Uncertainty Aversion and Systemic Risk*

David L. Dicks
Kenan-Flagler Business School
University of North Carolina

Paolo Fulghieri
Kenan-Flagler Business School
University of North Carolina
CEPR and ECGI

March 10, 2015

Abstract

We propose a new theory of systemic risk based on uncertainty (or "ambiguity") aversion (Knight, 1921). We show that, due to uncertainty aversion, bad news on one asset class induces investors to be more pessimistic about other asset classes as well. This means that idiosyncratic risk can create contagion and snowball into systemic risk. Furthermore, in a Diamond and Dybvig (1983) setting, we show that, surprisingly, uncertainty aversion causes investors to be less prone to run individual banks, but runs will be systemic. In addition, we show that bank runs are associated with stock market crashes and flight to quality. Finally, we show that increasing uncertainty makes the financial system more fragile and more prone to financial crises.

Keywords: Ambiguity Aversion, Systemic Risk, Financial Crises, Bank Runs

*We would like to thank Laura Bottazzi, Elena Carletti, Robert Connolly, Massimo Marinacci, Adam Reed, Jacob Sagi, Merih Sevilir, Anjan Thakor, Andrew Winton, and seminar participants at Bocconi University, the Corporate Finance Conference at Washington University, and the UNC Brown Bag for their helpful comments. All errors are only our own. We can be reached at David_Dicks@kenan-flagler.unc.edu and Paolo_Fulghieri@kenan-flagler.unc.edu

Uncertainty and waves of pessimism are the hallmark of financial crises. Financial crises and bank runs are often associated with periods of great uncertainty and sudden widespread pessimism on future returns of financial and real assets. In addition, a puzzling feature of several recent financial crises has been contagion among apparently unrelated asset classes. For example, the Asian financial crisis of 1997 spread to the Russian crisis of 1998, which eventually brought the fall of LTCM (see Allen and Gale, 1999). In addition, negative idiosyncratic shocks in one asset class can snowball into systemic shocks. For example, the recent crisis of 2008/2009 was triggered by negative shocks in the relatively small sub-prime mortgage market, and then rapidly spread to the general financial markets, leading to a near meltdown of the entire financial system. These events put into question the very notion (and assessment) of systemic risk, and raise the question of the mechanism that triggers such systemic contagions.

In this paper we propose a new theory of systemic risk based on uncertainty aversion. Our model builds on the distinction between risk, whereby investors know the probability distribution of assets' cash flows, and Knightian uncertainty (Knight, 1921), whereby investors lack such knowledge. The distinction between the known-unknown and the unknown-unknown is relevant since investors appear to display aversion to uncertainty (or "ambiguity") as originally suggested by Ellsberg (1961).

We study an economy where uncertainty-averse investors hold, either directly or through financial intermediaries (i.e., banks), a portfolio of risky assets. Investors are uncertain on the distribution of the returns on the risky assets.[1] We show that uncertainty-averse investors prefer to hold an uncertain asset only if they can also hold other uncertain assets in their portfolios, a feature that we denote *uncertainty hedging*. We show this implies that "bad news" on one asset class makes investors also more pessimistic on other asset classes as well. In this way, a shock to one asset class spreads to other asset classes, creating contagion even in cases where shocks are idiosyncratic. Thus, uncertainty aversion is *independently* a source of systemic risk.

We build on the classic Diamond and Dybvig (1983) model to include two banks, each with access to a bank-specific class of risky assets in addition to the safe asset. Following existing literature, banks are benevolent, maximizing the welfare of investors who are exposed to uninsurable liquidity

[1] This uncertainty represents, for example, incomplete knowledge on the structure of the economy that generates asset returns, i.e., it can be viewed as model uncertainty (see Hansen and Sargent, 2008).

shocks. Risk factors in each asset class are independent given the state of the economy, but the state of the economy differentially affects each asset class and provides the source of uncertainty.

In the absence of uncertainty aversion, both banks invest in risky assets. Banks provide investors with (partial) insurance against liquidity shocks, but runs are possible in equilibrium at the interim date. Runs, however, are not necessarily systemic. Formally, as in Diamond and Dybvig (1983) there are multiple equilibria, with and without runs. There are both panic runs, due to coordination failure among investors, and fundamental runs, due to the arrival of (idiosyncratic) bad news about a bank's expected profitability. In the absence on uncertainty aversion, however, there is no reason for bank runs to be systemic, that is to occur simultaneously on both banks.

With uncertainty aversion, however, investing in a class of risky assets is more valuable to investors if they hold other asset classes in their portfolio as well, due to uncertainty hedging. This feature has a number of important consequences. First, it generates two equilibria in banks' investment decisions. When banks decide how much to invest in the risky asset, each bank is willing to make such investments if and only if the other bank invests in its risky asset as well. This implies that investors' uncertainty aversion makes investment in risky assets strategic complements, with the possibility of a second Pareto-inferior equilibrium where both banks invest in the safe asset only, a situation that we denote as a "lending freeze." This second (inefficient) equilibrium represents a new type of equilibrium in a Diamond and Dybvig setting with multiple banks.

The second effect of uncertainty aversion is that it creates the possibility of contagion across banks. This happens because, if a late investor withdraws early from one bank, it can now become optimal for that investor to withdraw early from the other bank as well, even if no one else runs. Thus, negative idiosyncratic shocks at any one bank can generate a negative outlook onto other banks and, thus, cause runs on other banks, creating systemic risk. In other words, negative news specific to one asset class may create pessimism that spreads to other asset classes. In this way, uncertainty aversion generates endogenous contagion and becomes a source of systemic risk. Note that this new source of systemic risk is driven by investor preferences and its effect on the economy's stochastic discount factor, rather than by systemic shocks to economic fundamentals. We also show that, surprisingly, uncertainty aversion causes investors to be less prone to run individual banks, but runs will be systemic.

In our model, bank runs can also be associated with stock market crashes leading to a "flight to quality." Distinct from existing literature, contagion between the financial sector and the real economy is driven by investor preferences, creating a new channel through which a banking crisis can affect the real economy which is different, for example, from the adverse effects of liquidity crunches.

Finally, we show that increasing uncertainty makes the financial system more fragile and more prone to financial crises. Specifically, we show that for low levels of uncertainty idiosyncratic shocks at a single bank generate local runs, while for greater levels of uncertainty such shocks spread to other banks and become systemic. In addition, we show that for even greater levels of uncertainty a second equilibrium exists where banks only invest in the safe asset, generating "lending freezes." In addition, we show our results extend to a setting with multiple banks.

We conclude our paper with a discussion of the public policy implications of our model. First, we argue that greater transparency may be beneficial in periods of high perceived uncertainty by investors. Second, we suggest that, because the risky equilibrium is preferred to the safe equilibrium, regulatory attempts to limit risk taking can be harmful. We also suggest that bank bailouts and assets purchases by the central bank may involve not only the banks that are directly affected, but must also be extended to other banks that may be affected by the systemic nature of the financial crisis.

Our paper is related to several stands of literature. First is the theory of bank runs based on the liquidity provision/maturity transformation role of financial intermediation originating with Diamond and Dybvig (1983). This includes Jacklin (1987), Bhattacharya and Gale (1987), Jacklin and Bhattacharya (1988), Chari and Jaghannathan (1988), and Goldstein and Pauzner (2005), among many others. Allen, Carletti, and Gale (2009) argue that aggregate volatility can induce banks to stop trading among each other, effectively generating a lending freeze. Our paper shows that uncertainty aversion creates externalities and strategic complementarities across asset holdings which lead to a new safe (Pareto inferior) equilibrium where runs are not possible.

More importantly, our paper is also linked to the recent emerging literature on contagion and systemic risk. Allen and Gale (2000) generate contagion as the outcome of an imperfect interbank market for liquidity. Kodres and Pritsker (2002) model transmission (i.e., contagion) of idiosyncratic shocks across asset markets by investors' rebalancing their portfolios' exposures to shared macroeconomic risks among asset classes. Garleanu, Panageas, and Yu (2014) derive contagion across assets due to limited participation and overlapping portfolios of investors. Allen, Babus, and Carletti (2012) examine the impact of financial connections on systemic risk. Acharya, Mehran, and Thakor (2013) consider a model where regulatory forbearance induces banks to invest in correlated assets, thus creating systemic risk. Others papers include Freixas, Parigi, and Rochet (2000), Rochet and Vives (2004), Acharya and Yorulmazer (2008), Brusco and Castiglionesi (2008), among many others. More closely related to our work is the literature on uncertainty aversion. Caballero and Krishnamurthy (2008) examine a version of Diamond and Dybvig (1983) with uncertainty-averse investors. Uncertainty in their model concerns the extent of the investors' liquidity shocks (and not a bank's expected profitability, as in our model). Uncertainty aversion makes investors very pessimistic (that is, they "fear the worst") triggering a "flight-to-quality." In their model, uncertainty aversion acts as an "amplification mechanism."[2] Contagion (that is, the transmission mechanism) can happen, for example, through forced asset sales in unrelated asset markets due to investors' balance sheet constraints. In our paper, uncertainty aversion itself is a new source of contagion and systemic risk, by its impact on the stochastic discount factor.

Our paper is organized as follows. In Section 1, we briefly discuss the model of uncertainty aversion that underpins our analysis. In Section 2, we outline the model. In Section 3, we develop our theory of systemic risk based on uncertainty aversion. In Section 4, we discuss the contagion effect of bank runs on the stock market. In Section 5, we discuss the effect of increased uncertainty on fragility of the financial system. Results are extended to a multiple bank setting in Section 6. In Section 7, we discuss the policy implications of our model. Section 8 concludes. All proofs are in the Appendix.

1 Uncertainty aversion

A common feature of current economic models is to assume that all agents know the distribution of all possible outcomes.[3] An implication of this assumption is that there is no distinction between

[2] See also Khrishamurthy (2010) and, in a similar vein, Easley and O'Hara (2009).
[3] This section draws on Dicks and Fulghieri (2015).

the known-unknown and the unknown-unknown. However, the Ellsberg paradox shows that this implication is not warranted.[4] This introductory section briefly describes how various models have accounted for risk and uncertainty.

In traditional models, economic agents maximize their Subjective Expected Utility (SEU). Given a von-Neumann Morgenstern (vNM) utility function u and a probability distribution over wealth, μ, each player maximizes

$$U^e = E_\mu \left[u\left(w \right) \right]. \tag{1}$$

One limitation of the SEU approach is that it cannot account for aversion to uncertainty, or "ambiguity." In the SEU framework, economic agents merely average over the possible probabilities. Under uncertainty aversion, a player does not know the true prior, but only knows that the prior is from a given set, $\mathcal{M}$.

A common way for modeling uncertainty (or ambiguity) aversion is the minimum expected utility approach (MEU), promoted in Epstein and Schneider (2011). In this framework, economic agents maximize

$$U^a = \min_{\mu \in \mathcal{M}} E_\mu \left[u\left(w \right) \right]. \tag{2}$$

As shown in Gilboa and Schmeidler (1989), the MEU approach is a consequence of replacing the Sure-Thing Principle of Anscombe and Aumann (1963), with the Uncertainty Aversion Axiom.[5] This assumption captures the intuition that economic agents prefer risk to uncertainty – they prefer known probabilities to unknown. MEU has the intuitive feature that a player first calculates expected utility with respect to each prior, and then takes the worst-case scenario over all possible priors. In other words, the agent follows the maxim "Average over what you know, then worry

[4] A good illustration of the Ellsberg paradox is actually from Keynes (1921). There are two urns. Urn K has 50 red balls and 50 blue balls. Urn U has 100 balls, but the subject is not told how many of them are red (all balls are either red or blue). The subject will be given \$100 if the color of their choice is drawn, and the subject can choose which Urn is drawn from. Subjects typically prefer urn K, revealing aversion to uncertainty (this preference is shown to be strict if the subject receives \$101 from selecting Urn U but \$100 from Urn K being drawn). To see this, suppose the subject believes that the probability of drawing blue from Urn U is p_B. If $p_B < \frac{1}{2}$, the subject prefers to draw red from Urn U. If $p_B > \frac{1}{2}$, the subject prefers to draw blue from Urn U. If $p_B = \frac{1}{2}$, the subject is indifferent. Because subjects strictly prefer to draw from Urn K, such behavior cannot be consistent with a single prior on Urn U. This paradox provides the motivation for the use of multiple priors. Further, the subject's beliefs motivate the failure of additivity of asset prices: in this example, the subject believes that $p_B + p_R < p_{(B \cup R)} = 1$.

[5] Anscombe and Aumann (1963) is an extention of the Savage (1972) framework: the Anscombe and Aumann framework has both objective and subjective probabilities, while the Savage framework has only subjective probabilities.

about what you don't know."[6]

In this paper, we use the MEU approach with recursively defined utilities, as described in Epstein and Schnieder (2011). Formally, we model *sophisticated uncertainty-averse* economic agents with consistent planning. In this setting, agents are sophisticated: they correctly anticipate their future uncertainty aversion. Consistent planning accounts for the fact that agents take into account how they will actually behave in the future.[7] Our results are smooth (a.e.) because we explore a setting where we can apply a minimax theorem.

An important feature of uncertainty aversion that will play a critical role in our paper is that agents may benefit from diversification, a feature that we will refer to as *uncertainty hedging.* This feature can be seen as follows. Consider two random variables, y_k, $k \in \{1,2\}$, with distribution $\mu \in \mathcal{M}$, which is ambiguous to agents. Uncertainy-hedging is the property that uncertainty-averse agents prefer to pick the worst case scenario for a portfolio, rather than choosing the worst case scenario for each individual asset in its portfolio.[8]

Theorem 1 *Uncertainty-averse agents prefer uncertainty-hedging:*

$$\begin{aligned} q \min_{\mu \in \mathcal{M}} E_\mu \left[u\left(y_1\right)\right] + (1-q) \min_{\mu \in \mathcal{M}} E_\mu \left[u\left(y_2\right)\right] &\leq \\ \min_{\mu \in \mathcal{M}} \{ q E_\mu \left[u\left(y_1\right)\right] + (1-q) E_\mu \left[u\left(y_1\right)\right]\}, \quad \textit{for all } q &\in [0,1]. \end{aligned} \tag{3}$$

If agents are SEU, (3) holds as an equality.

This property will play a key role in our model. It implies that uncertainty-averse agents prefer to hold a portfolio of uncertain assets rather than a single uncertain asset, because investors can lower their exposure to uncertainty by holding a diversified portfolio. Alternatively, it suggests that an investor will be more "optimistic" about a portfolio than about a single asset. Thus, uncertainty hedging creates a complementarity between asset classes for investors so the value investors place

[6] Another approach is the smooth ambiguity model developed by Klibanoff, Marinacci, and Mukerji (2005). In their model, agents maximize expected felicity of expected utility. Agents are uncertainty averse if the felicity function is concave.

[7] Siniscalchi (2011) describes this framework as preferences over trees.

