Dynamic Corporate Liquidity*

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Abstract

We develop and structurally estimate a dynamic model of corporate liquidity and risk management. When external finance is costly, liquid funds provide corporations with instruments to absorb and react to shocks. Making optimal use of liquid funds means transferring them to times and states where they are most valuable. In the model, firms can transfer liquidity across time using cash and across states drawing on credit lines subject to debt capacity constraints. Optimal liquidity management arises as a trade-off between conditional liquidity with credit lines subject to collateral constraints and uncontingent liquidity using cash. The estimated model explains well the cross-sectional and time series patterns of corporate liquidity management: Small and constrained firms use cash to provide liquidity to fund investment opportunities, and large and unconstrained firms rely on credit lines. While equity issuances are used to replenish cash balances, credit lines fund unanticipated investment opportunities. To solve the model, we develop a novel and efficient approach to dynamic programming relying on linear programming, that is more widely applicable to high-dimensional dynamic models.

Keywords: Corporate liquidity, cash, credit lines, debt capacity, structural estimation

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1. Introduction

When external finance is costly, liquid funds provide corporations with instruments to absorb and react to shocks. Making optimal use of liquid funds means transferring them to times and states where they are most valuable. Liquid funds may be valuable because they aid financing of a profitable investment opportunity, or because they help covering cash shortfalls. Anticipations of such future states thus provide a rationale for corporate liquidity management and renders it inherently dynamic. One way to implement liquidity management is using uncontingent instruments, such as holding cash, which transfers liquid funds across all states symmetrically. We will refer to such policies as unconditional liquidity management. Alternative instruments, such as credit lines or derivatives, have a more state-contingent flavor in that corporations may draw on them to transfer funds to specific states only. We will refer to such policies as conditional liquidity management.

In practice, we see firms engaging in many combinations of conditional and unconditional liquidity management policies, yet there is relatively little work attempting to understand the determinants of these choices. In this paper, our objective is to take a step towards filling this gap. We do so by proposing a dynamic model of corporate policies that explicitly allows corporations to transfer liquid funds unconditionally using cash and conditionally by drawing on credit lines¹. The result is a quantitative theory of optimal liquidity management based on the trade-off between conditional liquidity subject to collateral constraints and unconditional, unconstrained liquidity. In the model, liquidity needs arise from stochastic investment opportunities and cash shortfalls in the context of high leverage. By estimating the model structurally by means of the simulated method of moments (SMM), we provide novel empirical predictions on the cross-sectional and time-series determinants of corporations' liquidity policies. We test these predictions empirically using data on credit lines from CapitalIQ and find strong support for them. The model thus provides a quantitatively and empirically successful framework rationalizing corporate investment, financing and liquidity policies and the joint occurrence of cash, debt and credit lines in the presence of capital market imperfections.

¹Credit lines play a first-order role for firm's financing. As Sufi (2009) points out, over 80 percent of bank debt held by public firms is in the form of lines of credit. Moreover, Colla, Ippolito, and Li (2013) report that the drawn part alone of credit lines accounts for more than 20 percent of the debt structure of US listed firms.

In the model, firms attempt to take advantage of profitable investment opportunities that arise stochastically. However, due to capital market imperfections, issuing equity entails costs such that firms will find it beneficial to exploit the tax benefits of leverage by issuing debt. However, we assume that debt needs to be collateralized by capital so that all debt is secured. This means that firms' debt capacity is endogenously bounded. In this context, a rationale for liquidity management arises. Firms can transfer liquidity unconditionally across all states by saving, that is, by holding cash. On the other hand, firms can preserve debt capacity in a state-contingent way by drawing on their credit lines as economic conditions dictate. This allows firms to transfer liquidity conditionally to specific states only. We show that the model predicts that firms will exploit conditional and unconditional liquidity management highly differentially both in the cross-section and in the time series. Estimating the model, we find that such differential use of liquidity management provides a coherent explanation for many stylized facts about firms joint investment, financing and liquidity policies.

Our model rationalizes the empirical evidence that firms simultaneously hold cash and debt, hence corroborating the notion that cash is not negative debt. Within the context of our model, the intuition is simple. While debt and credit lines jointly allow for state-contingency within the limits of debt capacity, holding cash allows to transfer liquidity beyond collateral constraints in case of high financing needs. Such high financing needs most likely arise when firms have many profitable investment opportunities. In this context, the model predicts that small firms and constrained firms (as measured by net worth) hold more cash, all else equal. This is a pattern well documented in the data, indicating that such firms mostly manage liquidity by means of unconditional instruments. On the other hand, large firms and relatively unconstrained firms are predicted to hold less cash and have more undrawn credit, indicating that they rely more conditional policies for liquidity management. We confirm this prediction using data on credit lines from CapitalIQ. Our model also replicates the well documented positive relationship between leverage and size.

