

and

$$PI_{\bar{K}}^G = \gamma_s\bar{e}\bar{X}_L^G + (1-\gamma_s\bar{e})\,\underline{X}_L^G$$

As in case **A1)**, the condition for $PI_{\bar{K}}^G \geq PI_{\underline{K}}^G$ is

$$c_e \geq (1-\gamma_s)\alpha\bar{e}[\bar{X}_L^G - \underline{X}_L^G]$$

Since $c_e \leq \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]$, this condition can only be satisfied if

$$\begin{aligned}(1-\gamma_s)\alpha\bar{e}[\bar{X}_L^G - \underline{X}_L^G] &\leq \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right] \\ \gamma_s &\geq 1 - \frac{1}{\alpha}\left[\frac{\bar{X}_H^G - \underline{X}_H^G}{\bar{X}_L^G - \underline{X}_L^G} - 1\right] := \tilde{\gamma}_s.\end{aligned}$$

Note that $\tilde{\gamma}_s \to 0$ as $\frac{\bar{X}_H^G - \underline{X}_H^G}{\bar{X}_L^G - \underline{X}_L^G} \to 1+\alpha$ and $\tilde{\gamma}_s \to 1$ as $\frac{\bar{X}_H^G - \underline{X}_H^G}{\bar{X}_L^G - \underline{X}_L^G} \to 1$ so $\gamma_s \in [0,1]$. Thus, for $\gamma_s < \tilde{\gamma}_s$ the maximum pledgeable income is $PI_{\underline{K}}^G$ for all $c_e \in \left(0, \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]\right)$. For $\gamma_s \geq \tilde{\gamma}_s$, the maximum pledgeable income is $PI_{\underline{K}}^G$ for $c_e \in \left(0, (1-\gamma_s)\alpha\bar{e}[\bar{X}_L^G - \underline{X}_L^G]\right)$ and $PI_{\bar{K}}^G$ for $c_e \in \left((1-\gamma_s)\alpha\bar{e}[\bar{X}_L^G - \underline{X}_L^G], \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]\right)$. To ensure debt overhang in the latter case, the face value associated with raising $F = PI_{\bar{K}}^G - \Delta x_1$ has to be larger than $\underline{K}$. As shown in case A.2 (step 4) above, this holds for $\Delta x_1 < \frac{\gamma_s}{\alpha}c_e$ which is again guaranteed by (18).

For $c_e \in \left(\bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right], \alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right]$, $\underline{K} = \bar{X}_L^G - \frac{c_e}{\alpha\bar{e}}$ as before and $\bar{K} = \bar{X}_L^G - \frac{c_e}{\alpha\bar{e}} + \frac{1}{\alpha}\left((\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right)$. Consequently,

$$PI_{\underline{K}}^G = \bar{e}\left(\bar{X}_L^G - \frac{c_e}{\alpha\bar{e}}\right) + (1-\bar{e})\,\underline{X}_L^G$$

and

$$PI_{\bar{K}}^G = \gamma_s\bar{e}\left[\bar{X}_L^G - \frac{c_e}{\alpha\bar{e}} + \frac{1}{\alpha}\left((\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right)\right] + (1-\gamma_s\bar{e})\,\underline{X}_L^G$$

Hence, $PI_{\bar{K}}^G \geq PI_{\underline{K}}^G$ holds if

$$\gamma_s\bar{e}\left[\bar{X}_L^G - \frac{c_e}{\alpha\bar{e}} + \frac{1}{\alpha}\left(\begin{array}{c}(\bar{X}_H^G - \underline{X}_H^G)\\ -(\bar{X}_L^G - \underline{X}_L^G)\end{array}\right)\right] + (1-\gamma_s\bar{e})\,\underline{X}_L^G \geq \bar{e}\left(\bar{X}_L^G - \frac{c_e}{\alpha\bar{e}}\right) + (1-\bar{e})\,\underline{X}_L^G$$

i.e.,

$$\gamma_s \geq \frac{\bar{X}_L^G - \underline{X}_L^G - \frac{c_e}{\alpha\bar{e}}}{\frac{1}{\alpha}\left((\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right) + \left(\bar{X}_L^G - \underline{X}_L^G - \frac{c_e}{\alpha\bar{e}}\right)} := \hat{\gamma}_s \in (0,1).$$

Thus, in the range $c_e \in \left(\bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right], \alpha\bar{e}(\bar{X}_L^G - \underline{X}_L^G)\right)$ the maximum pledgeable income is $PI_{\underline{K}}^G$ for $\gamma_s < \hat{\gamma}_s$ and $PI_{\bar{K}}^G$ for $\gamma_s \geq \hat{\gamma}_s$. To ensure debt overhang in the latter case, the face value associated with raising $F = PI_{\bar{K}}^G - \Delta x_1$ has to be larger than $\underline{K}$. As shown in case A.2 (step 4) above, this holds for $\Delta x_1 < \frac{\gamma_s}{\alpha}c_e$ which is again guaranteed by (18).

We now establish that $\tilde{\gamma}_s \geq \hat{\gamma}_s$. Suppose the reverse were true, i.e., $\tilde{\gamma}_s < \hat{\gamma}_s$ and consider $\gamma_s \in (\tilde{\gamma}_s, \hat{\gamma}_s)$ and effort costs immediately to the left and right of the threshold $\bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]$. Since $\gamma_s > \tilde{\gamma}_s$, for $c_e = \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right] - \epsilon$ for some small $\epsilon > 0$, $PI_{\bar{K}}^G > PI_{\underline{K}}^G$. Yet, since $\gamma_s < \hat{\gamma}_s$, for $c_e = \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right] + \epsilon$, $PI_{\bar{K}}^G < PI_{\underline{K}}^G$. Note that $PI_{\underline{K}}^G$ is given by $\bar{e}\left(\bar{X}_L^G - \frac{c_e}{\alpha\bar{e}}\right) + (1-\bar{e})\,\underline{X}_L^G$ for all c_e and decreases in c_e at the rate $1/\alpha$.