[8] Note that, as such, property (3) is reminiscent of the well-known feature that a portfolio of options is worth more than an option on a portfolio and, thus, that writing a portfolio of options is more costly than writing an option on a portfolio.

on any one type of asset is increasing in their portfolio exposure to other assets.[9]

A second critical feature of our model is that we do not impose rectangularity of beliefs (as in Epstein and Schneider 2003). Rectangularity of beliefs effectively implies that prior beliefs in the set of admissible priors can be chosen independently from each other.[10] In our model, the agent faces a restriction on the set of the core beliefs $\mathcal{M}$ over which the minimization problem (2) is taking place. These restrictions are justified by the observation that the nature of the economic problem imposes certain consistency requirements in the set of the core beliefs $\mathcal{M}$. In other words, we recognize that the "fundamentals" of the economic problem faced by the uncertainty-averse agent generates a loss of degree of freedom in the selection of prior beliefs.[11]

2 The model

We study a two-period model, with three dates, $t \in \{0, 1, 2\}$. The economy is endowed with three types of assets: a riskless asset (or "safe technology"), which will serve as our numeraire, and two classes (or types) of risky assets denominated by τ, with $\tau \in \{A, B\}$. Making an investment in a risky asset at the beginning of the first period, $t = 0$, generates at the end of the second period, $t = 2$, a random payoff denominated in terms of the riskless asset. Specifically, a unit investment in the type-τ asset produces at $t = 2$ a payoff R (success) with probability p_τ, and a payoff 0 (failure) with probability $1 - p_\tau$. A unit investment in the riskless asset, which can be made either at $t = 0$ or $t = 1$, yields a unit return in the second period, so that the (net) riskless rate of return is zero. We assume that returns on risky assets depends on the state of the overall economy, which provides the source of uncertainty in the model, as described below.

Our economy has two classes of players: investors and banks. The banking system is specialized: each bank can only invest in one asset class. Thus, banks of type τ can only invest in type-τ assets, for $\tau \in \{A, B\}$, at $t = 0$. This assumption captures the notion that banks in our economy are

[9] We will show that such portfolio complementarity will induce banks to exhibit strategic complementarity in their investment decisions, resulting in multiple equilibria. In addition, we will show that uncertainty hedging generates contagion across asset classes, and it will provide the new channel through which financial panics spread in the economy.

[10] Rectangularity of beliefs is commonly assumed to guarantee dynamic consistency. However, Aryal and Stauber (2014) show that, with multiple players, rectangularity of beliefs is not sufficient for dynamic consistency.

[11] For example, an uncertainty-averse producer may face uncertainty on the future consumption demand exerted by her customers. The beliefs held by the uncertainty-averse agent on consumer demand must be consistent with basic restrictions, such as the fact that the consumer choices must satisfy an appropriate budget constraint.

specialized lenders with a well-defined clientele. At $t = 1$, a bank has the choice of (partially) liquidating the project, allowing it to recover a fraction of the initial investment. Thus, liquidation at $t = 1$ of a fraction ℓ of the investment in risky asset τ will generate a payoff ℓ at $t = 1$, and $(1 - \ell)R$ with probability $p_\tau(\theta)$ at $t = 2$.

The economy is populated by a continuum of investors. Each investor is endowed at $t = 0$ with \$2 in the riskless asset and, as we will show later, in equilibrium will invest \$1 in Bank A and \$1 in Bank B. Following Diamond and Dybvig (1983), each investor faces at $t = 1$ a liquidity shock with probability λ.[12] Occurrence of the liquidity shock is privately observed by the investor and determines her "type." An investor hit with the liquidity shock, that is, a "short-term" investor, must consume immediately, and her utility is $u(c_1)$, with $u' > 0 > u''$, where c_1 is consumption at $t = 1$. An investor not impacted by the liquidity shock, that is a "long-term" investor, consumes only at $t = 2$. For analytical tractability we assume that long-term investors are risk neutral in wealth, that is, their utility is $u_2(c_2) = c_2$, where c_2 is consumption at $t = 2$.[13]

The model unfolds as follows. At the beginning of the period, $t = 0$, banks offer deposit contracts (described below) to investors. At $t = 1$, investors learn whether or not they are affected by the liquidity shock. Investors hit by a liquidity shock withdraw from the bank(s) where they made a deposit and consume all their wealth. Investors not hit by a liquidity shock must decide whether to keep their deposits in the bank(s) for later withdrawal, or to withdraw (part of) their deposits immediately from one or both banks, that is to "run" banks, and invest the proceeds in the storage technology for later consumption. At $t = 2$, cash flows from risky assets are realized and divided among investors remaining in the bank.

An important deviation from the traditional Diamond and Dybvig (1983) framework is that we assume investors are uncertainty averse. Following Dicks and Fulghieri (2015), we model uncertainty aversion by assuming that the success probability of an asset of type-τ depends on the value of an underlying parameter θ, and is denoted by $p_\tau(\theta)$. Uncertainty-averse agents treat the parameter θ as ambiguous, and believe that $\theta \in C \equiv \left[\hat{\theta}_0, \hat{\theta}_1\right] \subseteq [\theta_0, \theta_1]$, where C represents the set of "*core beliefs*". We posit that the parameter θ describes the state of the economy at $t = 2$, and that a greater value

[12] Liquidity shocks are statistically independent across investors. Differently from Wallace (1988, 1990), and Chari (1989), among others, there is neither aggregate risk nor uncertainty on the liquidity shock.

[13] While we make the assumption that the utility for consumption at $t = 2$ is linear for analytical tractability, numerical analysis of the concave utility case yields similar results to the ones presented in our paper.

of θ is "favorable" for asset A and "unfavorable" for asset B.[14] For analytical tractability, we assume that $p_A(\theta) = e^{\theta-\theta_1}$ and $p_B(\theta) = e^{\theta_0-\theta}$.[15] In this specification, greater values of the parameter θ increase the success probability of type A assets and decrease the success probability of type B assets. Also, for a given value of the parameter θ, the probabilities distributions $p_\tau(\theta)$, $\tau \in \{A, B\}$, are independent.[16]

Finally, we assume that the core of beliefs is symmetric, so that $\theta_1 - \hat{\theta}_1 = \hat{\theta}_0 - \theta_0$, and we let $\theta^e \equiv \frac{1}{2}(\theta_0 + \theta_1)$. We will at times benchmark the behavior of uncertainty-averse agents with the behavior of uncertainty-neutral, or SEU, agents, and we will assume that uncertainty-neutral investors believe that $\theta = \theta^e$, differently from uncertainty-averse investors who believe that $\theta \in \left[\hat{\theta}_0, \hat{\theta}_1\right]$. Finally, we assume throughout that $e^{\theta^e-\theta_1}R > 1$, which from the definition of θ^e, implies that $e^{\theta_0-\theta^e}R > 1$ as well. These inequalities imply that the expected profits from risky assets are sufficiently large to make an uncertainty-neutral investor willing to invest in such assets.

2.1 Deposit contracts

In our model, banks are benevolent and offer investors deposit contracts that maximize their welfare. Because, banks can make risky investments, departing from Diamond and Dybvig (1983) deposit contracts have three components, which determine the contractual return to the investor depending on the date of withdrawal and the realization of the investment in the risky asset. Thus, a deposit contract offered by Bank τ is a triplet $d_\tau \equiv \{d_{1\tau}, d^s_{2\tau}, d^r_{2\tau}\}$, as follows. Investors who withdraw at $t = 1$ receive an amount $d_{1\tau}$ of the safe asset; investors who remain in the bank until $t = 2$ receive an amount $d^s_{2\tau}$ of the safe asset and an amount $d^r_{2\tau}$ of type-τ asset.

We assume that banks offer incentive-compatible deposit contracts such that "no-run" equilibria exist, which will be the main focus of our paper.[17] Given a deposit contract $d_\tau \equiv \{d_{1\tau}, d^s_{2\tau}, d^r_{2\tau}\}$

[14] A simple example of our economy is one with two consumption goods, A, B. Consumers' preferences over the two consumption goods (that is, their relative valuation) is random and is characterized by the parameter θ. In this case, a higher (respectively, lower) value of θ represents a stronger consumer preference for good A (respectively, B) with respect to the other good.

[15] Note that this assumption allows us to dispense with rectangularity of beliefs in a tractable way, but is not necessary. For example, our paper's main results will go through for $p_\tau(\theta_\tau)$, with $\theta_\tau \in [\theta_0, \theta_1]$, as long as the core belief set C is a strictly convex, compact subset of $[\theta_0, \theta_1]^2$ with a smooth boundary, such that $\{\theta_A, \theta_B\} \in C$.

[16] Our model can easily be extended to the case where, given θ, the realization of the asset payoffs at the end of the period are correlated.

[17] As typical in this class of models, "run" equilibria also exist. In Section 3, in the spirit of Goldstein and Pauzner (2005) we will extend our basic model to have equilibria runs as well.

offered by Bank τ, for $\tau \in \{A, B\}$, investors' payoffs from holding contracts in the two banks are determined as follows. Absent a run, investors hit with the liquidity shock must withdraw early, and receive from the two banks a total payoff equal to $d_{1A} + d_{1B}$. Investors not hit with the liquidity shock, and who hold their initial deposits with both banks, have a payoff which depends on the realized return on each of the risky assets. If both asset classes are successful, investors receive a total payoff $d^s_{2A} + d^s_{2B} + R(d^r_{2A} + d^r_{2B})$; if only type τ assets are successful, they receive $d^s_{2A} + d^s_{2B} + Rd^r_{2\tau}$; if neither asset class is successful, they receive $d^s_{2A} + d^s_{2B}$. We let U_0 be the value function of investors at $t = 0$, and let U_L be the value function of investors who remain in the bank at $t = 1$, in the absence of run. Thus,

$$U_0 = \lambda u \left(d_{1A} + d_{1B}\right) + (1 - \lambda) U_L \left(\theta_L\right),$$

$$U_L \left(\theta_L\right) = d^s_{2A} + d^2_{2B} + e^{\theta_L - \theta_1} Rd^r_{2A} + e^{\theta_0 - \theta_L} Rd^r_{2B},$$

where θ_L is the belief held at time $t = 1$ about the state of the economy, determined next.

2.2 Endogenous Beliefs

An important implication of uncertainty aversion is that the investors' belief on the parameter θ depend on their overall exposure to the source of risk in the economy and, thus, on the structure of their portfolios.[18] Specifically, if a long-term investor does not run either bank, and both banks are solvent, the investor owns $d^s_{2A} + d^s_{2B}$ units of the safe asset and $d^r_{2\tau}$ units of type-τ assets, for $\tau \in \{A, B\}$. This means that the long-term investor holds an overall portfolio $\Pi = \{d^r_{2A}, d^r_{2B}, d^s_{2A} + d^s_{2B}\}$. Because of uncertainty aversion, the investor's belief at $t = 1$ on the state of the economy, θ^a, is the solution to the minimization problem:

$$\theta^a \left(\Pi\right) = \arg \min_{\theta \in C} U_L \left(\theta\right),$$

and is characterized in the following Lemma.

[18] For additional discussion, see Dicks and Fulghieri (2015).

Lemma 1 *Let*

$$\tilde{\theta}^a(\Pi) = \theta^e + \frac{1}{2}\ln\frac{d^r_{2B}}{d^r_{2A}}. \tag{4}$$

The belief held by an uncertainty-averse agent with portfolio $\Pi = \{d^r_{2A}, d^r_{2B}, d^s_{2A} + d^s_{2B}\}$ *is*

$$\theta^a(\Pi) = \begin{cases} \hat{\theta}_0 & \tilde{\theta}^a(\Pi) \leq \hat{\theta}_0 \\ \tilde{\theta}^a(\Pi) & \tilde{\theta}^a(\Pi) \in \left(\hat{\theta}_0, \hat{\theta}_1\right) \\ \hat{\theta}_1 & \tilde{\theta}^a(\Pi) \geq \hat{\theta}_1 \end{cases}. \tag{5}$$

Lemma 1 shows investors' beliefs on the parameter θ and, thus, on banks' expected profitability, as it is affected by the state of the economy, depend critically on the composition of their overall portfolio, Π. We will refer to $\tilde{\theta}^a(\Pi)$ as the "*porfolio-distorted*" belief. We will say that the investor has *interior beliefs* when $\theta^a \in \left(\hat{\theta}_0, \hat{\theta}_1\right)$. In this case, the investor's belief is equal to the portfolio-distorted belief, that is $\theta^a(\Pi) = \tilde{\theta}^a(\Pi)$. Otherwise, we will say that the investor holds "*corner beliefs.*" The following lemma can be immediately be verified.

Lemma 2 *Holding type-τ assets constant, a decrease in an investor's holding in type-τ' assets, $d^r_{2\tau'}$ with $\tau' \neq \tau$, makes the investor more pessimistic about type-τ assets, for $\tau \in \{A, B\}$. In addition, portfolio-distorted beliefs are homogeneous of degree zero in the holding of the risky assets, $\{d^r_{2A}, d^r_{2B}\}$.*

Lemma 2 shows that when a investor has a relatively greater proportion of her portfolio invested in asset τ (determined, for example, by a decrease in an investor's holding in type-τ'), she will be relatively more concerned about the priors that are less favorable to that asset. Thus, the investor will give more weight to the states of nature that are less favorable for that asset, that is, to the unfavorable values of the parameter θ. In other words, the investor will be more "pessimistic" about the return on that asset. Correspondingly, the investor will become more "optimistic" with respect to the other asset. Proportional changes in an investor's position in the risky assets will not affect her belief.

Lemma 1 will play a crucial role in our analysis. Specifically, it implies that (idiosyncratic) bad news about Bank-τ, which will induce a run on that bank, will make the investor also more pessimistic about Bank-τ' expected profitability, possibly triggering a run also on that bank. In this way, the presence of uncertainty aversion creates the possibility of contagion, and thus systemic risk.

2.3 Optimal deposit contracts

We now examine the optimal deposit contracts offered by banks. Because liquidity shocks are privately observable only to investors at the interim date, $t = 1$, deposit contracts offered by a bank must satisfy appropriate incentive compatibility constraints. Early investors must consume immediately, since they gain no utility from $t = 2$ consumption. Late investors, in contrast, may pretend to be early investors and withdraw their deposits from either (or both) banks and invest in the safe technology for later consumption. Thus, to prevent runs on one (or both) banks, deposit contracts must satisfy three incentive compatibility constraints for late consumers, as follows.

First, late investors must prefer keeping their deposits in both banks rather than running on both of them:

$$U_L\left(\theta^a\right) \geq d_{1A} + d_{1B}. \tag{6}$$

Second, they must find it optimal to not run only Bank A:

$$U_L\left(\theta^a\right) \geq d_{1A} + d^s_{2B} + e^{\theta_0 - \hat{\theta}_1} R d^r_{2B}, \tag{7}$$

and they must find it optimal to not run only Bank B:

$$U_L\left(\theta^a\right) \geq d_{1B} + d^s_{2A} + e^{\hat{\theta}_0 - \theta_1} R d^r_{2A}. \tag{8}$$

Note that the incentive compatibility constraint (7) reflects the fact that, if a long term investor runs on Bank A and not on Bank B, she will have a portfolio that includes risky assets of type-B only. This implies that she will be concerned only with the states of the economy that are least favorable to asset B and, thus, will set $\theta = \hat{\theta}_1$. If the long-term investor runs on Bank B, a similar argument leads the investor to hold belief $\hat{\theta}_0$, and thus to (8). Finally, the deposit contract offered by Bank τ must satisfy the bank's budget constraint

$$\lambda d_{1\tau} + (1-\lambda)\left[d_{2\tau}^{s} + d_{2\tau}^{r}\right] \leq 1. \tag{9}$$

In an equilibrium without bank runs, the optimal deposit contract offered by Bank A, $d_A = \{d_{1A}, d_{2A}^{s}, d_{2A}^{r}\}$, maximizes U_0 subject to (6), (7), and (9); similarly, the optimal deposit contract offered by Bank B, $d_B = \{d_{1B}, d_{2B}^{s}, d_{2B}^{r}\}$, maximizes U_0 subject to (6), (8), and (9) We will also assume the following:

(A_0): *Regularity conditions:*

$$u'(2) > e^{\theta^e - \theta_1} R > u'\left(2\frac{e^{\theta^e-\theta_1}R}{\lambda e^{\theta^e-\theta_1}R + (1-\lambda)}\right). \tag{10}$$

The first inequality ensures that the optimal deposit contract offered by banks to uncertainty-neutral investors provides (partial) insurance against liquidity shocks, while the second inequality ensures that the optimal deposit contracts satisfy the incentive compatibility constraint (6), that is, that the constraint is not binding in the optimal contract.[19]

As a benchmark we consider first the case in which agents are uncertainty-neutral, as follows.