An important implication of the model is that empirically we carefully need to distinguish between small firms (as measured by the capital stock) and constrained firms (as measured by net worth). Indeed, these variables are the two relevant (endogenous) state variables in the model. While the two variables are indeed somewhat correlated, we document the need of distinguishing them by means of panel regressions of the amount of transferred liquidity, and of the fraction of liquidity transferred conditionally through undrawn credit on capital and net worth. These regressions suggest that the main driver of cash holdings is capital, while financial constraints matter less. Since low capital implies valuable growth opportunities (in a model with decreasing returns to scale), this suggests that unconditional liquidity management mostly serves to transfer funds to states with high investment opportunities. On the other hand, the amount of undrawn credit significantly varies with net worth, controlling for capital. Indeed, unconstrained firms have more slack on their credit lines, so that the transfer more funds to valuable states conditionally. Symmetrically, constrained firms mostly exhaust their debt capacity. This is consistent with the notion, developed in Rampini and Viswanathan (2010), and Rampini and Viswanathan (2012a), that constrained firms hedge less, and that if they do, they do it unconditionally using cash. We find strong support for these predictions in the data, suggesting the need to distinguish between size and financial constraints, in contrast to most commonly used financial constraint indicators in empirical work. Moreover, these findings suggest that cross-sectionally we can distinguish firms whose liquidity management is mostly dictated by preserving liquidity for investment opportunities. which we label 'upstate hedging', as opposed to firms preserving liquidity in order to cover cash shortfalls, which we label 'downstate hedging'. In particular, our findings suggest that different instruments serve such liquidity needs better. Figure 1 illustrates our results.

Our analysis points to the importance of examining financing and liquidity policies in the context of investment opportunities, and in particular, investment frictions. While it is well known that financing policies in dynamic investment models exhibit considerable sensitivity to the specification of investment technologies, we reinforce such results in the context of measures of firms' liquidity management. Obstructions to frictionless adjustment of the capital stock in dynamic corporate models are most commonly represented by means of a convex (quadratic) adjustment cost. Our results clearly indicate that fixed costs of adjustment are important to understand liquidity management at the firm level, and cash holdings in particular.

From a computational viewpoint, we introduce linear programming methods into dynamic corporate finance. Accounting for conditional liquidity management by means of state-contingent policies introduces a large number of control variables into our setup which would render our model subject to the curse of dimensionality for standard computational methods. We exploit and extend linear programming methods to circumvent this problem and efficiently solve for the value and policy functions in this class of problems. Linear programming methods, while common in operations research, have been introduced into economics and finance by Trick and Zin (1993, 1997). We extend their methods to setups common in corporate finance. More specifically, we exploit a separation oracle, an auxiliary mixed integer programming problem, to deal with large state spaces and find efficient implementations of Trick and Zin's constraint generation algorithm.

Our paper is at the intersection of several converging lines of literature. In particular we interpret the quantitative literature on dynamic investment and financing (as started by Gomes (2001), Hennessy and Whited (2005), and Hennessy and Whited (2007)) further in light of the recently emerged literature on dynamic risk management and hedging in the context of collateralized debt (Rampini and Viswanathan (2010), Rampini and Viswanathan (2012a)). We build on Rampini and Viswanathan by modeling state-contingent debt subject to collateral constraints. While Rampini and Viswanathan operate in a dynamic optimal contracting framework, we take the form of the contracts as exogenously given and interpret them in the wider context of commonly used frictions in the dynamic financing literature, such as equity issuance costs and investment frictions. Most importantly, we allow firms to use cash as a form of liquidity management. While these leads to a distinct set of empirical predictions, we moreover view our paper as contributing more to the quantitative and empirical literature rather than the one on optimal security design.

Our paper is closely related to the emerging literature on firm policies and cash holdings. A non-exhaustive list includes Gamba and Triantis (2008), Nikolov and Whited (2009), Morellec and Nikolov (2009), Hugonnier, Malamud and Morellec (2011), Bolton, Chen, and Wang (2011), Falato, Kadyrzhanova, and Sim (2013), Bolton, Chen, and Wang (2012), and Eisfeldt and Muir (2013). Our main departure from this line of literature is that we allow for conditional liquidity management that we interpret in the context of credit lines. Our empirical results suggest that this is a relevant model feature. In this context, our paper is most closely related to Bolton, Chen and Wang (2011, 2012), who allow firms to access credit lines and hedge aggregate shocks using derivatives. On the other hand, for tractability, these authors operate within an AK-framework which allows to reduce the number of state variables and to obtain analytical solutions up to an ordinary differential equation. However, our empirical results suggest that distinguishing between the capital stock and net worth as state variables is empirically relevant.