In contrast, for $c_e \in \left[\bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right] - \epsilon, \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]\right)$, $PI_{\bar{K}}^G$ is given by $\gamma_s\bar{e}\bar{X}_L^G + (1-\gamma_s\bar{e})\,\underline{X}_L^G$ which is invariant with c_e. For $c_e \in \left(\bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right], \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right] + \epsilon\right]$, $PI_{\bar{K}}^G$ is given by $\gamma_s\bar{e}\left[\bar{X}_L^G - \frac{c_e}{\alpha\bar{e}} + \frac{1}{\alpha}\left((\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right)\right] + (1-\gamma_s\bar{e})\,\underline{X}_L^G$ which decreases in c_e at the rate γ_s/α, i.e., more slowly than $PI_{\underline{K}}^G$ in the same interval. Thus if $PI_{\bar{K}}^G > PI_{\underline{K}}^G$ for $c_e = \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right] - \epsilon$ it must also be true that $PI_{\bar{K}}^G > PI_{\underline{K}}^G$ for $c_e = \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right] + \epsilon$, a contradiction.

To summarize our findings, we have three regions in terms of γ_s:

1. If $\gamma_s < \hat{\gamma}_s$, then $PI_{\bar{K}}^G < PI_{\underline{K}}^G$ for the full relevant range of c_e and there is no debt overhang.

2. If $\hat{\gamma}_s \leq \gamma_s < \tilde{\gamma}_s$, then for $c_e \in \left(0, \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]\right)$ we have $PI_{\bar{K}}^G < PI_{\underline{K}}^G$ and no debt overhang, while for $c_e \in \left(\bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right], \alpha\bar{e}(\bar{X}_L^G - \underline{X}_L^G)\right)$ we have $PI_{\bar{K}}^G > PI_{\underline{K}}^G$ and debt overhang.

3. If $\tilde{\gamma}_s \leq \gamma_s$, then for $c_e \in \left(0, (1-\gamma_s)\alpha\bar{e}[\bar{X}_L^G - \underline{X}_L^G]\right)$ we have $PI_{\bar{K}}^G < PI_{\underline{K}}^G$ and no debt overhang, while for $c_e \in \left((1-\gamma_s)\alpha\bar{e}[\bar{X}_L^G - \underline{X}_L^G], \alpha\bar{e}(\bar{X}_L^G - \underline{X}_L^G)\right)$ we have $PI_{\bar{K}}^G > PI_{\underline{K}}^G$ and debt overhang.

It remains to check that it is in the investor's interest to retain a good fund. In all three regions of γ_s where $PI_{\underline{K}}^G > PI_{\bar{K}}^G$ the analysis of the retention decision is identical to case A.1 (step 4). In the regions $\gamma_s < \hat{\gamma}_s$ and $\gamma_s \geq \tilde{\gamma}_s$ where $PI_{\bar{K}}^G > PI_{\underline{K}}^G$ the constraint $\bar{K} = \bar{X}_L^G$ binds, and the analysis of the retention decision is identical to case A.2. (step 4). In the region $\gamma_s \in [\hat{\gamma}_s, \tilde{\gamma}_s)$ where $PI_{\bar{K}}^G > PI_{\underline{K}}^G$ the constraint $\bar{K} = \bar{X}_L^G - \frac{c_e}{\alpha\bar{e}} + \frac{1}{\alpha}\left((\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right)$ binds. The corresponding face value of debt K^{***} is obtained by setting

$$\begin{aligned}\gamma_s\bar{e}\left[\bar{X}_L^G - \tfrac{c_e}{\alpha\bar{e}} + \tfrac{1}{\alpha}\left((\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right)\right] \\ + (1-\gamma_s\bar{e})\,\underline{X}_L^G - \Delta x_1\end{aligned} = \gamma_s\bar{e}K^{***} + (1-\gamma_s\bar{e})\,\underline{X}_L^G,$$

giving

$$K^{***} = \bar{X}_L^G - \frac{c_e}{\alpha\bar{e}} + \frac{1}{\alpha}\left((\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right) - \frac{\Delta x_1}{\gamma_s\bar{e}}.$$

Hence, the investor's payoff from retaining the fund is

$$(1-\alpha)\left[\bar{X}_L^G - \left(\bar{X}_L^G - \frac{c_e}{\alpha\bar{e}} + \frac{1}{\alpha}\left((\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right) - \frac{\Delta x_1}{\gamma_s\bar{e}}\right)\right] - w$$

and retention is in the investor's interest if

$$\left[\frac{c_e}{\alpha\bar{e}} - \frac{1}{\alpha}\left((\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right) + \frac{\Delta x_1}{\gamma_s\bar{e}}\right] \geq \frac{w}{(1-\alpha)}$$

Since $c_e > \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]$, this condition is satisfied given $\Delta x_1 > \frac{w}{(1-\alpha)}$. This concludes the analysis of the consequences of borrowing to separate for case B.■