Theorem 2 *If investors are uncertainty neutral, the optimal deposit contract,* $d_{\tau}^{R*} \equiv \{d_{1\tau}^{*}, d_{2\tau}^{s*}, d_{2\tau}^{r*}\}$, *has:*

$$d_{2\tau}^{s*} = 0, \quad 1 < d_{1\tau}^{*} < e^{\theta^e-\theta_1} R d_{2\tau}^{r*}, \text{ for } \tau \in \{A, B\}, \tag{11}$$

that is, banks provide partial insurance against liquidity shocks and are exposed to runs. Finally, it is optimal WLOG for investors to invest equally in both banks.

Theorem 2 shows that, as in Diamond and Dybvig (1983), a symmetric equilibrium with $d_{1A} = d_{1B}$ and $d_{2A}^{r} = d_{2B}^{r}$ always exists, whereby banks provide investors with (partial) insurance against liquidity shocks. In addition, insurance provision implies that, in equilibrium, banks are illiquid and, thus, exposed to runs. It is, however, important to note that bank runs are not necessarily

[19] Note that the regularity conditions (A_0) have the same role as the assumptions in Diamond and Dybvig (1983) that investors have a coefficient of RRA greater than 1 and that $\rho R > 1$, which together ensure that in the optimal deposit contract in their model, $\{d_1^*; d_2^*\}$, satisfies $1 < d_1^* < d_2^* < R$.

systemic: a run on one bank does not necessarily induce a run on the other bank. Thus, the banking system is not necessarily fragile.

These results change dramatically when investors are uncertainty averse. From Lemma 1 we know that, because of uncertainty aversion, the investors' belief on the future state of the economy and, thus, on banks' expected solvency, depends on their overall portfolio composition. In this way, uncertainty aversion creates a direct link between investor's desired holding in each asset class, making asset holdings effectively complementary. The strategic complementarity due to uncertainty aversion generates the possibility of multiple equilibria.

There are two types of equilibria when investors are uncertainty averse. The first type of equilibrium has the same properties as the one in which investors are uncertainty neutral, as described in Theorem 2. In this equilibrium, banks invest in the risky assets, offer partial insurance to investors, are illiquid and exposed to runs. We will denote this equilibrium as the "risky" equilibrium. In the second equilibrium, banks invest only in the riskless asset, making the banking system effectively immune to runs, an equilibrium we will denote as the "safe" equilibrium. In this second "safe" equilibrium, banks refrain from investing in the (potentially) more profitable risky assets and, rather, invest only in the safe asset. Since investment in risky assets typically consists in carrying out banks' ordinary lending activity, we interpret this equilibrium as a "lending freeze."

We will make the following additional assumption:

$$(A_1): \; e^{\hat{\theta}_0 - \theta_1} R \; < 1.$$

This inequality implies that in the core beliefs set there are beliefs such that an investor holding such beliefs is not willing to invest in risky project A. In addition, since $\theta_1 - \hat{\theta}_1 = \hat{\theta}_0 - \theta_0$, this also implies that $e^{\theta_0 - \hat{\theta}_1} R \; < 1$ and, thus, that there are, in the core beliefs set, also (other) beliefs such that an investor holding such beliefs is not willing to invest in risky project B. As will become apparent below, (A_1) implies that an uncertainty-averse investor would not be willing to invest in a risky asset individually, while she may still be willing to invest in a portfolio of risky assets. The equilibrium with uncertainty-averse investors is characterized in the following.

Theorem 3 *If investors are uncertainty averse and (A1) holds, there are both a "risky" equilib-*

rium, where the optimal deposit contract is again d_τ^{R*} characterized in (11), and a "safe" equilibrium, in which both banks invest only in the safe technology and offer a safe deposit contract, d_τ^{S*}, with no insurance against liquidity risk: $d_{2A}^r = d_{2B}^r = 0$. Investors optimally invest equally in both banks. Furthermore: (i) The "risky" equilibrium Pareto dominates the "safe" equilibrium; (ii) runs are not possible in the "safe" equilibrium, but runs are possible in the "risky" equilibrium. (iii) All runs will be systemic.

Theorem 3 shows that the presence of uncertainty aversion has the effect of creating a second equilibrium in addition to the one prevailing in an economy populated by SEU agents. In addition to the equilibrium where banks invest in risky technology and offer (partial) insurance against liquidity shocks that prevails when investors are uncertainty neutral, there is also a "lending freeze" equilibrium in which banks refrain from investing in risky assets. In this second "lending freeze" equilibrium, banks invest only in the riskless asset and, thus, cannot provide any insurance against liquidity risk.

Existence of the "lending freeze" equilibrium depends critically on the fact that an uncertainty-averse investor is willing to deposit funds in one type of banks and, thus, be exposed to one type of risk, only if she can invest also in the other bank and, thus, be exposed to the other source of risk as well. This implies that if one bank offers only the safe contract, the other bank will only offer the safe deposit contract as well. Thus, uncertainty aversion creates a strategic externality in the deposit-offering policy of banks: investors invest in one bank only if they have the opportunity to invest in the other bank as well. This externality creates the potential of a "coordination failure" among banks that leads to the possibility of multiple equilibria. In addition, the second "safe" equilibrium is Pareto dominated by the "risky" equilibrium where banks invest in both risky assets.

A second important effect of uncertainty aversion is that a run on a class of banks also causes a run on the other class of banks. A run by long-term investors on a bank of any given risk class shifts the composition of risky assets in their portfolios in favor of the other risk class. From Lemma 2, this change of portfolio composition causes the investors to become more pessimistic on the asset class still in their portfolios, triggering a run on that asset class as well. Thus, uncertainty aversion creates systemic risk.

3 Uncertainty aversion and systemic risk

There are two distinct categories of runs in our economy: panic runs and fundamental runs. Panic runs occur when investors run a bank, even though the bank would still be solvent if they did not run, and investors would prefer the outcome of no one running. Panic runs are essentially due to a coordination failure among agents in an otherwise solvent economy. A fundamental run occurs when there is a shock to fundamentals large enough so that it ceases to be optimal for a long-term investor to remain invested in the bank, even if everyone else stays in the bank. Since in the "safe" equilibria bank runs are not possible, we will focus on the (symmetric) "risky" equilibrium.

A further important distinction is that bank runs can either be "local runs," that is, involving only one bank, or "systemic runs," that is, runs that involve both banks. As shown in Theorem 2 and Theorem 3, runs are always possible in a "risky" equilibrium. However, when investors are uncertainty neutral, runs may not necessarily spread from one bank to the other. In contrast, if investors are uncertainty averse, all runs will be systemic.

To model the possibility of equilibrium runs, following Goldstein and Pauzner (2005), we now assume that, at $t = 1$, investors receive public signals, s_τ, $\tau \in \{A, B\}$, that are informative on the return on the risky assets at time $t = 2$. Specifically, we assume that $R_\tau = s_\tau R$, with $s_\tau \in \{\phi, 1\}$ and $\phi < 1$. We also assume that with probability $\varepsilon > 0$ investors observe "bad news" about type τ assets only, $s_\tau = \phi$ and $s_{\tau' \neq \tau} = 1$, for $\tau \in \{A, B\}$, while with probability Δ, investors observe "bad news" about both type A and type B assets, $s_\tau = s_{\tau' \neq \tau} = \phi$, and with probability $1 - 2\varepsilon - \Delta$, investors learn that both asset classes are unaffected, $s_\tau = s_{\tau' \neq \tau} = 1$. Because "bad news" about both banks generate the expected and arguably uninteresting outcome of fundamental systemic runs, we set $\Delta = 0$. For tractability, we now assume that investors' utility function, u, is piece-wise affine. Specifically,

$$u(w) = \begin{cases} \psi w & w \leq \tilde{c} \\ \psi \tilde{c} + (w - \tilde{c}) & w > \tilde{c} \end{cases} \tag{12}$$

where $\psi > e^{\frac{1}{2}(\theta_0 - \theta_1)} R$ and $\tilde{c} \in \left(2, 2\frac{e^{\theta^e - \theta_1} R}{\lambda e^{\theta^e - \theta_1} R + (1-\lambda)}\right)$. This utility function captures the notion that early investors value lower consumption levels, up to $\tilde{c}$, relatively more than larger consumption. It also implies that early investors, who are subject to the liquidity shock, value consumption more than late investors, preserving the value of insurance against the liquidity shock.

In this section, we focus on fundamental runs, and we assume that investors run on a bank only if it is no longer profitable to stay in the bank, effectively ruling out panic-based runs. We start the analysis by establishing the possibility of systemic runs under uncertainty aversion for given (arbitrary) deposit contracts $d_\tau = \{d_{1\tau}, d^s_{2\tau}, d^r_{2\tau}\}$, $\tau \in \{A, B\}$. We will then characterize the optimal deposit contracts.

Theorem 4 *Let $d_\tau = \{d_{1\tau}, d^s_{2\tau}, d^r_{2\tau}\}$, $\tau \in \{A, B\}$ be symmetric deposit contracts with $d^s_{2A} = d^s_{2B} = 0$ and $d^r_{2\tau} > 0$ (i.e, risky deposit contracts) so that investors strictly prefer staying in both banks in the absence of bad news. If investors are not uncertainty averse, they will run Bank τ following bad news about type τ assets if $d_{1\tau} > \phi p_\tau(\theta^e) R d^r_{2\tau}$, but investors will not run Bank $\tau' = \tau$. If investors are uncertainty averse, they will run both banks if $d_{1\tau} > \phi^{\frac{1}{2}} p_\tau(\theta^e) R d^r_{2\tau}$.*

Theorem 4 uncovers a new source of systemic risk that is due to uncertainty aversion, and provides one of the key results of our paper. The theorem shows that, in the presence of uncertainty-averse investors, bad news at one bank, say Bank A, while it generates a fundamental run on that bank, also induces investors to run on the other bank, Bank B, even in the absence of bad news at the latter bank. Thus, bad news on one bank can create a systemic run; in other words, idiosyncratic risk can indeed generate systemic risk.

The mechanism behind the systemic risk described in Theorem 4 is the uncertainty hedging motive due to uncertainty aversion (see Theorem 1). As discussed earlier, investors' demand for a risky asset depends on their overall portfolio. In particular, an uncertainty-averse investor is willing to be invested in one bank, and to be exposed to the risk of one type of assets, provided that she is also exposed to the other type of risky assets as well. This implies that, if the investor learns bad news about one risky-asset class, say $\tau = A$, inducing a run on Bank A, the investor's portfolio will become overly exposed to the other risky asset class, $\tau = B$. From Lemma 2, we know that the resulting portfolio imbalance causes a shift in the investor's beliefs against the other asset class, B, making the investor relatively more pessimistic about risky asset B. Thus, a run on Bank B may happen even if that bank was not affected by bad news. Thus, bad news about Bank A spills over to Bank B causing contagion and, thus, systemic risk. Note that this source of contagion and systemic risk is entirely driven by uncertainty aversion and is novel in the literature. It will be denoted as "uncertainty-based" systemic risk, which generates "uncertainty-based" systemic runs.

Theorem 4 describes investors' behavior in response to negative shocks, given the contract that they are in. Banks, however, offer ex-ante optimal deposit contracts that anticipate such behavior.

Lemma 3 *Let early investors have piecewise affine utility as in (12) and ε be small enough. (i) If investors are not uncertainty averse, the unique equilibrium is a "risky equilibrium" where banks invest in the risky technology and provide insurance against the liquidity shock by offering the deposit contract:*

$$d_{1\tau} = \frac{1}{2}\tilde{c},\ d^s_{2\tau} = 0, \quad \textit{and}\ \ d^r_{2\tau} = \frac{1-\lambda d_{1\tau}}{1-\lambda},\ \textit{for}\ \tau \in \{A, B\}.$$

(ii) If investors are uncertainty averse, there are two equilibria: the "risky equilibrium" described in part (i), and a "safe" equilibrium where banks hold only the risk-free asset and the deposit contract is a safe deposit contract: $d_{1\tau} = d^s_{2\tau} = 1$, $\tau \in \{A, B\}$.

Lemma 3 shows that the equilibrium contracts mimic those described in Theorem 2 and Theorem 3.[20] However, the presence of a public signal on the return on the risky assets, and thus on the banks' expected profitability, generates the possibility of fundamental bank runs, as follows.

Theorem 5 *Suppose early investors have utility as in (12), and banks invest in risky assets. If investors are not uncertainty averse, they run Bank-τ after observing bad news on that bank ($s_\tau = \phi$) iff $\phi < \underline{\phi} \equiv \frac{(1-\lambda)\tilde{c}}{e^{\frac{1}{2}(\theta_0-\theta_1)}R(2-\lambda\tilde{c})}$, with $0 < \underline{\phi} < 1$, but investors will not run the other bank. If investors are uncertainty averse, they will run both banks after observing bad news on either of the two banks, that is $s_\tau = \phi$ or $s_{\tau'\neq\tau} = \phi$, iff $\phi < \underline{\phi}^2$.*

Theorem 5 describes the two effects of uncertainty aversion on bank runs and systemic risk. First, as discussed in Theorem 4, the presence of uncertainty aversion creates the possibility of systemic runs even in cases where such runs would not occur under SEU. Thus, uncertainty aversion is a source of contagion and systemic risk. However, under uncertainty aversion, investors are *slower*

[20] Note that in the optimal contract in the "risky" equilibrium, banks provide (partial) insurance against the liquidity shock, since the marginal utility of early consumption (measured by ψ) is sufficiently large. Insurance is limited (late investors strictly prefer not mimicking early investors) because $\tilde{c}$ is not too large.

to run after observing bad news on a bank than SEU investors. This happens because uncertainty-averse investors value their investment in a risky asset more if they hold the other risky asset in their portfolio as well. This means that an uncertainty-averse investor is more reluctant to run a bank after observing bad news on that bank. However, if the bad news is sufficiently bad to induce a run, the run spreads to the other bank. Thus, uncertainty-averse investors are less prone to bank runs, but when they run they generate a systemic run.[21]

4 Bank runs and the stock market

In the previous sections, we discussed the effect of uncertainty aversion on the systemic risk of the banking sector. An important question is the potential connection between bank runs and the performance of other parts of the financial sectors such as the stock market. For example, in the recent financial crisis the near collapse of the (shadow) banking system was also associated with a substantial drop of the stock market. This observation raises the question of the transmission mechanism between the banking sector and the "real" sector. In this section, we show the contagion effect that we described in the previous sections can spread beyond the banking sector and spill over to the stock market as well.

We modify our basic model as follows. Suppose that Bank A is still a bank, which now represents the overall banking sector, but Bank B is now a stock company (or a mutual fund), denoted as Firm B, which now represents the stock market. In this new interpretation, the stock company has access to type-B assets. In the spirit of Jacklin (1987), we posit that Firm B promises to pay investors a dividend Δ_{1B} at time $t = 1$, and holds a portfolio $\{\sigma_{2B}, \rho_{2B}\}$ of the safe asset and type-B asset, until $t = 2$. Similar to our discussion in the previous section, Bank A offers a contract that gives investors the choice between receiving d_{1A} at $t = 1$ and receiving d^s_{2A} of the riskless asset of d^r_{2A} of type A assets at $t = 2$. Investors still face the possibility of a liquidity shock, so they

[21] It should be noted, however, that Theorem 5 depends on the assumption that utility is piecewise affine, as in (12). Affine utility guarantees that banks set the intermediate cashflow at the kink, so $d_{1\tau} = \frac{1}{2}\tilde{c}$. Thus, the optimal contract does not change when investors anticipate learning news. If u were strictly concave, results are similar but banks would decrease $d_{1\tau}$, unless there is an Inada condition for u. Because sufficient bad news induces a run on both banks, it would be possible for early households to receive 0, so banks would drastically change contracts to avoid that state even for very small probability events if there were an Inada condition. Also, banks would have to decide if they were going to avert a fundamental run, or to allow a fundamental run (optimally choosing the contract with the risk of a run in mind). In either scenario, banks decrease the insurance provided to early type, $d_{1\tau}$.

would like to have insurance against it. For tractability, we will assume again that early investors have affine utility as in (12).

Lemma 4 *The stock company implements incentive-compatible cash flow of* $\{d_{1B}, d_{2B}^{s}, d_{2B}^{r}\}$ *by setting* $\Delta_{1B} = \lambda d_{1B}$*,* $\sigma_{2B} = (1-\lambda)\, d_{2B}^{s}$*, and* $\rho_{2B} = (1-\lambda)\, d_{2B}^{r}$*. Late investors use the dividend to buy shares from the late consumers for price* $P_{1B} = (1-\lambda) d_{1B}$*.*

Lemma 4 follows directly from the line of reasoning described in Jacklin (1987). The stock company, Firm B, can duplicate the payouts of a bank by committing to pay investors a certain dividend at $t = 1$. Early investors, because they must consume at $t = 1$, finance consumption using the dividend plus the proceeds from the sale of Firm-B shares to late investors. Late investors, in turn, use the dividend they receive from Firm B to purchase shares from selling early investors, and then consume at $t = 2$ the liquidating dividend they receive from Firm B. Investors' portfolio allocation between banks and the stock market is as follows.

Lemma 5 *Each investors deposits half of their wealth in the bank and buys one share of equity with the other half. If investors are uncertainty neutral, the "risky" equilibrium will be implemented. If investors are uncertainty averse, there are both the "safe" equilibrium and the "risky" equilibrium.*

Lemma 5 shows that the equilibrium from Lemma 3 is not sensitive to the institutional structure. In the spirit of Jacklin (1987), if no bad news arrives, the equilibrium allocation is identical whether the intermediaries are stock companies or banks. What happens if there is bad news?

Theorem 6 *Idiosyncratic risk leads to systemic risk iff investors are uncertainty averse. That is, bad news about the bank harms the market value of the stock, and bad news about the stock can produce a run on the bank, iff investors are uncertainty averse.*

Theorem 6 establishes a new mechanism for bad news to spread across segments of the financial sectors in an economy. Specifically, uncertainty aversion generates complementarity among different asset classes in the economy. Because of asset complementarity, bad news spreads directly across asset classes, due to investor preferences. This means that systemic risk extends to the broader financial sector, generating fragility for the whole financial sector.

Theorem 6 implies that a run on the banking sector is associated with a negative performance of the stock market as well, that it leads to a "market crash." Our model also implies that investors would run to redeem their shares in mutual funds that have demandable features, such as money market funds, leading to a "breaking of the buck." Also, our model proposes a new channel through which financial crises spread from the banking sector to the real sector. Note that this new channel is driven by the impact of a bank run on investors' beliefs, generating a negative effect on stock market valuations. Thus, our theory differs from the more traditional view that a crisis in the banking sector affects negatively banks' lending and, thus, the real sector and stock market valuations.

Theorem 6 also implies that sufficiently negative news on the stock markets, which leads to a stock market "crash," also induces a run on the banking system. The bank run is then followed by a subsequent rebalancing of the long term investors' portfolios with a reinvestment of their holdings in the safe asset. Thus, a bank run generates a "flight to quality," as often observed in reality.

5 Increased uncertainty and financial crises

In this section, we examine the impact of the "extent" of uncertainty on financial system fragility and contagion. We show that increasing uncertainty makes the financial system more fragile and more prone to contagions and, thus, more vulnerable to systemic risk.

We measure the extent of uncertainty by the size of investors' core belief set, as follows. Let $\alpha \equiv \theta^e - \hat{\theta}_0$. We interpret an increase of α as characterizing "greater uncertainty." In this paper we take as exogenous the factors that may induce time series variations of the parameter α. However, Epstein and Schneider (2011) suggests that such variations in uncertainty may be the product of learning by uncertainty-averse agents. The impact of increasing uncertainty is characterized in the following.

Theorem 7 *There are critical values $\{\underline{\alpha}, \bar{\alpha}\}$ such that*

1. *for $\alpha \leq \underline{\alpha}$ the only equilibrium is the "risky equilibrium," and there is no contagion;*

2. *If $\underline{\alpha} < \alpha < \bar{\alpha}$ the only equilibrium is the "risky equilibrium," but there is contagion and all runs are systemic;*

3. If $\alpha \geq \bar{\alpha}$, there both "risky" equilibria, with the possibility of contagion and systemic runs, and "safe" equilibria with a "lending freeze."

Theorem 7 shows that greater uncertainty leads to a more fragile financial system. When uncertainty is low, that is, for $\alpha \leq \underline{\alpha}$, the only equilibrium is the "risky" equilibrium that mimics the usual Diamond and Dybvig (1993) scenario. In this case, fundamental runs are possible following bad news on a bank's future expected profitability, but runs are local and do not create contagion. At intermediate levels of uncertainty, that is, for $\underline{\alpha} < \alpha < \bar{\alpha}$, bad news from one bank can spread to the other bank, thus creating contagion and systemic risk. At even greater levels of uncertainty, that is, for $\alpha \geq \bar{\alpha}$, "safe" equilibria are also possible. In this case, the financial system may retrench in a "safety mode" whereby banks invest only in the safe asset. This equilibrium may emerge when banks expect other banks to be in the "safety mode" as a consequence of the increased uncertainty. This is a "lending freeze" equilibrium, which is Pareto-inferior to the more ordinary "risky" equilibrium (see Section 2.3).

6 Multiple Banks

In this section, we will show that the main results of this paper are unchanged when there are multiple banks. Similar to Section 2, our economy is endowed with $N+1$ types of assets: N classes of risky assets and a riskless asset. Specifically, making an investment in a risky asset at $t = 0$ generates at $t = 2$ a random payoff in terms of the riskless asset: a unit investment in type τ asset produces at $t = 2$ a payoff of R with probability p_τ and a payoff of 0 with probability $1-p_\tau$. Similar to Section 2, risky assets have an early liquidation option.

Departing from Section 2, there is uncertainty over the vector $\overrightarrow{\theta} = \{\theta_\tau\}_{\tau=1}^{N}$, where $p_\tau(\theta_\tau) = e^{\theta_\tau - \theta_{Max}}$. To capture the notion that beliefs are not rectangular, we will assume that $\theta_\tau \in [\theta_L, \theta_H] \subset [\theta_{\min}, \theta_{Max}]$, and that $\Sigma_{\tau=1}^{N}\theta_\tau = N\theta^e + \kappa$, where κ is the aggregate shock to the economy. Further, assume that $\kappa \in [-A, A]$, and that $N\theta_L < N\theta^e - A$ and $N\theta_H > N\theta^e + A$.

Similar to Section 2, Bank τ offers the contract $\{d_{1\tau}, d_{2\tau}^s, d_{2\tau}^r\}$. If an investor holds \$1 in each bank, they receive utility

$$U = \Sigma_{\tau=1}^{N}\lambda d_{1\tau} + (1-\lambda)\min_{\overrightarrow{\theta}} U_L\left(\overrightarrow{\theta}\right)$$

where

$$U_L\left(\overrightarrow{\theta}\right) = \Sigma_{\tau=1}^{N}\left[d_{2\tau}^{s} + p_{\tau}\left(\theta_{\tau}\right) R d_{2\tau}^{r}\right].$$

We need to consider the endogenous beliefs.

Lemma 6 *Uncertainty-averse investors will have interior beliefs if banks offer contracts that have similar risky payoffs. Investors will believe the worst about the aggregate state, $\kappa = -A$, so their worst-case scenario will be*

$$\theta_{\tau} = \theta^{e} - \frac{A}{N} + \frac{1}{N}\sum_{\tau'=1}^{N} \ln d_{2\tau'}^{r} - \ln d_{2\tau}^{r}.$$

For ease of exposition, we write the expression for beliefs in the statement Lemma 6 only in the case when beliefs are interior. Beliefs are derived in full generality if the proof of Lemma 6. The general expression for endogenous beliefs is uglier than that in Lemma 1 due to higher dimensionality, but is qualitatively similar. Indeed, similar to Lemma 2, we still achieve the spillover effect of uncertainty aversion (increasing the portfolio position of a given asset type makes the investor more optimistic about other asset types).

To show that the results of the paper extend to the multiple bank case, we make the following regularity assumptions. First, assume that $e^{\theta^{e}-\frac{A}{N}-\theta_{Max}}R > 1 > e^{\frac{1}{N-1}(N\theta^{e}-A-\theta_{H})-\theta_{Max}}$. The first inequality guarantees that it is a positive NPV project to invest in the risky asset, if all of the banks invests. The second inequality guarantees that it is a negative NPV project to invest in the risky asset if one of the other banks does not.

Theorem 8 *In the absence of uncertainty aversion, the only equilibrium is the risky equilibrium, and local shocks stay local. In the presence of uncertainty aversion, there are both the risky equilibrium (where all banks invest in risky assets) and the safe equilibrium (where no banks invest in the risky asset), and all runs will be systemic.*

Thus, the results of this paper extend to the case of multiple banks.

7 Policy implications

The role of regulation to curb systemic risk and promote financial stability has been the object of extensive discussion in the recent academic and public policy debate. To implement effective stabilization polices and regulations, it is critical to understand the source of systemic risk. In this section, we outline the implications of our model for such debate.

1. *Uncertainty and transparency.* One of the basic results of our paper is that uncertainty harms stability and creates the possibility of systemic runs. The financial system is more fragile in times of greater uncertainty. In these cases, regulatory authorities may wish to release relevant information that reduces such uncertainty.

2. *Banks' equity recapitalization.* Negative idiosyncratic shocks at any one bank will have a negative effect on that bank's equity capitalization. Our paper suggest that such negative shock may affect other banks' equity capitalizations as well triggering a widespread banking crisis. In other words, an idiosyncratic shock on one bank depresses its equity value, and the negative sentiment spreads to other banks which may now see distressed equity valuations. This may result in banks facing binding minimum equity requirements and may force banks to raise new equity at distressed equity prices. Thus, honoring excessive equity requirements would be very costly to banks.

3. *Bank bailout strategies.* If the central bank wants to prevent bank runs, what kind of bailout policy must it implement to prevent them? Our paper shows that, if investors are uncertainty-averse, the central bank must worry about idiosyncratic shocks that affect individual banks, since these shocks can have systemic effects. In addition, the implementation of the bailout policy depends on the size of the shocks affecting the banking sector. For sufficiently small shocks, the central bank can avert a run by bailing out just the affected bank. If the shock is large enough, however, the central bank must also bail out unaffected (potentially solvent) banks to avoid a systemic crisis. In contrast, if investors are not ambiguity averse, the central bank only needs to bailout the affected bank.

4. *Bailouts and Asset Sales.* Suppose the central bank wants to prevent a systemic run and wants

to find the most efficient way to do so. The central bank can either provide capital directly to the banks to fund their short term liquidity needs (a bailout, discussed above), or it can buy a bank's risky assets and replace them with the safe asset (asset sales). Thus, bailouts inject liquidity without changing a bank's balance sheet, while asset sales change the risk structure of the bank's portfolio. If investors are uncertainty averse, and the shock is large enough, our paper suggests that the optimal intervention policy involves asset sales. However, the central bank must purchase assets from the *unaffected* bank, not from the affected bank. Bad news to one bank effectively shifts the composition of investors' portfolios toward the other bank's holdings, so investors become more pessimistic about the unaffected bank's holdings. The central bank will be able to purchases these assets at distressed prices, which means that, ex post, the central bank will make large profits from these asset sales. If investors are ambiguity neutral, there is no place for asset sales.

5. *The Volcker Rule.* Our paper has also implications for the Volcker Rule. We interpret here the Volcker Rule as forcing banks to be robust to a run at other banks, that is, to systemic runs. Our paper shows that to be immune to runs at other banks, this requirement means that banks must invest only in the safe asset, effectively ruling out the Pareto-superior "risky" equilibrium. Thus, regulation aimed that ensuring systemic stability may infect imply efficiency losses.

   The upside of the Volcker Rule, however, is that regulating traditional banks can kill the shadow banking sector (similar to Section 4, model the shadow banking sector as Bank B). This could be optimal if runs impose large negative externalities on the economy as a whole.

8 Conclusion

In this paper, we propose a new theory of systemic risk based on uncertainty aversion. We show that uncertainty aversion creates complementarities among investors' asset holdings, a feature that we denote uncertainty hedging. Because of uncertainty hedging, bad news on an asset class may spread to other asset classes, generating systemic risk. A second implication of uncertainty hedging is that banks may individually refrain from investing in risky assets even if, collectively, it would be beneficial to do so. In these situations, risky asset are valued by investors at distressed prices, and banks invest only in the safe assets, a feature that we describe as a "lending freeze." Finally, we derive public policy implications of our model.

References

[1] Acharya, V., Mehran, H., and Thakor, A. (2013) "Caught between Scylla and Charybdis? Regulating Bank Leverage When There is Rent Seeking and Risk Shifting," ECGI - Finance Working Paper 365/2013, SSRN Working Paper 1786637.

[2] Acharya, V. and Yorulmazer, T. (2008) "Information Contagion and Bank Herding," *Journal of Money, Credit and Banking*, **40**: 215-231.

[3] Allen, F., Babus, A., and Carletti, E. (2012) "Asset Commonality, Debt Maturity and Systemic Risk," *Journal of Financial Economics*, **104**: 519-534.

[4] Allen, F., Carletti, E., and Gale, D. (2009) "Interbank Market Liquidity and Central Bank Intervention," *Journal of Monetary Economics*, **56**(5), 639–652.

[5] Allen F. and Gale, D. (1999) "The Asian Crisis and the Process of Financial Contagion," *Journal of Financial Regulation and Compliance*, **7**(3): 243-249.

[6] Allen F. and Gale, D. (2000) "Financial Contagion," *Journal of Political Economy*, **108**: 1-34.