Investor participation constraint for $c_e \in \left(0, (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$:
Since (as shown in the proof of Proposition 4) $K^{**} > \underline{X}_L^G$, if the investors invest $1+w$ in the hedge fund (w is used for fees and 1 is invested in the block) then they receive the following expected payoffs:

$$\gamma_\theta\left[E\left(x_1^G\right) + \bar{e}\left(\bar{X}_L^G - \frac{c_e}{\alpha\bar{e}}\right) + (1-\bar{e})\,\underline{X}_L^G - \Delta x_1 - w + (1-\alpha)\left(\frac{c_e}{\alpha} + \Delta x_1\right)\right] + (1-\gamma_\theta)\,\underline{X}_L^B.$$

Hence, participation requires

$$\begin{aligned}\gamma_\theta\left[E\left(x_1^G\right) + \bar{e}\left(\bar{X}_L^G - \tfrac{c_e}{\alpha\bar{e}}\right) + (1-\bar{e})\,\underline{X}_L^G - \Delta x_1 - w + (1-\alpha)\left(\tfrac{c_e}{\alpha} + \Delta x_1\right)\right] \\ + (1-\gamma_\theta)\,\underline{X}_L^B > 1 + w\end{aligned}$$

which is clearly satisfied as long as $\bar{X}_L^G$ is high enough.

Proof of Proposition 5: To separate, the good fund must pay out enough to prevent mimicking by the bad fund. The good fund always prefers to pay out liquid assets Y_0 in the first period (that would anyway go to creditors in the second period) because, holding fixed the separation payout, replacing the paying out of Y_0 with additional borrowing is costly: For each dollar borrowed the good fund must pay back either $1/\gamma_s\bar{e}$ (if debt overhang arises) or $1/\bar{e}$ (otherwise) in the second period. Both are costly to the hedge fund's payoff, as it receives a second period carry. This establishes that Y_0 is fully paid out in any separating equilibrium. The remaining steps mirror those of the proof of Proposition 4, and are thus stated in brief.

Given pre-existing debt K_0 and all liquid assets Y_0 paid out, there is debt overhang in $s = L$ if the face value of debt satisfies $K > \underline{K}_{K_0} \equiv \bar{X}_L^G - K_0 - \frac{c_e}{\alpha\bar{e}}$, and no debt overhang in $s = H$ if $K < \bar{X}_L^G - K_0 - \frac{c_e}{\alpha\bar{e}} + \frac{1}{\alpha}\left(\bar{X}_H^G - \bar{X}_L^G\right)$. As before, non-verifiability imposes an upper bound $K \leq \bar{K}_{K_0} \equiv \bar{X}_L^G - K_0$. As in the leading Case A of the baseline analysis, as long as $\bar{X}_H^G \geq (1+\alpha)\bar{X}_L^G$, it is this latter constraint which binds. We restrict attention to this case. For $c_e \in \left[(1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - K_0\right], \alpha\bar{e}\bar{X}_L^G\right]$, it is easy to check that $PI^G_{\bar{K}_{K_0}} > PI^G_{\underline{K}_{K_0}}$. Thus, separation requires an amount of borrowing equal to $PI^G_{\bar{K}_{K_0}} - \Delta x_1 = \gamma_s\bar{e}\left(\bar{X}_L^G - K_0\right) - \Delta x_1$, with corresponding face value $K^*_{K_0} = \bar{X}_L^G - K_0 - \frac{\Delta x_1}{\gamma_s\bar{e}}$. For consistency we need $K^*_{K_0} > \underline{K}_{K_0}$, which is always satisfied as long as $\bar{X}_L^G - K_0 > \frac{\Delta x_1}{\gamma_s(1-\gamma_s)\bar{e}}$, which is a very similar condition to the baseline model.

Next we check that the investor wants to retain a good hedge fund. Since w paid at $t = 1$ is sunk and the investor has already received $D_1^* = x_1^G + Y_0 + \gamma_s\bar{e}\left(\bar{X}_L^G - K_0\right) - \Delta x_1$, the investor retains the good fund if $(1-\alpha)\gamma_s\bar{e}\left(\bar{X}_L^G - K_0 - K^*_{K_0}\right) \geq w$, i.e., if $(1-\alpha)\Delta x_1 > w$ as in the baseline model. Note that for $c_e \in \left[(1-\gamma_s)\alpha\bar{e}\bar{X}_L^G, \alpha\bar{e}\bar{X}_L^G\right]$, if $K_0 = 0$, and Y_0 is paid out in the first period, the analysis of the baseline model implies that debt overhang arises in the low state in the SEML. Since

$$\left[(1-\gamma_s)\alpha\bar{e}\bar{X}_L^G, \alpha\bar{e}\bar{X}_L^G\right] \subset \left[(1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - K_0\right], \alpha\bar{e}\bar{X}_L^G\right],$$

we can conclude that for $c_e \in \left[(1-\gamma_s)\alpha\bar{e}\bar{X}_L^G, \alpha\bar{e}\bar{X}_L^G\right]$, for $\bar{X}_L^G$ and Δx_1 large enough debt overhang arises in the low state in the SEML in levered and unlevered target firms.

Finally, we can compare (i) the payoffs to equity holders in firms with and without pre-existing debt in the presence of hedge fund activists and (ii) the payoffs to pre-existing creditors in levered target firms in the presence and absence of hedge fund activists.