[7] Anscombe, F. and Aumann, R. (1963) "A Definition of Subjective Probability," *The Annals of Mathematical Statistics*, **34**: 199-205.

[8] Aryal, G. and Stauber, R. (2014) "A Note on Kuhn's Theorem with Ambiguity Averse Players," *Economics Letters*, **125**: 110-114.

[9] Bhattacharya S. and Gale, D. (1987) "Preference Shocks, Liquidity and Central Bank Policy," in W. Barnett and K. Singleton (eds.), *New Approaches to Monetary Economics*, Cambridge University Press, 69-88.

[10] Brusco S. and Castiglionesi, F. (2007) "Liquidity Coinsurance, Moral Hazard and Financial Contagion," *Journal of Finance*, **62**: 2275-2302.

[11] Caballero, R. and Krishnamurthy, A. (2008) "Collective Risk Management in a Flight to Quality Episode," *Journal of Finance*, **63**: 2195-2230.

[12] Chari, V. (1989) "Banking Without Deposit Insurance or Bank Panics: Lessons from a Model of the U.S. National Banking System," *Federal Reserve Bank of Minneapolis Quarterly Review,* **13**(3): 3-19.

[13] Chari, V. and Jagannathan, R. (1988) "Banking Panics, Information and Rational Expectations Equilibrium," *Journal of Finance*, **43**: 749-763.

[14] Diamond, D. and Dybvig, P. (1983) "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, **91**: 401-419.

[15] Dicks, D. and Fulghieri, P. (2015) "Ambiguity, Disagreement, and Allocation of Control in Firms," ECGI - Paper 396/2013, UNC Kenan-Flagler Research Paper No. 2357599, SSRN Working Paper 2357599.

[16] Easley, D. and O'Hara, M. (2009) "Ambiguity and Nonparticipation: The Role of Regulation," *Review of Financial Studies*, **22**: 1817-1843.

[17] Ellsberg, D. (1961) "Risk, Ambiguity, and the Savage Axioms." *Quarterly Journal of Economics,* **75**: 643-669.

[18] Epstein, L. and Schneider, M. (2003) "Recursive Multiple Priors," *Journal of Economic Theory*, **113**: 1-31.

[19] Epstein, L. and Schneider, M. (2011) "Ambiguity and Asset Markets," *Annual Review of Financial Economics*, **2**: 315-346.

[20] Freixas, X., Parigi, B., and Rochet, J. (2000) "Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank," *Journal of Money, Credit and Banking*, **32:** 611-38.

[21] Gârleanu, N., Panageas, S., and Yu, J. (2014) "Financial Entanglement: A Theory of Incomplete Integration, Leverage, Crashes, and Contagion," *American Economic Review*, forthcoming.

[22] Gilboa, I. and Schmeidler, D. (1989) "Maxmin Expected Utility with a Non-Unique Prior," *Journal of Mathematical Economics*, **18**: 141-153.

[23] Goldstein, I. and Pauzner, A. (2005) "Demand-Deposit Contracts and the Probability of Bank Runs," *Journal of Finance*, **60**: 1293-1327.

[24] Hansen, L. and Sargent, T. (2008) *Robustness.* Princeton University Press.

[25] Jacklin, C. (1987) "Demand Deposits, Trading Restrictions, and Risk Sharing." In *Contractual Arrangements for Intertemporal Trade*, eds. E.C. Prescott and N. Wallace. Minneapolis, MN: University of Minnesota Press: 26–47.

[26] Jacklin C. and Bhattacharya, S. (1988) "Distinguishing Panics and Information-based Bank Runs: Welfare and Policy," *Journal of Political Economy*, **96**: 568-592.

[27] Keynes, J. (1921) *A Treatise on Probability.* London: Macmillan.

[28] Klibanoff, P., Marinacci, M. and Mukerji, S. (2005) "A Smooth Model of Decision Making under Ambiguity," *Econometrica*, **73**: 1849–1892.

[29] Knight, F. (1921) *Risk, Uncertainty, and Profit.* Boston: Houghton Mifflin.

[30] Kodres, L., and Pritsker, M. (2002) "A Rational Expectations Model of Financial Contagion," *Journal of Finance,* **57**: 769-799.

[31] Krishnamurthy, A. (2010) "Amplification Mechanisms in Liquidity Crises," *American Economic Journal: Macroeconomics* **2**(3): 1-30.

[32] Rochet, J. and Vives, X. (2004) "Coordination Failures and the Lender of Last Resort: Was Bagehot Right after All?" *Journal of the European Economic Association*, **2:** 1116-1147.

[33] Savage, L. J. (1972) *The Foundations of Statistics* (2nd revised edition). New York: Dover Publications.

[34] Siniscalchi, M. (2011) "Dynamic Choice under Ambiguity," *Theoretical Economics*, **6**: 379–421.

[35] Wallace, N. (1988) "Another attempt to explain an illiquid banking system: The Diamond and Dybvig model with sequential service taken seriously" *Federal Reserve Bank of Minneapolis Quarterly Review* **12**(4): 3-16.

[36] Wallace, N. (1990) "A Banking Model in Which Partial Suspension Is Best" *Federal Reserve Bank of Minneapolis Quarterly Review* **14**(4): 11-23.

A Appendix

Proof of Theorem 1. Let $V_\mu = qE_\mu [u(y_1)] + (1-q)E_\mu [u(y_1)]$, and define $\mu_1 = \arg\min E_\mu [u(y_1)]$, $\mu_2 = \arg\min E_\mu [u(y_2)]$, and $\mu_q = \arg\min V_\mu$. Thus, $E_{\mu_1}[u(y_1)] \le E_{\mu_q}[u(y_1)]$ and $E_{\mu_2}[u(y_2)] \le E_{\mu_q}[u(y_2)]$, so $qE_{\mu_1}[u(y_1)] + (1-q) E_{\mu_2}[u(y_2)] \le qE_{\mu_q}[u(y_1)] + (1-q) E_{\mu_q}[u(y_2)] = \min V_\mu$. Thus, (3) holds. Because uncertainty-neutral agents can be modeled as uncertainty-averse agents with a singleton for their core of beliefs, the inequality holds with equality in the absence of uncertainty aversion. ■

Proof of Lemma 1. Note $U_L'(\theta) = R\left\{e^{\theta-\theta_1} d_{2A}^r - e^{\theta_0-\theta} d_{2B}^r\right\}$. Because $U_L'' > 0$, U_L is convex in θ, so first order conditions are sufficient for a minimum. Note $U_L' = 0$ iff $\theta = \tilde{\theta}^a$ where

$$\tilde{\theta}^a(\Pi) = \frac{1}{2}(\theta_0+\theta_1) + \frac{1}{2}\ln\frac{d_{2B}^r}{d_{2A}^r}.$$

Thus, if $\tilde{\theta}^a(\Pi) \in \left[\hat{\theta}_0, \hat{\theta}_1\right]$, $\theta^a = \tilde{\theta}^a$ (because $\tilde{\theta}^a$ is minimizes U_L). If $\tilde{\theta}^a < \hat{\theta}_0$, $U_L' > 0$ for all $\theta \in \left[\hat{\theta}_0, \hat{\theta}_1\right]$, so $\theta^a = \hat{\theta}_0$. Similarly, if $\tilde{\theta}^a > \theta_1$, $U_L' < 0$ for all $\theta \in \left[\hat{\theta}_0, \hat{\theta}_1\right]$, so $\theta^a = \hat{\theta}_1$. Therefore, (5) corresponds to the worst-case scenario for investors. ■

Proof of Theorem 2. We will guess that (7) and (8) are lax, solve the relaxed problem then show that these two constraints are satisfied. Because investors are not uncertainty averse, they believe $C = \{\theta^e\}$. For $\tau \in \{A, B\}$, Bank τ's problem is

$$\begin{aligned}
&\max U_0 \\
&\lambda d_{1\tau} + (1-\lambda)\left[d_{2\tau}^s + d_{2\tau}^r\right] \le 1 \\
&U_L(\theta^e) \ge d_{1A} + d_{1B},
\end{aligned}$$

where

$$U_0 = \lambda u(d_{1A} + d_{1B}) + (1-\lambda) U_L(\theta^e),$$

and

$$U_L(\theta) = d_{2A}^s + d_{2B}^s + p_A(\theta^e) R d_{2A}^r + p_B(\theta^e) R d_{2B}^r.$$

Let $\kappa_{\tau 1}$ be the multiplier for the budget constraint of Bank τ, let $\kappa_{\tau 2}$ be the multiplier for the incentive compatibility constraint of Bank τ, and let L_τ be the Lagrangian function for Bank τ, for $\tau \in \{A, B\}$. Thus, the FOCs for Bank τ are

$$\begin{aligned}
\frac{dL_\tau}{dd_{1\tau}} &= \lambda u'(d_{1A} + d_{1B}) - \lambda\kappa_{\tau 1} - \kappa_{\tau 2}, \\
\frac{dL_\tau}{dd_{2\tau}^s} &= (1-\lambda) - (1-\lambda)\kappa_{\tau 1} + \kappa_{\tau 2},
\end{aligned}$$

and

$$\frac{dL_\tau}{dd_{2\tau}^r} = (1-\lambda) p_\tau(\theta^e) R - (1-\lambda)\kappa_{\tau 1} + p_\tau(\theta^e) R\kappa_{\tau 2}.$$

Because $\theta^e = \frac{1}{2}(\theta_0+\theta_1)$, $p_A(\theta^e) = p_B(\theta^e) = e^{\frac{1}{2}(\theta_0-\theta_1)}$. Because $e^{\frac{1}{2}(\theta_0-\theta_1)} R > 1$, $\frac{dL_\tau}{dd_{2\tau}^r} > \frac{dL_\tau}{dd_{2\tau}^s}$. Because there is no upper bound on $d_{2\tau}^s$ or $d_{2\tau}^r$, this implies that $d_{2\tau}^s = 0$.

It can be quickly verified that the constraints will be identical for both banks.[22] If the IC constraint binds

[22] We have four equations ($\frac{dL_A}{dd_{1A}} = 0$, $\frac{dL_A}{dd_{2A}^r} = 0$, $\frac{dL_B}{dd_{1B}} = 0$, and $\frac{dL_B}{dd_{2A}^r} = 0$) which are linear in four multipliers $(\kappa_{A1}, \kappa_{A2}, \kappa_{B1}, \kappa_{B2})$. Thus, there will be a unique set of multipliers that satisfy the FOCs because the system of equations is full rank. Because the equations are symmetric ($u'(d_{1A}+d_{1B})$ is the same for both banks and $e^{\theta^e-\theta_1} = e^{\theta_0-\theta^e}$ by definition of θ^e), $\kappa_{A1} = \kappa_{B1}$ and $\kappa_{A2} = \kappa_{B2}$. Therefore, the same constraints will bind at both banks. The budget constraint always binds (monotonicity). Either the IC constraint binds at both banks or it is lax at both banks.

$(\kappa_{A2} = \kappa_{B2} > 0)$, the equilibrium is given by

$$\lambda d_{1A} + (1-\lambda)\, d_{2A}^{r} = 1$$
$$\lambda d_{1B} + (1-\lambda)\, d_{2B}^{r} = 1$$
$$e^{\theta^e - \theta_1} R d_{2A}^{r} + e^{\theta_0 - \theta^e} R d_{2B}^{r} = d_{1A} + d_{1B},$$

Solving for $d_{1A} + d_{1B}$, it follows that

$$d_{1A} + d_{1B} = 2\frac{e^{\theta^e - \theta_1} R}{e^{\theta^e - \theta_1} R\lambda + (1-\lambda)}$$

However, we assumed that $u'\left(2\frac{e^{\theta^e-\theta_1}R}{\lambda e^{\theta^e-\theta_1}R+(1-\lambda)}\right) < p_\tau(\theta^e)\, R$, so this cannot be an equilibrium: banks would decrease d_{1A} and d_{1B}.

Thus, the IC is lax at both banks: $\kappa_{A2} = \kappa_{B2} = 0$. $\frac{dL_A}{dd_{1A}} = 0$ implies that $\kappa_{A1} = u'(d_{1A} + d_{1B})$, and $\frac{dL_A}{dd_{2A}^{r}} = 0$ implies that $\kappa_{A1} = e^{\theta^e-\theta_1}R$, so $u'(d_{1A} + d_{1B}) = e^{\theta^e-\theta_1}R$. Thus, banks set $d_{1A} + d_{1B} = (u')^{-1}\left(e^{\theta^e-\theta_1}R\right)$ and sets $d_{2A}^{r} = \frac{1-\lambda d_{1A}}{1-\lambda}$ and $d_{2B}^{r} = \frac{1-\lambda d_{1B}}{1-\lambda}$. Because $u'\left(2\frac{e^{\theta^e-\theta_1}R}{\lambda e^{\theta^e-\theta_1}R+(1-\lambda)}\right) < e^{\theta^e-\theta_1}R$, $d_{1A} + d_{1B} < 2\frac{e^{\theta^e-\theta_1}R}{\lambda e^{\theta^e-\theta_1}R+(1-\lambda)}$ (because u is concave). Therefore, (6) is lax:

$$e^{\theta^e - \theta_1} R d_{2A}^{r} + e^{\theta_0 - \theta^e} R d_{2B}^{r} > d_{1A} + d_{1B},$$

We need to verify that (7) and (8) are satisfied. Because the core of beliefs is a singleton, $C = \{\theta^e\}$, (7) holds iff $e^{\theta^e-\theta_1} d_{2A}^{r} \geq d_{1A}$, while (8) holds iff $e^{\theta_0-\theta^e} d_{2B}^{r} \geq d_{1B}$. Therefore, both of these constraints are satisfied at the symmetric outcome: $d_{1A} = d_{1B}$ and $d_{2A}^{r} = d_{2B}^{r}$. Because $u'(2) > e^{\theta^e-\theta_1}R$, it follows that $d_{1A} + d_{1B} > 2$, so banks provide insurance against the liquidity shock. This insurance is imperfect because (6) is lax. WLOG, it is optimal for investors to invest equally in both banks.

Thus, $e^{\theta^e-\theta_1}Rd_{2A}^{r} > d_{1A} > 1$. Because $d_{1A} > 1$, runs are possible (the bank will be insolvent if all investors run). Because $e^{\theta^e-\theta_1}Rd_{2A}^{r} > d_{1A}$, investors prefer to stay in the bank if all other late investors stay. Thus, there are four equilibria at $t = 1$. The efficient equilibrium is that late investors stay in both banks. There is also a second equilibrium where late investors run only Bank A at $t = 1$, and a third where investors run only Bank B. Finally, there is an equilibrium where investors run both. Because a bank run on only one bank is an equilibrium, runs are not necessarily systemic. ■

Proof of Theorem 3. We will guess that (7) and (8) are lax, solve the relaxed problem then show that these two constraints are satisfied. Investors are uncertainty averse: they believe $C \in \left[\hat{\theta}_0, \hat{\theta}_1\right]$. For $\tau \in \{A, B\}$, Bank τ's simplified problem is

$$\max U_0$$
$$\lambda d_{1\tau} + (1-\lambda)\left[d_{2\tau}^{s} + d_{2\tau}^{r}\right] \leq 1$$
$$\min_{\theta \in C} U_L(\theta) \geq d_{1A} + d_{1B},$$

where

$$U_0 = \lambda u(d_{1A} + d_{1B}) + (1-\lambda)\min_{\theta\in C} U_L(\theta),$$

and

$$U_L(\theta) = d_{2A}^{s} + d_{2B}^{s} + p_A(\theta)\, R d_{2A}^{r} + p_B(\theta)\, R d_{2B}^{r}.$$

Let $\kappa_{\tau 1}$ be the multiplier for the budget constraint of Bank τ, let $\kappa_{\tau 2}$ be the multiplier for the incentive compatibility constraint of Bank τ, and let L_τ be the Lagrangian function for Bank τ, for $\tau \in \{A, B\}$. Thus, the FOCs for Bank τ are

$$\frac{dL_\tau}{dd_{1\tau}} = \lambda u'(d_{1A} + d_{1B}) - \lambda\kappa_{\tau 1} - \kappa_{\tau 2},$$

$$\frac{dL_\tau}{dd_{2\tau}^s} = (1-\lambda) - (1-\lambda)\kappa_{\tau 1} + \kappa_{\tau 2},$$

and

$$\frac{dL_\tau}{dd_{2\tau}^r} = (1-\lambda)\, p_\tau(\theta^a) R - (1-\lambda)\kappa_{\tau 1} + p_\tau(\theta^a) R\kappa_{\tau 2}.$$

The final FOC is found from applying minimax theorem, because $U_L'(\theta)\frac{d\theta^a}{dd_{2A}^r} = 0$ uniformly. Note that, if $p_\tau(\theta^a) R < 1$, $\frac{dL_\tau}{dd_{2\tau}^r} < \frac{dL_\tau}{dd_{2\tau}^s}$, so $d_{2\tau}^r = 0$. Similarly, if $p_\tau(\theta^a) R > 1$, $\frac{dL_\tau}{dd_{2\tau}^r} > \frac{dL_\tau}{dd_{2\tau}^s}$, so $d_{2\tau}^s = 0$. Finally, if $p_\tau(\theta^a) R = 1$, $\frac{dL_\tau}{dd_{2\tau}^r} = \frac{dL_\tau}{dd_{2\tau}^s}$, so $d_{2\tau}^s \geq 0$ and $d_{2\tau}^r \geq 0$.

Safe Equilibrium: Suppose that Bank B sets $d_{2B}^r = 0$. Then, by Lemma 1, $\theta^a = \hat{\theta}_0$ for all $d_{2A}^r > 0$. However, $e^{\hat{\theta}_0 - \theta_1} R < 1$, so Bank A finds it optimal to set $d_{2A}^r = 0$. Similarly, if Bank A sets $d_{2A}^r = 0$, Bank B optimally sets $d_{2B}^r = 0$, because $e^{\theta_0 - \hat{\theta}_1} R < 1$. Because $u'(2) > 1$, banks provide as much insurance against the liquidity shock as possible, so the IC binds. Thus, $d_{1A} = d_{2A}^s = 1$ and $d_{1B} = d_{2B}^s = 1$. The restriction that investors not run each bank individually, (7) and (8), simplify to $d_{2A}^s \geq d_{1A}$ and $d_{2B}^s \geq d_{1B}$, which are both satisfied (because they are all 1).

Risky Equilibrium: The symmetric equilibrium described in Theorem 2 is also an equilibrium here. Guess that all IC constraints are lax, so $\kappa_{\tau 2} = 0$ for $\tau \in \{A, B\}$. $\frac{dL_\tau}{dd_{1\tau}} = 0$ requires that $\kappa_{\tau 1} = u'(d_{1A} + d_{1B})$, so $\kappa_{A1} = \kappa_{B1}$. Because $\frac{dL_\tau}{dd_{2\tau}^r} = 0$ for $\tau \in \{A, B\}$, this implies that $e^{\theta^a - \theta_1} R = e^{\theta_0 - \theta^a} R$, or equivalently, that $\theta^a = \theta^e$. By Lemma 1, $\theta^a = \theta^e$ iff $d_{2A}^r = d_{2B}^r$. Because $e^{\theta^e - \theta_1} R > 1$, this implies $d_{2A}^s = d_{2B}^s = 0$. The budget constraints are symmetric, so $d_{1A} = d_{1B}$. By identical logic as the proof of Theorem 2, (6) is lax.

The constraint that investors will not run only one bank is lax as well. Note that $e^{\theta^e - \theta_1} R d_{2A}^r + e^{\theta_0 - \theta^e} R d_{2B}^r > d_{1A} + d_{1B}$ but $d_{1A} = d_{1B} > 1$. Further, $d_{2A}^r = \frac{1 - \lambda d_{1A}}{1 - \lambda}$. Because $d_{1A} > 1$, $d_{2A}^r < 1$. If a investor runs only Bank B, they will assume the worst-case scenario for their remaining portfolio: $\theta^a = \hat{\theta}_0$, so the investor's utility of long-term assets in Bank A is $e^{\hat{\theta}_0 - \theta_1} R d_{2A}^r < 1 < d_{1A}$. Therefore,

$$d_{1A} + d_{1B} > d_{1B} + e^{\hat{\theta}_0 - \theta_1} R d_{2A}^r,$$

implying

$$e^{\theta^e - \theta_1} R d_{2A}^r + e^{\theta_0 - \theta^e} R d_{2B}^r > d_{1B} + e^{\hat{\theta}_0 - \theta_1} R d_{2A}^r.$$

Therefore, investors will not run only Bank B. By identical logic, investors would also refuse to run only Bank A.

In the risky equilibrium, investors strictly prefer to exit one bank if they cannot access the other because $d_{1A} > 1 > e^{\hat{\theta}_0 - \theta_1} R d_{2A}^r$. Thus, an investor will either run both banks or not run either one. Thus, under the risky equilibrium, there are two ex post equilibria: investors run both banks or they run neither.

In the safe equilibrium, $d_{1A} = d_{2A}^s = 1$, so investors are indifferent to running Bank A and not running, no matter what happens at Bank B. Because the safe equilibrium does not provide insurance against the liquidity shock, $d_{1A} = 1$, other investors running the bank does not harm those who stay in the bank (the bank will be solvent for sure). Thus, there are no runs in the safe equilibrium.

Finally, we must show that investors find it optimal to invest equally in both banks in the risky equilibrium. Investing w_A in Bank A gives an investor the choice, at $t = 1$, between $d_{1A} w_A$ at $t = 1$ or $d_{2A}^r w_A$ of type A assets. Similarly, investing in w_B in Bank B gives an investor the choice between $d_{1B} w_B$ at $t = 1$ or $d_{2B}^r w_B$ of type B assets. Also, the investor may want to save extra funds, $2 - w_A - w_B$, on their own.

Given that the investor withdraws early iff they are affected by the liquidity shock, by investing w_A in Bank A and w_B in Bank B, the investor earns payoff

$$U_0 = \lambda u(d_{1A} w_A + d_{1B} w_B + 2 - w_A - w_B) + (1 - \lambda) \min_{\theta \in C} U_L(\theta)$$

where

$$U_L(\theta) = e^{\theta - \theta_1} R d_{2A}^r w_A + e^{\theta_0 - \theta} R d_{2B}^r w_B + 2 - w_A - w_B.$$

with the constraint that $w_A + w_B \leq 2$. Define L as the Lagrangian, and let γ be the multiplier on the constraint.

Applying the minimax theorem:

$$\frac{\partial L}{\partial w_\tau} = \lambda u' \left(d_{1A} w_A + d_{1B} w_B + 2 - w_A - w_B\right) \left(d_{1\tau} - 1\right) + (1 - \lambda) \left(p_\tau \left(\theta^a\right) R d_{2\tau}^r - 1\right) - \gamma$$

for $\tau \in \{A, B\}$. Because $d_{1A} = d_{1B} > 1$, the first term is strictly positive. Because $d_{2A}^r = d_{2B}^r$, $p_\tau\left(\theta^e\right) R d_{2A}^r > 1$ for $\tau \in \{A, B\}$, if w_A and w_B are not too far from each other, $p_\tau\left(\theta^a\right) R d_{2A}^r > 1$ for $\tau \in \{A, B\}$ (one of these holds with strict inequality). Therefore, $\gamma > 0$, so $w_A + w_B = 2$. Because $d_{1A} = d_{1B}$, $\frac{\partial L}{\partial w_A} = \frac{\partial L}{\partial w_B} = 0$ implies that $e^{\theta^a - \theta_1} = e^{\theta_0 - \theta^a}$, which implies that $\theta^a = \theta^e$. Therefore, the investor optimally sets $w_A = w_B$. ■

Proof of Theorem 4. First, suppose investors are SEU, so they believe $p_A = p_B = e^{\frac{1}{2}(\theta_0 - \theta_1)}$. Following the shock to type A assets, the payoff to the investor of staying in both banks is $p_A \phi R d_{2A}^r + p_B R d_{2B}^r$. The payoff of running only Bank A is $d_{1A} + p_B R d_{2B}^r$, while the payoff of running only Bank B, is $p_A \phi R d_{2A}^r + d_{1B}$. Finally, the payoff of running both banks is $d_{1A} + d_{1B}$. Note the IC is lax, $p_A R d_{2A}^r + p_B R d_{2B}^r > d_{1A} + d_{1B}$, and the contract is symmetric ($d_{2A}^r = d_{2B}^r$ and $d_{1A} = d_{1B}$). Because $p_B R d_{2B}^r > d_{1B}$, any run on Bank B would be a panic-based run. However, it would be a fundamental run for investors to run Bank A if $d_{1A} > p_A \phi R d_{2A}^r$.

Suppose instead that investors are MEU. Suppose that bad news about type A assets arrives, yet the investor thinks all other investors are staying in both banks. The payoff to the investor of staying in both banks is

$$\begin{aligned}
\min_{\theta \in \left[\hat{\theta}_0, \hat{\theta}_1\right]} \left\{p_A(\theta) \phi R d_{2A}^r + p_B(\theta) R d_{2B}^r\right\} &= 2 e^{\frac{1}{2}(\theta_0 - \theta_1)} R \left[\phi d_{2A}^r d_{2B}^r\right]^{\frac{1}{2}} \\
&= 2 e^{\frac{1}{2}(\theta_0 - \theta_1)} \phi^{\frac{1}{2}} R d_{2A}^r.
\end{aligned}$$

The first equality follows from Lemma 1, assuming that beliefs are interior. The second equality holds by symmetry, because $d_{2A}^r = d_{2B}^r$. If the investor runs only Bank A, they receive payoff $d_{1A} + e^{\theta_0 - \hat{\theta}_1} R d_{2B}^r$, while if the investor runs only Bank B, they receive payoff $d_{1B} + e^{\hat{\theta}_0 - \theta_1} \phi R d_{2A}^r$. If the investor runs both banks, they receive payoff $d_{1A} + d_{1B}$.

If the investor runs Bank A, they will run Bank B as well. Recall that $e^{\theta_0 - \hat{\theta}_1} R < 1$, and the budget constraint implies that $d_{2B}^r = \frac{1 - \lambda d_{1B}}{1 - \lambda}$. $d_{1B} > 1$, so $d_{2B}^r < 1$, so $e^{\theta_0 - \hat{\theta}_1} R d_{2B}^r < 1 < d_{1B}$. Thus, $d_{1A} + e^{\theta_0 - \hat{\theta}_1} R d_{2B}^r < d_{1A} + d_{1B}$. Therefore, it is better to run both banks than only Bank A. Because Bank A received the bad news, it is worse to run only Bank B than only Bank A. Therefore, the investor will either run both banks or run neither.

So far, we have guessed that the bad news still resulted in interior beliefs. We will now show that any shock bad enough to cause corner beliefs will induce a run, thus proving that beliefs are interior at the cutoff. Following bad news on type A assets, investors believe, from Lemma 1, $\theta^a = \theta^e + \frac{1}{2} \ln \frac{1}{\phi}$. Thus, $\theta^a = \hat{\theta}_1$ iff $\phi \le e^{-2\left(\hat{\theta}_1 - \theta^e\right)}$. At the boundary, $\phi = e^{-2\left(\hat{\theta}_1 - \theta^e\right)}$ and $\theta^a = \hat{\theta}_1$, the payoff to staying in both banks is

$$p_A\left(\hat{\theta}_1\right) e^{-2\left(\hat{\theta}_1 - \theta^e\right)} R d_{2A}^r + p_B\left(\hat{\theta}_1\right) R d_{2B}^r = d_{2A}^r R \left[e^{\hat{\theta}_1 - \theta_1} e^{-2\left(\hat{\theta}_1 - \theta^e\right)} + e^{\theta_0 - \hat{\theta}_1}\right]$$

Note $\hat{\theta}_1 - \theta_1 - 2\left(\hat{\theta}_1 - \theta^e\right) = -\hat{\theta}_1 - (\theta_1 - \theta_0)$. Thus, the payoff to staying in both is $R d_{2A}^r e^{\theta_0 - \hat{\theta}_1} \left[e^{-\theta_1} + 1\right]$. Because $\hat{\theta}_1 > 0$, $e^{-\theta_1} < 1$. Also, $e^{\theta_0 - \hat{\theta}_1} R < 1$ and $d_{2A}^r < 1$, so we know that the RHS is less than 2, which is less than $d_{1A} + d_{1B}$, so the investor will run both banks if news is bad enough to give her corner beliefs.

Thus, uncertainty-averse investors run the bank if $d_{1A} > \phi^{\frac{1}{2}} p_A\left(\theta^e\right) R d_{2A}^r$, and they will run both banks if they run either. ■

Proof of Lemma 3. To find the optimal contract, we must find the optimal contract that allows a run, the optimal contract that deters a run, then find which one is best. The assumption of affine utility guarantees that the optimal contract is the one that allows a run.

Consider what happens during a run on Bank A (runs on Bank B are symmetric). Applying the sequential service constraint, the first $\frac{1}{d_{1A}}$ receive d_{1A} while the remaining $\left(1 - \frac{1}{d_{1A}}\right)$ receive nothing. Because $d_{1A} + d_{1B} \le \tilde{c}$ (we will prove this to be optimal later), early types receive expected utility ψ in a run, while late investors receive expected utility of 1 in a run.