(i) **Payoffs to equity holders:** With pre-existing leverage of K_0, target shareholders receive an expected payoff of

$$\gamma_\theta\left(E\left(x_1^G\right) + Y_0 + \gamma_s\bar{e}\left(\bar{X}_L^G - K_0\right) - \Delta x_1\right) + (1-\gamma_\theta)E\left(x_1^B\right)$$

in the first period and $\gamma_\theta \Delta x_1$ in the second period. Without leverage, target shareholders receive an expected payoff of

$$\gamma_\theta \left(E\left(x_1^G\right) + Y_0 + \gamma_s \bar{e} \bar{X}_L^G - \Delta x_1\right) + (1-\gamma_\theta) E\left(x_1^B\right)$$

in the first period and $\gamma_\theta \Delta x_1$ in the second period. Thus, leverage reduces first period payoffs to target shareholders without affecting second period payoffs.
(ii) **Payoffs to pre-existing creditors:** In the absence of the hedge fund activists, creditors would have expected to receive K_0 in the second period in either state (since $Y_0 > K_0$). In the presence of hedge fund activists, the same creditors can expect to receive K_0 in the second period in the high state with probability $\bar{e}$ but nothing otherwise. Thus, the presence of activist hedge funds expropriates pre-existing creditors.■

Proof of Proposition 6: To separate, the good type has to pay out $D_1^*(G) = x_1^B + Y_0$ and can therefore retain at most Δx_1 liquid assets. Given $K_0 > \Delta x_1$, the incentive compatibility constraint in state $s = L$

$$\alpha \bar{e}(\bar{X}_L^G - (K_0 - \Delta x_1)) > c_e$$

is violated for $c_e \in \left(\alpha \bar{e}\left[\bar{X}_L^G - (K_0 - \Delta x_1)\right], \alpha \bar{e} \bar{X}_L^G\right]$. By contrast, it is easy to see that the incentive compatibility constraint in state $s = H$

$$\alpha \bar{e}\left(\bar{X}_L^G - (K_0 - \Delta x_1)\right) + \bar{e}\left(\bar{X}_H^G - \bar{X}_L^G\right) \geq c_e$$

is slack provided that $\bar{X}_H^G > (1+\alpha)\bar{X}_L^G$.■

Proof of Proposition 8: We begin by assuming that

$$\left(\bar{X}_H^B - \bar{X}_L^B\right)\left(1 - \frac{\bar{k}}{\tau}\right) > \alpha \bar{X}_L^G, \tag{20}$$

which implies that

$$\left(\bar{X}_H^B - \bar{X}_L^B\right)\left(1 - \frac{\bar{k}}{\tau}\right) > \frac{c_e}{\bar{e}} \text{ for all } c_e \leq \alpha \bar{e} \bar{X}_L^G.$$

As will become clear later, this formalizes the sense in which we need $\bar{X}_H^B$ to be big enough and effectively restricts us to the equivalent of case A in the baseline model.

First we compute the debt capacity of the bad type, PI^B. Since leverage is observable, mimicking requires that $k = \Delta x_1$. Given $c_e > \alpha \bar{e} \bar{X}_L^B$, the bad type does not make an effort in state $s = L$ and his debt capacity is determined by his potential output in state $s = H$. The face value K^B that makes the bad type indifferent between exerting effort in state $s = H$ is determined by

$$\alpha \bar{e}\left(\bar{X}_L^B\left(1 - \frac{\Delta x_1}{\tau}\right) - K^B\right) + \bar{e}\left(\bar{X}_H^B - \bar{X}_L^B\right)\left(1 - \frac{\Delta x_1}{\tau}\right) - c_e = 0,$$

i.e.,

$$K^B = \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right) + \frac{1}{\alpha}\left(\bar{X}_H^B - \bar{X}_L^B\right)\left(1 - \frac{\Delta x_1}{\tau}\right) - \frac{c_e}{\alpha \bar{e}}.$$

However, the non-verifiability of macro states implies that

$$K^B \leq \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right).$$

As long as

$$\left(\bar{X}_H^B - \bar{X}_L^B\right)\left(1 - \frac{\Delta x_1}{\tau}\right) > \frac{c_e}{\bar{e}},$$

which is guaranteed by (20), the latter constraint is binding and the bad fund's debt capacity is

$$PI^B\left(k = \Delta x_1\right) = \gamma_s \bar{e} \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right).$$

Consequently, the good type has to borrow $\gamma_s \bar{e} \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right) + \epsilon$ in the first period to separate, and there are two possibilities:

Case 1: Borrowing $\gamma_s \bar{e} \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right)$ induces debt overhang in state $s = L$.

Case 2: Borrowing $\gamma_s \bar{e} \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right)$ does not induce debt overhang in state $s = L$.

Case 1

With debt overhang in state $s = L$, the face value $\bar{K}^G$ associated with borrowing $\gamma_s \bar{e} \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right)$ solves:

$$\gamma_s \bar{e} \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right) = \gamma_s \bar{e} \bar{K}^G + \gamma_s \left(1 - \bar{e}\right) 0 + \left(1 - \gamma_s\right) 0.$$

Consistency requires that $\bar{K}^G = \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right)$ leads to debt overhang in state $s = L$ but not in state $s = H$. The former implies

$$\alpha \bar{e} \left(\bar{X}_L^G - \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right)\right) < c_e.$$