We will first consider when the optimal contract that allows runs when investors are SEU. As shown in Lemma

5, runs only the affected bank when they hear bad news about that bank, so the expected payoff if there is bad news about type A assets is

$$U_B = \lambda\left[\frac{1}{d_{1A}}u\left(d_{1A}+d_{1B}\right)+\left(1-\frac{1}{d_{1A}}\right)u\left(d_{1B}\right)\right]+\left(1-\lambda\right)\left[1+d_{2B}^s+p_B\left(\theta^e\right)Rd_{2B}^r\right],$$

the expected payoff if there is bad news about type B assets is

$$U_A = \lambda\left[\frac{1}{d_{1B}}u\left(d_{1A}+d_{1B}\right)+\left(1-\frac{1}{d_{1B}}\right)u\left(d_{1A}\right)\right]+\left(1-\lambda\right)\left[d_{2A}^s+p_A\left(\theta^e\right)Rd_{2A}^r+1\right],$$

and the expected payoff in the absence of bad news is

$$U_{AB} = \lambda u\left(d_{1A}+d_{1B}\right)+\left(1-\lambda\right)\left[d_{2A}^s+p_A\left(\theta^e\right)Rd_{2A}^r+d_{2B}^s+p_B\left(\theta^e\right)Rd_{2B}^r\right].$$

Therefore, the contract provides investors with expected utility of

$$U=\left(1-2\varepsilon\right)U_{AB}+\varepsilon U_A+\varepsilon U_B.$$

Bank τ maximizes investor utility, subject to the budget constraint

$$\lambda d_{1\tau}+\left(1-\lambda\right)\left(d_{2\tau}^s+d_{2\tau}^r\right)\leq 1,$$

for $\tau\in\{A,B\}$. Let κ_τ be the multiplier on Bank τ's budget constraint, and let L_τ be the the Langrangian functions for Bank τ (we will ignore the incentive compatibility constraints, then check them later). Note

$$\frac{\partial L_A}{\partial d_{1A}}=\left(1-2\varepsilon\right)\frac{\partial U_{AB}}{\partial d_{1A}}+\varepsilon\frac{\partial U_A}{\partial d_{1A}}+\varepsilon\frac{\partial U_B}{\partial d_{1A}}-\lambda\kappa_A.$$

Note $\frac{\partial U_{AB}}{\partial d_{1A}}=\lambda u'\left(d_{1A}+d_{1B}\right)$, $\frac{\partial U_A}{\partial d_{1A}}=\lambda\left[\frac{1}{d_{1B}}u'\left(d_{1A}+d_{1B}\right)+\left(1-\frac{1}{d_{1B}}\right)u'\left(d_{1A}\right)\right]$, and $\frac{\partial U_B}{\partial d_{1A}}=\frac{\lambda}{d_{1A}^2}\left\{d_{1A}u'\left(d_{1A}+d_{1B}\right)-\left[u\left(d_{1A}+d_{1B}\right)-u\left(d_{1B}\right)\right]\right\}$. Thus,

$$\begin{aligned}\frac{\partial L_A}{\partial d_{1A}} &= \left(1-2\varepsilon\right)\lambda u'\left(d_{1A}+d_{1B}\right)+\varepsilon\lambda\left[\frac{1}{d_{1B}}u'\left(d_{1A}+d_{1B}\right)+\left(1-\frac{1}{d_{1B}}\right)u'\left(d_{1A}\right)\right]\\ &\quad+\varepsilon\frac{\lambda}{d_{1A}^2}\left\{d_{1A}u'\left(d_{1A}+d_{1B}\right)-\left[u\left(d_{1A}+d_{1B}\right)-u\left(d_{1B}\right)\right]\right\}-\lambda\kappa_A.\end{aligned}$$

Similarly,

$$\frac{\partial L_A}{\partial d_{2A}^s}=\left(1-2\varepsilon\right)\left(1-\lambda\right)+\varepsilon\left(1-\lambda\right)-\left(1-\lambda\right)\kappa_A,$$

and

$$\frac{\partial L_A}{\partial d_{2A}^r}=\left(1-2\varepsilon\right)\left(1-\lambda\right)p_A\left(\theta^e\right)R+\varepsilon\left(1-\lambda\right)p_A\left(\theta^e\right)R-\left(1-\lambda\right)\kappa_A.$$

Because $p_A\left(\theta^e\right)R>1$, $\frac{\partial L}{\partial d_{2A}^r}>\frac{\partial L}{\partial d_{2A}^s}$. By complementary slackness, $\frac{\partial L}{\partial d_{2A}^r}=0>\frac{\partial L}{\partial d_{2A}^s}$, so $d_{2A}^s=0$. Thus, $\kappa_A=\left(1-\varepsilon\right)p_A\left(\theta^e\right)R$. Because utility is piecewise affine, for $d_{1A}+d_{1B}<\tilde{c}$, $\frac{\partial L_A}{\partial d_{1A}}$ simplifies to

$$\frac{\partial L_A}{\partial d_{1A}}=\lambda\left(1-\varepsilon\right)\left(\psi-p_AR\right),$$

and for $d_{1A}+d_{1B}>\tilde{c}>\max\{d_{1A},d_{1B}\}$,

$$\begin{aligned}\frac{\partial L_A}{\partial d_{1A}} &= \left(1-\varepsilon\right)\lambda\left(1-p_A\left(\theta^e\right)R\right)+\varepsilon\lambda\left(1-\frac{1}{d_{1B}}\right)\left(\psi-1\right)\\ &\quad-\varepsilon\frac{\lambda}{d_{1A}^2}\left(\psi-1\right)\left(\tilde{c}-d_{1B}\right).\end{aligned}$$

The first term is negative, the second term is positive, and the third term is negative. To guarantee that $\frac{\partial L_A}{\partial d_{1A}} < 0$ for all $d_{1A} + d_{1B} > \tilde{c}$, it is sufficient that $\varepsilon < \frac{p_A(\theta^e)R-1}{\psi+p_A(\theta^e)R-2}$. Note this condition on ε is sufficient, but not necessary.[23]

Therefore, Bank A selects d_{1A} so that $d_{1A} + d_{1B} = \tilde{c}$ and sets $d^r_{2A} = \frac{1-\lambda d_{1A}}{1-\lambda}$. By symmetry, Bank B selects d_{1B} so that $d_{1A} + d_{1B} = \tilde{c}$ and sets $d^r_{2B} = \frac{1-\lambda d_{1B}}{1-\lambda}$. Because investors treat investment in the different asset classes as perfect substitutes, they are indifferent to all feasible[24] choices of d_{1A} and d_{1B}, so WLOG banks set $d_{1A} = d_{1B} = \frac{1}{2}\tilde{c}$.

In order to deter a run, the banks must offer a contract so that investors do not run following bad news. That is, they must set $d_{1A} \leq \phi p_A(\theta^e) R d^r_{2A}$ and $d_{1B} \leq \phi p_B(\theta^e) R d^r_{2B}$. After some messy algebra, it can be verified that banks will implement contracts that allow a run, because $\psi > e^{\frac{1}{2}(\theta_0-\theta_1)}R$ and $\phi < 1$.

Let us consider the optimal contract that allows runs when investors are MEU. As shown in Lemma 4, investors will run both banks when they receive bad news about either bank. When there is bad news, investors run both banks, so they earn an expected payoff of

$$U_R = \lambda U_{ER} + (1-\lambda) U_{LR}$$

where

$$U_{LR} = \frac{1}{d_{1A}d_{1B}}(d_{1A}+d_{1B}) + \frac{1}{d_{1A}}\left(1-\frac{1}{d_{1B}}\right)(d_{1A}) + \left(1-\frac{1}{d_{1A}}\right)\frac{1}{d_{1B}}(d_{1B})$$

and

$$U_{ER} = \frac{1}{d_{1A}d_{1B}}u(d_{1A}+d_{1B}) + \frac{1}{d_{1A}}\left(1-\frac{1}{d_{1B}}\right)u(d_{1A}) + \left(1-\frac{1}{d_{1A}}\right)\frac{1}{d_{1B}}u(d_{1B}).$$

Note that U_{LR} simplifies to $U_{LR} = 2$. Because u is piecewise affine, if $d_{1A} + d_{1B} \leq \tilde{c}$, $U_{ER} = 2\psi$. If $d_{1A} + d_{1B} > \tilde{c} > \max\{d_{1A}, d_{1B}\}$, $U_{ER} = 2\psi - (\psi-1)\frac{1}{d_{1A}d_{1B}}[d_{1A}+d_{1B}-\tilde{c}]$.[25]

If there is no run, investors earn a payoff of

$$U_S = \lambda u(d_{1A}+d_{1B}) + (1-\lambda)\min_{\theta\in[\hat{\theta}_0,\hat{\theta}_1]} U_L(\theta)$$

where

$$U_L(\theta) = d^s_{2A} + d^s_{2B} + e^{\theta-\theta_1}Rd^r_{2A} + e^{\theta_0-\theta}Rd^r_{2B}.$$

Therefore, investor welfare is given by

$$U = (1-2\varepsilon)U_S + 2\varepsilon U_R.$$

Therefore, κ_A be the multiplier on the budget constraint for Bank A, and let L_A be the respective Lagrangian functions of Bank A.

$$\frac{\partial L_A}{\partial d_{1A}} = (1-2\varepsilon)\frac{\partial U_S}{\partial d_{1A}} + 2\varepsilon\frac{\partial U_R}{\partial d_{1A}} - \lambda\kappa_A,$$

$$\frac{\partial L_A}{\partial d^s_{2A}} = (1-2\varepsilon)\frac{\partial U_S}{\partial d^s_{2A}} + 2\varepsilon\frac{\partial U_R}{\partial d^s_{2A}} - (1-\lambda)\kappa_A,$$

and

$$\frac{\partial L_A}{\partial d^r_{2A}} = (1-2\varepsilon)\frac{\partial U_S}{\partial d^r_{2A}} + 2\varepsilon\frac{\partial U_R}{\partial d^r_{2A}} - (1-\lambda)\kappa_A.$$

FOCs of optimality for Bank B are symmetric. Because late investors receive an average of 2 in a run, and this does not depend on the contract they receive, $\frac{\partial U_R}{\partial d^s_{2A}} = \frac{\partial U_R}{\partial d^r_{2A}} = 0$. Further, $\frac{\partial U_S}{\partial d^s_{2A}} = 1-\lambda$. If $d^r_{2B} = 0$, $\frac{\partial U_S}{\partial d^r_{2A}} =$

[23] We derived this cutoff by ignoring the third piece, which is strictly negative, and ignoring the $\frac{1}{d_{1B}}$, because that will be a negative term as well. Finally, if $d_{1B} > \tilde{c}$, the third term disappears, but $\frac{\partial L_A}{\partial d_{1A}} < 0$ so long as ε is less than this cutoff.

[24] Banks cannot make either of the promised intermediate cash flow too big, so that late investors do not run following no bad news at $t = 1$, which requires that $p_A(\theta^e) R d^r_{2A} > d_{1A}$ and $p_B(\theta^e) R d^r_{2B} > d_{1B}$.

[25] Similarly, if $d_{1A} > \tilde{c} > d_{1B}$, $U_{ER} = (\psi-1)\frac{1}{d_{1A}}(\tilde{c}-1) + 1 + \psi$. Finally, if $\tilde{c} < \min\{d_{1A}, d_{1B}\}$, $U_{ER} = (\psi-1)\frac{\tilde{c}}{d_{1A}d_{1B}}(d_{1A}+d_{1B}-1) + 2$. It will never be optimal to set d_{1A} and d_{1B} this large, however, because ε is small.

$(1-\lambda)\, e^{\hat{\theta}_0-\theta_1} R < 1-\lambda$, so $\frac{\partial L_A}{\partial d_{2A}^r} < \frac{\partial L_A}{\partial d_{2A}^s}$, which implies by complementary slackness, that $\frac{\partial L_A}{\partial d_{2A}^r} < \frac{\partial L_A}{\partial d_{2A}^s} = 0$, so $d_{2A}^r = 0$. Thus, the safe equilibrium is an equilibrium (the IC constraint requires that $d_{1A} = 1$ and $d_{2A}^s = 1$).

Similar to the proof of Theorem 3, we can also show that the risky equilibrium is an equilibrium here. Note that $\frac{\partial U_S}{\partial d_{2A}^r} = e^{\theta^a-\theta_1} R$. If it is optimal to set $d_{2A}^r > 0$, $\frac{\partial L_A}{\partial d_{2A}^r} = 0$, so $\kappa_A = (1-2\varepsilon)\, e^{\theta^a-\theta_1} R$. In order for it to be optimal for bank B to set $d_{2B}^r > 0$, $\frac{\partial L_B}{\partial d_{2B}^r} = 0$, so $\kappa_B = (1-2\varepsilon)\, e^{\theta_0-\theta^a} R$. For it to be optimal to set $d_{1A} + d_{1B} = \tilde{c}$, it must be that $\frac{\partial L_A}{\partial d_{1A}} > 0$ for $d_{1A} + d_{1B} < \tilde{c}$ and $\frac{\partial L_A}{\partial d_{1A}} < 0$ for $d_{1A} + d_{1B} > \tilde{c}$. It can be easily shown that $\frac{\partial U_R}{\partial d_{1A}} = 0$ for $d_{1A} + d_{1B} < \tilde{c}$, but $\frac{\partial U_R}{\partial d_{1A}} < 0$ for $d_{1A} + d_{1B} > \tilde{c}$. Further, $\frac{\partial U_S}{\partial d_{1A}} = \psi$ for $d_{1A} + d_{1B} < \tilde{c}$ but $\frac{\partial U_S}{\partial d_{1A}} = 1$ for $d_{1A} + d_{1B} > \tilde{c}$. Therefore, $\frac{\partial L_A}{\partial d_{1A}} > 0$ for $d_{1A} + d_{1B} < \tilde{c}$ but $\frac{\partial L_A}{\partial d_{1A}} < 0$ for $d_{1A} + d_{1B} > \tilde{c}$. Though there are many risky equilibria, the symmetric risky equilibrium referred to in the text is the most efficient (it provides the highest payoff for investors). After some messy algebra, it can be shown that banks will implement this contract, rather than the contract which deters runs, because $\psi > e^{\frac{1}{2}(\theta_0-\theta_1)} R$ and $\phi < 1$. ■

Proof of Theorem 5. The claim follows directly from Theorem 4 and Lemma 3. ■

Proof of Lemma 4. The stock company agrees to pay dividend Δ_{1B} at $t = 1$, and hold portfolio $\{\sigma_{2B}, \rho_{2B}\}$, which are risk-free and type-B assets respectively, until $t = 2$. Suppose that the stock is traded at $t = 1$ for price P_{1B}. Because households are identical, they will all behave the same at $t = 0$, so all investors hold one share of the stock at $t = 0$.

Early investors are willing to sell their share of the stock for any $P_{1B} > 0$, because they place no value on $t = 2$ consumption. Thus, they will consume $\Delta_{1B} + P_{1B}$ at $t = 1$. They will sell to the late investors. Late investors are willing to buy the shares from the early investors iff it improves their utility. If investors are SEU, they value shares of the mutual fund at $\sigma_{2B} + e^{\theta_0-\theta^e} R\rho_{2B}$, so they are willing to buy iff $\sigma_{2B} + e^{\theta_0-\theta^e} R\rho_{2B} > P_{1B}$. Because they invested all their funds at $t = 0$, late investors can only reinvest the dividend, so market clearing requires that $\lambda P_{1B} \leq (1-\lambda)\, \Delta_{1B}$. If Δ_{1B} is not too large, this binds, so $P_{1B} = \frac{1-\lambda}{\lambda} \Delta_{1B}$. Thus, the early type receives $\Delta_{1B} + P_{1B} = \frac{1}{\lambda} \Delta_{1B}$, while the late type receives $\frac{1}{1-\lambda} \left[\sigma_{2B} + e^{\theta_0-\theta^e} R\rho_{2B}\right]$. Thus, the late type is willing to use all of their fund to buy if $\sigma_{2B} + e^{\theta_0-\theta^e} R\rho_{2B} \geq \frac{1-\lambda}{\lambda} \Delta_{1B}$. Note that this collapses to the incentive compatibility constraint that the late type does not want to run the bank.

Therefore, the stock company can implement the same cash flows as the banking contract $\{d_{1B}, d_{2B}^s, d_{2B}^r\}$ by setting $\Delta_{1B} = \lambda d_{1B}$, $\sigma_{2B} = (1-\lambda)\, d_{2B}^s$, and $\rho_{2B} = (1-\lambda)\, d_{2B}^r$. The stock will trade at price $P_{1B} = (1-\lambda)\, d_{1B}$.

The case with MEU investors follows with similar logic, except that MEU investors are even more willing to buy the shares, because different asset classes are complements. ■

Proof of Lemma 5. Lemma 4 showed that the stock company can implement contract $\{d_{1B}, d_{2B}^s, d_{2B}^r\}$ by promising to pay dividend $\Delta_{1B} = \lambda d_{1B}$, holding risk-free assets $\sigma_{2B} = (1-\lambda)\, d_{2B}^s$, and type-$B$ assets of $\rho_{2B} = (1-\lambda)\, d_{2B}^r$. By identical logic to Theorem 2, it can be shown that the optimal contract to offer is sets $d_{1\tau} = \frac{1}{2}\tilde{c}$, $d_{2\tau}^s = 0$, and $d_{2\tau}^r = \frac{1-\frac{\lambda}{2}\tilde{c}}{1-\lambda}$ where $\tilde{c}$ satisfies $u'(\tilde{c}) = e^{\frac{1}{2}(\theta_0-\theta_1)} R$ with concave u, or $\tilde{c}$ is the kink in u as in (12). All that remains to show is that the it is optimal for investors to invest \$1 in the bank and \$1 in the stock. Similar to the proof of Theorem 2, uncertainty-neutral investors are indifferent between asset classes, the claim holds WLOG. Similarly, the case with uncertainty-averse investors follows by identical logic to Theorem 3. ■

Proof of Theorem 6. Two things need to be shown to prove the claim about uncertainty averse investors. First, we must show that a bank run harms the stock market. Second, we need to show that a sufficiently big shock to the stock market induces a bank run.

Consider first stock valuation. Late investors are willing to buy from early investors only if $P_{1B} \leq e^{\theta_0-\theta^a} R\rho_{2B}$, where θ^a is determined by the overall portfolio of late investors, as in Lemma 1. Because $\rho_{2B} = (1-\lambda)\, d_{2B}^r$, and because the initial allocation is incentive compatible, $d_{1B} < e^{\theta_0-\theta^e} R d_{2B}^r$, this constraint is lax in the absence of bad news, because $\theta^a = \theta^e$. Thus, if there is no bad news, $P_{1B} = (1-\lambda)\, d_{1B}$. If there is a run on the bank, however, $\theta^a = \hat{\theta}_1$ (because late investors only hold type B assets), so $P_{1B}^{Run} = e^{\theta_0-\hat{\theta}_1} R\rho_{2B}$, because $d_{1B} > 1 > e^{\theta_0-\hat{\theta}_1} R d_{2B}^r$. Because $P_{1B}^{Run} < P_{1B}$, a run on the bank harms stock market valuation.

Bad news on the bank can harm the stock even if the bad news is not sufficiently bad that it produces a run. As shown above, stock valuation is depressed iff $e^{\theta_0-\theta^a} R d_{2B}^r < d_{1B}$. If there is bad news about type A assets, but not sufficiently strong bad news to induce a run,[26] Lemma 1 implies that $e^{\theta_0-\theta^a}Rd_{2B}^r = e^{\theta_0-\theta^e}R\phi^{\frac{1}{2}}\left[d_{2A}^r d_{2B}^r\right]^{\frac{1}{2}}$. By symmetry of the optimal contract, the stock is harmed by bad news to the bank iff $\phi < \underline{\phi}^2$, where $\underline{\phi}$ is defined in Theorem 5.

It is optimal to run on the bank, however, if

$$d_{1A} + e^{\theta_0-\hat{\theta}_1}Rd_{2B}^r \geq 2e^{\frac{1}{2}(\theta_0-\theta_1)}\phi^{\frac{1}{2}}R\left[d_{2A}^r d_{2B}^r\right]^{\frac{1}{2}}.$$

Because the optimal contract is symmetric, it is optimal to run the bank if

$$\phi^{\frac{1}{2}} \leq \frac{1}{2}\left[\underline{\phi} + e^{\theta^e-\hat{\theta}_1}\right].$$

By (A_1), it can be quickly verified that $e^{\theta^e-\hat{\theta}_1} < \underline{\phi}$. Because this implies a strictly smaller cutoff for ϕ, it is possible to have bad news about the bank that harms stock valuation without triggering a run (a sufficiently bad shock induces a run and harms the stock). Also, this implies that the bank is less prone to runs when paired with a stock than when paired with another bank.

When there is bad news about the stock, it may be optimal to run the bank. Following bad news about the stock, if late investors stay in the bank, by Lemma 1, $\theta^a = \theta^e + \frac{1}{2}\ln\left(\phi d_{2B}^r\right) - \frac{1}{2}\ln d_{2A}^r$, so staying in the bank provides utility $2e^{\frac{1}{2}(\theta_0-\theta_1)}\phi^{\frac{1}{2}}R\left[d_{2A}^r d_{2B}^r\right]^{\frac{1}{2}}$. Running the bank provides late investors with utility $d_{1A}+e^{\theta_0-\hat{\theta}_1}\phi Rd_{2B}^r$, so it is optimal to run iff $d_{1A}+e^{\theta_0-\hat{\theta}_1}\phi Rd_{2B}^r \geq 2e^{\frac{1}{2}(\theta_0-\theta_1)}\phi^{\frac{1}{2}}R\left[d_{2A}^r d_{2B}^r\right]^{\frac{1}{2}}$, or, by applying symmetry, if $e^{\theta^e-\hat{\theta}_1}\phi - 2\phi^{\frac{1}{2}} + \frac{d_{1A}}{e^{\theta_0-\theta^e}Rd_{2A}^r} \geq 0$. Applying the quadratic formula, it can be quickly shown that it is optimal to run the bank if $\phi < \left[\frac{1-\sqrt{1-e^{-\alpha}\underline{\phi}}}{e^{-\alpha}}\right]^2$, where $\alpha = \hat{\theta}_1 - \theta^e$ and $\underline{\phi} = \frac{d_{1A}}{e^{\theta_0-\theta^e}Rd_{2A}^r}$ as in Theorem 5. Therefore, sufficiently bad news about the stock spreads to the banks and causes runs.

Finally, note that there is no contagion when investors are uncertainty neutral. When the investors are uncertainty neutral, they always believe $\theta = \theta^e$, so, by Theorem 4, they run the bank iff there is bad news on type A assets with $\phi \leq \underline{\phi} \equiv \frac{(1-\lambda)\tilde{c}}{e^{\frac{1}{2}(\theta_0-\theta_1)}R(2-\lambda\tilde{c})}$, and this will not affect the stock market. Similarly, bad news in the stock market will depress stock prices, to $e^{\theta_0-\theta^e}\phi R\rho_{2B}$ if ϕ is small enough, but will not affect the bank. ■

Proof of Theorem 7. If there is run on Bank B, late investors receive d_{1A} by running Bank A, but they receive $e^{\theta^e-\alpha-\theta_1}Rd_{2B}^r$ if they stay in Bank A (and so do all the other late investors), because late investors hold only type A assets, so they believes $\theta^a = \theta^e - \alpha$. Thus, it is an equilibrium for late investors to stay in Bank A only if $d_{1A} \leq e^{-\alpha}e^{\theta^e-\theta_1}Rd_{2A}^r$, or equivalently, only if $\alpha \leq \ln\frac{e^{\frac{1}{2}(\theta_0-\theta_1)}Rd_{2A}^r}{d_{1A}}$. The tradeoff in Bank B, is symmetric, so define $\underline{\alpha} \equiv \ln\frac{e^{\frac{1}{2}(\theta_0-\theta_1)}Rd_{2A}^r}{d_{1A}}$. Alternatively, if $\alpha > \underline{\alpha}$, all runs will be systemic.

Similarly, the safe equilibrium arises iff assumption (A_1) holds: $e^{\hat{\theta}_0-\theta_1}R < 1$. Because $\hat{\theta}_0 = \theta^e - \alpha$, the safe equilibrium arises iff $\alpha > \ln R - \frac{1}{2}\left(\theta_1 - \theta_0\right)$. Thus, define $\bar{\alpha} \equiv \ln R - \frac{1}{2}\left(\theta_1 - \theta_0\right)$. Because $d_{1A} > d_{2A}^r$, $\bar{\alpha} > \underline{\alpha}$. ■

Proof of Lemma 6. Bank τ offers contract $\{d_{1\tau}, d_{2\tau}^s, d_{2\tau}^r\}$. Note that uncertainty only affects the payoff of the risky portion of the portfolio. Thus, investors' worst-case scenario is

$$\min_{\theta}\sum_{\tau=1}^{N} e^{\theta_\tau-\theta_{Max}}d_{2\tau}^r$$

subject to $\theta_\tau \in [\theta_L, \theta_H]$ and $\sum_{\tau=1}^N \theta_\tau \in [N\theta^e - A, N\theta^e + A]$. Because $N\theta_L < N\theta^e - A$, $\exists\tau$ s.t. $\theta_\tau > \theta_L$. Similarly, because $N\theta_H > N\theta^e + A$, $\exists\tau$ s.t. $\theta_\tau < \theta_H$. Because the bank offers only long contracts, $d_{2\tau}^r \geq 0$, increasing θ helps the agent, so the minization problem sets the sum of θ as low as possible. Thus, $\kappa = -A$. Let λ be the multiplier on the constraint that $\sum_{\tau=1}^N \theta_\tau = N\theta^e - A$, let $\gamma_{\tau L}$ and $\gamma_{\tau H}$ be the respective constraints on θ_τ, and let L be the

[26] This assumes that the shock is not sufficiently bad to induce corner beliefs. If the shock is sufficiently bad to induce corner beliefs, however, investors will always run, because $d_{1A} > 1 > e^{\hat{\theta}_0-\theta_1}Rd_{2A}^r$.

Lagrangian for the minization problem. Thus,

$$\frac{\partial L}{\partial \theta_\tau} = -e^{\theta_\tau - \theta_{Max}} d^r_{2\tau} + \lambda + \gamma_{\tau L} - \gamma_{\tau H}.$$

If $\gamma_{\tau H} > 0$, $\theta_\tau = \theta_H$. This holds iff $-e^{\theta_H - \theta_{Max}} d^r_{2\tau} + \lambda > 0$, or equivalently, if $d^r_{2\tau} < \underline{D} \equiv e^{\theta_{Max} - \theta_H}\lambda$. Similarly, if $\gamma_{\tau L} > 0$, $\theta_\tau = \theta_L$. This holds iff $d^r_{2\tau} > \bar{D} \equiv e^{\theta_{Max} - \theta_L}\lambda$. If $\gamma_{\tau H} = \gamma_{\tau L} = 0$, $\theta_\tau \in (\theta_L, \theta_H)$, so $e^{\theta_\tau - \theta_{Max}} d^r_{2\tau} = \lambda$, so $\theta_\tau = \theta_{Max} + \ln \frac{\lambda}{d^r_{2\tau}}$. Define $A_L = \left\{\tau : d^r_{2\tau} \geq \bar{D}\right\}$, $A_H = |\{\tau : d^r_{2\tau} \leq \underline{D}\}|$, and $A_I = \left\{\tau : d^r_{2\tau} \in \left(\underline{D}, \bar{D}\right)\right\}$. Similarly, define $N_L = |A_L|$, $N_H = |A_H|$, and $N_I = |A_I|$. Because $\sum_{\tau=1}^N \theta_\tau = N\theta^e - A$, this implies $\sum_{\tau \in A_I} \theta_\tau = N\theta^e - A - N_H\theta_H - N_L\theta_L$. We can also express $\sum_{\tau \in A_I} \theta_\tau = N_I\theta_{Max} + N_I \ln\lambda - \sum_{\tau \in A_I} \ln d^r_{2\tau}$, so $\ln\lambda = \frac{1}{N_I}\left[N\theta^e - A - N_H\theta_H - N_L\theta_L\right] - \theta_{Max} + \frac{1}{N_I}\sum_{\tau \in A_I} \ln d^r_{2\tau}$. This implies that

$$\theta_\tau = \frac{1}{N_I}\left[N\theta^e - A - N_H\theta_H - N_L\theta_L\right] + \frac{1}{N_I}\sum_{\tau' \in A_I} \ln d^r_{2\tau'} - \ln d^r_{2\tau}.$$

This gives the general expression for endogenous beliefs. Investors hold extremely pessimistic beliefs, $\theta_\tau = \theta_L$, on their largest risky positions, $d^r_{2\tau} \geq \bar{D}$. They hold very optimistic beliefs, $\theta_\tau = \theta_L$, on their smallest risky positions, $d^r_{2\tau} \leq \underline{D}$. They hold interior beliefs on the beliefs between.

Finally, consider the case when all beliefs are interior: $\theta_\tau \in (\theta_L, \theta_H)$ for all τ. In this case, $N_H = N_L = 0$, $N_I = N$, and all $\tau \in A_I$, so investor beliefs are

$$\theta_\tau = \theta^e - \frac{A}{N} + \frac{1}{N}\sum_{\tau'=1}^{N} \ln d^r_{2\tau'} - \ln d^r_{2\tau}.$$

Note that an increase in $d^r_{2\tau}$ decreases θ_τ and increases $\theta_{\tau'}$ for all $\tau' \neq \tau$ (this holds weakly for corner beliefs). ■

Proof of Theorem 8. For uncertainty-neutral investors, the proof is obvious: they believe $\theta = \theta^e$, so investment in risky assets is positive NPV because $e^{\theta^e - \theta_{Max}} > e^{\theta^e - \frac{A}{N} - \theta_{Max}} > 1$. Thus, banks set $d_{1\tau}$ optimally: so $u'(Nd_{1\tau}) = e^{\theta^e - \theta_{Max}}$ in the case of concave utility, and $Nd_{1\tau} = \tilde{c}$ in the case of piece-wise affine utility, as in Theorem 2 and Lemma 3, respectively. Local shocks stay local by identical reasoning to Theorem 4.

If investors are uncertainty averse, by identical reasoning to Theorem 3, Theorem 4, and Lemma 3, it is sufficient to show that investment has a positive NPV iff investors can invest in all the other uncertain assets as well. If all banks select the same risky payoff, $d^r_{2\tau} = d^{r*}_2$, $\theta_\tau = \theta^e - \frac{A}{N}$ by Lemma 6. Because $e^{\theta^e - \frac{A}{N} - \theta_{Max}}R > 1$, it is positive NPV to invest in all the uncertain assets, so the risky equilibrium is an equilibrium. If all banks except one select the same risky payoff: $d^r_{2\tau} = d^{r*}_2$ for all $\tau \neq \tau'$ and $d^r_{2\tau'} = 0$, investors will believe $\theta_{\tau'} = \theta_H$, and $\theta_\tau = \frac{1}{N-1}\left[N\theta^e - A - \theta_H\right]$. Because $e^{\frac{1}{N-1}\left[N\theta^e - A - \theta_H\right]}R < 1$, it is a negative NPV project to invest in any of the uncertain projects, so the safe equilibrium is an equilibrium and all runs will spread.[27] ■

[27] This cutoff is sufficient, but not necessary. Because banks want to insure against the liquidity shock, $d_{1\tau} > 1 > d^r_{2\tau}$ in the risky equilibrium. For runs to spread, it must be the case that $e^{\frac{1}{N-1}\left[N\theta^e - A - \theta_H\right]}Rd^r_{2\tau} < d_{1\tau}$, by identical reasoning to Theorem 5. Similarly, if $(N-1)\theta_H + \theta_L \geq N\theta^e - A$, then the safe equilibrium is an equilibrium iff $e^{\theta_L - \theta_{Max}}R < 1$.