Since $\alpha \bar{e} \left(\bar{X}_L^G - \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right)\right) < \alpha \bar{e} \bar{X}_L^G$ for any $\bar{X}_L^G$, the constraints on $\bar{X}_L^G$ below do not affect the existence of a positive measure of effort costs $\left(\alpha \bar{e} \left(\bar{X}_L^G - \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right)\right), \alpha \bar{e} \bar{X}_L^G\right]$ for which borrowing $\gamma_s \bar{e} \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right)$ induces debt overhang in state $s = L$. Effort exertion in $s = H$ requires that

$$\alpha \bar{e} \left(\bar{X}_L^G - \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right)\right) + \bar{e}\left(\bar{X}_H^G - \bar{X}_L^G\right) \geq c_e,$$

i.e.,

$$\bar{X}_H^G \geq \left(1 - \alpha\right) \bar{X}_L^G + \alpha \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right) + \frac{c_e}{\bar{e}},$$

which can be guaranteed by the following condition:

$$\bar{X}_H^G \geq (1-\alpha)\,\bar{X}_L^G + \alpha \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right) + \alpha \bar{X}_L^G = \bar{X}_L^G + \alpha \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right). \tag{21}$$

A good fund separates only if investors retain a fund that separates. Since the payoff from closing the fund down is 0 (since $\underline{X}_s^\theta = 0$ for all θ, s), investors retain the fund if

$$-w + (1-\alpha)\,\gamma_s \bar{e} \left(\bar{X}_L^G - \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right)\right) \geq 0,$$

i.e.,

$$\bar{X}_L^G \geq \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right) + \frac{w}{\gamma_s \bar{e}\,(1-\alpha)}. \tag{22}$$

Finally, it must be verified that the good fund prefers to use leverage to monetization.[24] For any monetization, leverage combination (k_G, L_G) by the good type the bad type will aim to imitate by choosing $(k_B = k_G + \Delta x_1, L_B = L_G)$. Thus, unless type G sets $k_G > \bar{k} - \Delta x_1$, his only option is to separate using leverage, and thus have a monetization-leverage combination of $\left(k_G, L_G = PI^B\,(k_G + \Delta x_1) + \epsilon\right)$. Above we have solved for the case where $k_G = 0$, and now examine whether a good fund can realize a higher payoff by choosing $k_G \in \left(0, \bar{k} - \Delta x_1\right)$ combined with the corresponding separating leverage. Increasing k_G gives raise to two conflicting effects: On the one hand a larger k_G destroys cash flows, thereby reducing the good fund's payoff. On the other hand, a larger k_G raises $k_B = k_G + \Delta x_1$ which reduces the pledgable income of the bad fund and thus the leverage required for separation which in turn increases the good fund's payoff.

We start with $\left(k_G = 0, L_G = PI^B(k = \Delta x_1\right)$ and increase k_G slightly to $k > 0$, assuming that there is still enough leverage to generate debt overhang in state $s = L$. Arguments that parallel the computation of $PI^B\,(k = \Delta x_1)$ above imply that, as long as

$$\left(\bar{X}_H^B - \bar{X}_L^B\right)\left(1 - \frac{k + \Delta x_1}{\tau}\right) > \frac{c_e}{\bar{e}},$$

which is guaranteed by (20), $K^B \leq \bar{X}_L^B \left(1 - \frac{k+\Delta x_1}{\tau}\right)$ binds and

$$PI^B\,(k + \Delta x_1) = \gamma_s \bar{e} \bar{X}_L^B \left(1 - \frac{k + \Delta x_1}{\tau}\right).$$

Given this amount of borrowing leads to debt overhang in state $s = L$, the corresponding face value K^G is

$$K^G = \bar{X}_L^B \left(1 - \frac{k + \Delta x_1}{\tau}\right).$$

[24] Since leverage is publicly observable, another way to rule out monetization is to choose off equilibrium beliefs suitably. However, the argument here shows that we do not need to resort to off equilibrium beliefs to rule out monetization by the good fund.

Under this deviation, the expected payoff to the good fund is:

$$\alpha\gamma_s\bar{e}\left(\bar{X}_L^G\left(1-\frac{k}{\tau}\right)-\bar{X}_L^B\left(1-\frac{k+\Delta x_1}{\tau}\right)\right)+\gamma_s\bar{e}\left(\bar{X}_H^G-\bar{X}_L^G\right)\left(1-\frac{k}{\tau}\right)-\gamma_s c_e.$$

In contrast, in equilibrium, the expected payoff to the good fund is:

$$\alpha\gamma_s\bar{e}\left(\bar{X}_L^G-\bar{X}_L^B\left(1-\frac{\Delta x_1}{\tau}\right)\right)+\gamma_s\bar{e}\left(\bar{X}_H^G-\bar{X}_L^G\right)-\gamma_s c_e.$$

Thus, the deviation is unprofitable as long as:

$$\gamma_s\bar{e}\frac{k}{\tau}\left[\alpha\left(\bar{X}_L^G-\bar{X}_L^B\right)+\left(\bar{X}_H^G-\bar{X}_L^G\right)\right]>0,$$

which is always true.

Now, consider a larger increase $\hat{k}$ such that, due to the reduction in PI^B, the implied face value of debt does not lead to debt overhang in state $s=L$ for the good fund, while the bad fund still does not exert effort in state $s=L$. As before, given condition (20), the pledgeable income of the bad fund is given by

$$PI^B\left(\hat{k}+\Delta x_1\right)=\gamma_s\bar{e}\bar{X}_L^B\left(1-\frac{\hat{k}+\Delta x_1}{\tau}\right),$$

while the corresponding face value of debt solves

$$\gamma_s\bar{e}\bar{X}_L^B\left(1-\frac{\hat{k}+\Delta x_1}{\tau}\right)=\gamma_s\bar{e}K^G+\left(1-\gamma_s\right)\bar{e}K^G,$$

i.e.,

$$K^G=\gamma_s\bar{X}_L^B\left(1-\frac{\hat{k}+\Delta x_1}{\tau}\right)$$

Under this deviation, the expected payoff to the good fund is:

$$\alpha\bar{e}\left(\bar{X}_L^G\left(1-\frac{\hat{k}}{\tau}\right)-\gamma_s\bar{X}_L^B\left(1-\frac{\hat{k}+\Delta x_1}{\tau}\right)\right)+\gamma_s\bar{e}\left(\bar{X}_H^G-\bar{X}_L^G\right)\left(1-\frac{\hat{k}}{\tau}\right)-c_e.$$

In contrast, in equilibrium, the expected payoff to the good fund is:

$$\alpha\gamma_s\bar{e}\left(\bar{X}_L^G-\bar{X}_L^B\left(1-\frac{\Delta x_1}{\tau}\right)\right)+\gamma_s\bar{e}\left(\bar{X}_H^G-\bar{X}_L^G\right)-\gamma_s c_e.$$

Thus, the deviation is unprofitable as long as:

$$\alpha\bar{e}\left(\bar{X}_L^G\left(\gamma_s-\left(1-\frac{\hat{k}}{\tau}\right)\right)-\gamma_s\bar{X}_L^B\frac{\hat{k}}{\tau}\right)+\gamma_s\bar{e}\left(\bar{X}_H^G-\bar{X}_L^G\right)\frac{\hat{k}}{\tau}+\left(1-\gamma_s\right)c_e>0, \tag{23}$$

which holds as long as $\bar{X}_H^G$ is large enough.

Since the bad fund never exerts effort in state $s = L$ the set of cases considered so far is exhaustive. Thus, when $c_e \in \left(\alpha\bar{e}\left(\bar{X}_L^G - \bar{X}_L^B\left(1-\frac{\Delta x_1}{\tau}\right)\right), \alpha\bar{e}\bar{X}_L^G\right]$ and (20), (21), (22), and (23) hold, the SEML involves debt overhang in state $s = L$.

Case 2

Without debt overhang in state $s = L$,the face value $\underline{K}^G$ associated with borrowing $\gamma_s\bar{e}\bar{X}_L^B\left(1-\frac{\Delta x_1}{\tau}\right)$ solves:

$$\gamma_s\bar{e}\bar{X}_L^B\left(1-\frac{\Delta x_1}{\tau}\right) = \gamma_s\bar{e}\underline{K}^G + \gamma_s(1-\bar{e})\,0 + (1-\gamma_s)\,\bar{e}\underline{K}^G + (1-\gamma_s)(1-\bar{e})\,0,$$

i.e.

$$\underline{K}^G = \gamma_s\bar{X}_L^B\left(1-\frac{\Delta x_1}{\tau}\right).$$

Consistency requires that the good type exerts effort in both states when borrowing with a promised repayment amount of $\underline{K}^G = \gamma_s\bar{X}_L^B\left(1-\frac{\Delta x_1}{\tau}\right)$. Effort exertion in state $s = L$ requires that

$$\alpha\bar{e}\left(\bar{X}_L^G - \gamma_s\bar{X}_L^B\left(1-\frac{\Delta x_1}{\tau}\right)\right) \geq c_e.$$

The non-emptyness of this effort cost region is guaranteed by:

$$\begin{aligned}\alpha\bar{e}\left(\bar{X}_L^G - \gamma_s\bar{X}_L^B\left(1-\frac{\Delta x_1}{\tau}\right)\right) &> \alpha\bar{e}\bar{X}_L^B,\\ \bar{X}_L^G &> \bar{X}_L^B\left(1+\gamma_s\left(1-\frac{\Delta x_1}{\tau}\right)\right).\end{aligned}$$

which is implied by condition (22) because

$$\bar{X}_L^B\left(1-\frac{\Delta x_1}{\tau}\right) + \frac{w}{\gamma_s\bar{e}(1-\alpha)} > \bar{X}_L^B\left(1-\frac{\Delta x_1}{\tau}\right) + \bar{X}_L^B > \bar{X}_L^B\left(1+\gamma_s\left(1-\frac{\Delta x_1}{\tau}\right)\right).$$

where the first inequality follows from assumption (12). The exertion of effort in state $s = H$ is guaranteed by

$$\alpha\bar{e}\left(\bar{X}_L^G - \gamma_s\bar{X}_L^B\left(1-\frac{\Delta x_1}{\tau}\right)\right) + \bar{e}\left(\bar{X}_H^G - \bar{X}_L^G\right) \geq c_e,$$

which is implied by (21).

Retention by investors conditional on separation requires that

$$-w + (1-\alpha)\,\bar{e}\left(\bar{X}_L^G - \gamma_s\bar{X}_L^B\left(1-\frac{\Delta x_1}{\tau}\right)\right) \geq 0,$$

which is implied by (22).

As before we conclude with checking that the good fund prefers to use leverage to monetization. The good type never finds it desirable to monetize enough to induce debt overhang in state $s = L$. This would increase the face value of debt, reducing the carry and – in addition – the good fund would receive the carry only in state $s = H$. Thus, the only possibility that we need to consider is an increase to $\hat{k}$ which does not lead to debt overhang in state $s = L$. As before, given condition (20), the pledgeable income of the bad type in this case is given by

$$PI^B\left(\hat{k} + \Delta x_1\right) = \gamma_s \bar{e} \bar{X}_L^B \left(1 - \frac{\hat{k} + \Delta x_1}{\tau}\right).$$

while the corresponding face value of debt solves

$$\gamma_s \bar{e} \bar{X}_L^B \left(1 - \frac{\hat{k} + \Delta x_1}{\tau}\right) = \gamma_s \bar{e} K^G + (1 - \gamma_s)\, \bar{e} K^G,$$

so that

$$K^G = \gamma_s \bar{X}_L^B \left(1 - \frac{\hat{k} + \Delta x_1}{\tau}\right)$$

Under this deviation, the expected payoff to the good fund is:

$$\alpha \bar{e} \left(\bar{X}_L^G \left(1 - \frac{\hat{k}}{\tau}\right) - \gamma_s \bar{X}_L^B \left(1 - \frac{\hat{k} + \Delta x_1}{\tau}\right)\right) + \gamma_s \bar{e} \left(\bar{X}_H^G - \bar{X}_L^G\right) \left(1 - \frac{\hat{k}}{\tau}\right) - c_e.$$

In contrast, in equilibrium, the expected payoff to the good fund is:

$$\alpha \bar{e} \left(\bar{X}_L^G - \gamma_s \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right)\right) + \gamma_s \bar{e} \left(\bar{X}_H^G - \bar{X}_L^G\right) - c_e.$$

Thus, the deviation is unprofitable as long as:

$$\alpha \bar{e} \frac{\hat{k}}{\tau} \left(\bar{X}_L^G - \bar{X}_L^B\right) + \gamma_s \bar{e} \left(\bar{X}_H^G - \bar{X}_L^G\right) \frac{\hat{k}}{\tau} > 0, \tag{24}$$

which is always true.

Thus, when $c_e \in \left(\alpha \bar{e} \bar{X}_L^B, \alpha \bar{e} \left(\bar{X}_L^G - \gamma_s \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right)\right)\right]$ and (20), (21) and (22) hold, the SEML involves no debt overhang in $s = L$.

Combining the analysis for Cases 1 and 2, we note that for

$$c_e \in \left(\alpha \bar{e} \left(\bar{X}_L^G - \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right)\right), \alpha \bar{e} \left(\bar{X}_L^G - \gamma_s \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right)\right)\right]$$

the SEML may or may not involve debt overhang in $s = L$. In order to consider only essential instances of debt overhang we thus unify the two cases as follows: When (20), (21), (22), and (23) hold, there exist SEML without debt overhang in $s = L$ for $c_e \in \left(\alpha \bar{e} \bar{X}_L^B, \alpha \bar{e} \left(\bar{X}_L^G - \gamma_s \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right)\right)\right]$ while for $c_e \in \left(\alpha \bar{e} \left(\bar{X}_L^G - \gamma_s \bar{X}_L^B \left(1 - \frac{\Delta x_1}{\tau}\right)\right), \alpha \bar{e} \bar{X}_L^G\right]$, the SEML involves debt overhang in $s = L$.■

References

Admati, A. R., P. Pfleiderer, and J. Zechner, 1994, "Large Shareholder Activism, Risk Sharing, and Financial Market Equilibrium," *Journal of Political Economy*, 102(6), 1097–1130.

Admati, A. R., and P. C. Pfleiderer, 2009, "The "Wall Street Walk" and Shareholder Activism: Exit as a Form of Voice," *Review of Financial Studies*, 22(7), 2445–2485.

Agarwal, V., N. D. Daniel, and N. Y. Naik, 2009, "Role of Managerial Incentives and Discretion in Hedge Fund Performance," *Journal of Finance*, 64(5), 2221–2256.

Armour, J., and B. R. Cheffins, 2009, "The Rise and Fall (?) of Shareholder Activism by Hedge Funds," *ECGI - Law Working Paper No. 136/2009. Available at SSRN: http://ssrn.com/abstract=1489336*.

Aslan, H., and H. Maraachlian, 2009, "Wealth Effects of Hedge Fund Activism," *Available at SSRN: http://ssrn.com/abstract=1343403.*

Axelson, U., T. Jenkinson, P. Stromberg, and M. S. Weisbach, 2013, "Borrow cheap, buy high? The determinants of leverage and pricing in buyouts," *Journal of Finance*, 68, 2223–2267.

Axelson, U., P. Stromberg, and M. Weisbach, 2009, "Why are buyouts levered? The financial structure of private equity firms," *Journal of Finance*, 64, 1549–1582.

Baquero, G., and M. Verbeek, 2009, "A Portrait of Hedge Fund Investors: Flows, Performance and Smart Money," *EFA 2006 Zurich Meetings. Available at SSRN: http://ssrn.com/abstract=773384*.

Becht, M., J. Franks, J. Grant, and H. Wagner, 2014, "The Returns to Hedge Fund Activism: An International Study," *ECGI - Finance Working Paper No. 402/2014*.

Becht, M., J. Franks, C. Mayer, and S. Rossi, 2009, "Returns to Shareholder Activism: Evidence from a clinical study of the Hermes UK Focus Fund," *Review of Financial Studies*, 22, 3093–3129.

Bolton, P., and E.-L. von Thadden, 1998, "Blocks, Liquidity, and Corporate Control," *Journal of Finance*, 53(1), 1–25.

Boyson, N. M., and R. M. Mooradian, 2011, "Corporate Governance and Hedge Fund Activism," *Review of Derivatives Research*, 14.

Brav, A., W. Jiang, and H. Kim, 2010, "Hedge fund activism: A review," *Foundations and Trends in Finance*, 4, 1–66.

Brav, A. P., W. Jiang, F. Partnoy, and R. S. Thomas, 2008, "Hedge Fund Activism, Corporate Governance, and Firm Performance," *Journal of Finance*, 63(4), 1729–1775.

Burkart, M., D. Gromb, and F. Panunzi, 1997, "Large Shareholders, Monitoring, and the Value of the Firm," *The Quarterly Journal of Economics*, 112(3), 693–728.

Byrd, F., D. Hambly, and M. Watson, 2007, "Short-term shareholder activists degrade creditworthiness of rated companies," *Special Comment, Moody's Investors Service*.

Chung, J., B. A. Sensoy, L. Stern, and M. Weisbach, 2012, "Pay for performance from future fund flows: The case of private equity," *Review of Finacial Studies*, 25, 3295–3304.

Clifford, C. P., 2008, "Value creation or destruction? Hedge funds as shareholder activists," *Journal of Corporate Finance*, 14, 323–336.

Dasgupta, A., and G. Piacentino, 2011, "The Wall Street Walk when Blockholders Compete for Flows," *Available at SSRN: http://ssrn.com/abstract=1848001*.

Dasgupta, A., and A. Prat, 2008, "Information Aggregation in Financial Markets with Career Concerns," *Journal of Economic Theory*, 143(1), 83–113.

Dasgupta, A., A. Prat, and M. Verardo, 2011, "The Price Impact of Institutional Herding," *Review of Financial Studies*, 24(3), 892–925.

Edmans, A., 2009, "Blockholder Trading, Market Efficiency, and Managerial Myopia," *Journal of Finance*, 64(6), 2481–2513.

Edmans, A., and G. Manso, 2011, "Governance Through Trading and Intervention: A Theory of Multiple Blockholders," *Review of Financial Studies*, 24(7), 2395–2428.

Faure-Grimaud, A., and D. Gromb, 2004, "Public Trading and Private Incentives," *The Review of Financial Studies*, 17(4), pp. 985–1014.

Fung, W., D. A. Hsieh, N. Y. Naik, and T. Ramadorai, 2008, "Hedge funds: Performance, risk, and capital formation," *The Journal of Finance*, LXIII, 1777–1803.

Gantchev, N., and P. Jotikasthira, 2013, "Hedge Fund Activists: Do They Take Cues from Institutional Exit?," *Available at SSRN: http://ssrn.com/abstract=2139482*.

Gillan, S. L., and L. T. Starks, 2007, "The Evolution of Shareholder Activism in the United States," *Journal of Applied Corporate Finance*, 19(1), 55–73.

Goldman, E., and G. Strobl, 2011, "Large shareholder trading and the complexity of corporate investments," *Journal of Financial Intermediation, Forthcoming.*

Grossman, S. J., and O. D. Hart, 1980, "Takeover Bids, the Free-Rider Problem, and the Theory of the Corporation," *Bell Journal of Economics*, 11(1), 42–64.

Guerrieri, V., and P. Kondor, 2011, "Fund Managers, Career Concerns, and Asset Price Volatility," *American Economic Review, Forthcoming.*

Innes, R., 1990, "Limited liability and incentive contracting with ex ante action choices," *Journal of Economic Theory*, 52, 45–67.

Jensen, M., and W. Meckling, 1976, "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure," *Journal of Financial Economics*, 3, 305–360.

Kadyrzhanova, D., and M. Rhodes-Kropf, 2013, "Governing Misvalued Firms," *Working paper.*

Kahn, C., and A. Winton, 1998, "Ownership Structure, Speculation, and Shareholder Intervention," *The Journal of Finance*, 53(1), 99–129.

Klein, A., and E. Zur, 2009, "Entrepreneurial Shareholder Activism: Hedge Funds and Other Private Investors," *Journal of Finance*, 64(1), 187–229.

——— , 2011, "The impact of hedge fund activism on the target firm's existing bondholders," *The Review of Financial Studies*, 24, 1735–1771.

Korinek, A., 2009, "Systemic Risk-Taking: Amplification effects, externalities and regulatory responses," *Working paper.*

Krishnamurthy, A., 2003, "Collateral constraints and the amplification mechanism," *Journal of Economic Theory*, 111, 277–292.

Li, Y., and J. Xu, 2010, "Hedge Fund Activism and Bank Loan Contracting," *AFA 2011 Denver Meetings Paper. Available at SSRN: http://ssrn.com/abstract=1573217.*

Lim, J., B. A. Sensoy, and M. S. Weisbach, 2013, "Indirect Incentives of Hedge Fund Managers," *NBER Working Paper No. 18903.*

Malenko, A., and N. Malenko, 2014, "A theory of LBO activity based on repeated debt-equity conflicts," *Working paper.*

Martos-Vila, M., M. Rhodes-Kropf, and J. Harford, 2013, "Financial vs strategic buyers," *Working paper.*

Maug, E., 1998, "Large Shareholders as Monitors: Is There a Trade-Off between Liquidity and Control?," *Journal of Finance*, 53(1), 65–98.

Noe, T. H., 2002, "Investor Activism and Financial Market Structure," *Review of Finacial Studies*, 15(1), 289–319.

Ross, S., 1977, "The Determination of Financial Structure: The Incentive Signalling Approach," *Bell Journal of Economics*, 8, 23–40.

Shiller, R., 1998, *Macro Markets*. Oxford University Press.

Shleifer, A., and R. W. Vishny, 1986, "Large Shareholders and Corporate Control," *Journal of Political Economy*, 94(3), 461–88.

Sunder, J., S. V. Sunder, and W. Wongsunwai, 2010, "Debtholder Responses to Shareholder Activism: Evidence from Hedge Fund Interventions," *Available at SSRN: http://ssrn.com/abstract=1737962*.

Tirole, J., 2001, "Corporate Governance," *Econometrica*, 69(1), 1–35.