This paper is structured as follows. We present a simple example illustrating the key mechanisms at work in section 2. We then integrate these mechanisms into a dynamic, quantitative model of firm financing, investment and liquidity management that we describe in section 3. We introduce our estimation procedure along with empirical results in section 4. Section 5 concludes. Appendices collect various results related to our computational solution technique.

2. A simple example

In this section we present a numerical example that illustrates the key economic tradeoff between conditional and unconditional liquidity that we embed in a standard dynamic model of investment and financing. Although this example is highly stylized, it explains both the basic insight of this work, and how firms can implement their conditional and unconditional liquidity choices using existing securities, namely lines of credit and cash.

A risk-neutral firm has an initial net worth of 10\$ in cash. At this stage, we ignore intertemporal discounting and interest payments. The firm decides how much physical capital k to purchase today to invest in a project. After the investment possible scenarios can occur with equal probability. In the bad scenario, the project is not successful and generates zero profits. In the good scenario, the project generates an intertemporal profit equal to 0.3 $k^{0.85}$, and the firm will be able to reinvest an amount k_G and obtain additional certain profits equal to 0.3 $k^{0.85}_G$. When profits are realized, the amounts of physical capital k and k_G depreciate at a rate equal to 10%.

First consider the baseline case summarized in Panel A of Figure 1. The firm can either borrow an amount b to increase the amount k invested and to be repaid after the uncertainty about the project prospects resolves, or hoard an amount of cash c that can be used to reinvest to a larger scale k_G in case the good scenario occurs. Borrowing is subject to collateral constraints. Specifically, the amount the firm can credibly repay is a fraction $\theta = 0.5$ of the physical capital k the firm can pledge as collateral, that is

$$b \le 0.5 \ k$$

In this standard case, the firm can transfer liquidity only unconditionally using cash. The firm's goal is to maximize the expected discounted value V of dividends, that is²:

$$\frac{1}{2} \quad \underbrace{(0.3 \ k_G^{0.85} + 0.9 \ k_G)}_{\text{good scenario}} + \frac{1}{2} \quad \underbrace{(0.9 \ k + c - b)}_{\text{bad scenario}}$$

²We assume, without loss of generality, that dividends are paid only at the end of the project's life.

where the initial budget constraint and the budget constraint in the good scenario are respectively

and

$$10\$ + b = k + c$$

$$k_G = 0.3 \ k^{0.85} + 0.9 \ k + c - b$$

The optimal firm's choice is k = 6.47, b = 0, c = 3.53, $k_G = 10.82$, and the resulting firm's value is V = 10.68. In this standard setup, cash is negative debt, that is it is never optimal for the firm to simultaneously hold cash and debt³. Intuitively, the firm prefers to reduce the scale of the initial investment k to transfer liquidity unconditionally using cash, in that this liquidity will be valuable in the good scenario.

Consider instead a second case, summarized in Panel B of Figure 1. The firm now arranges a state-contingent contract with the lender. The lender is risk-neutral, and agrees to lend an amount b equal to the expected value of the future repayments in the good scenario b_G and in the bad scenario b_B , that is $b = 0.5 b_G + 0.5 b_B$. As in the previous case, the collateral constraints make the debt risk free and limit the amount the firm can credibly repay in each scenario, that is

$$\begin{cases} b_G \le 0.5 \ k \\ b_B \le 0.5 \ k \end{cases}$$

In practice, the state-contingent payments b_G and b_B can be implemented combining standard securities as standard, state-uncontingent, debt and lines of credit. Suppose the firm initially has the same initial net worth of 10\$ of the previous case. However, net worth is now composed of 15\$ of cash and of an available credit line with an amount already drawn equal to $CL_0 = 5$ \$ and to be repaid at the end of the project's lifetime. For example, the firm might invest k = 20\$, raise b = 5\$ today and effectively repay $b_G = 0$ \$ in the good state and $b_B = 10$ \$ is the bad state by arranging a 5\$ uncontingent loan, drawing 5\$ from the credit line in the good scenario, and restoring 5\$ on the credit line in the bad scenario. In this way, the firm would effectively transfer liquidity conditionally only to the good state, by exhausting its debt capacity in the bad state, and keeping slack on its collateral constraint in the good state.

³See, for example, Strebulaev and Whited (2012).

For illustration purposes, suppose initially that the firm cannot transfer liquidity unconditionally using cash at all, that is c is set to zero. The firm's objective is to maximize

$$V = \frac{1}{2} \quad \underbrace{(0.3 \ k_G^{0.85} + 0.9 \ k_G - CL_G)}_{\text{good scenario}} + \frac{1}{2} \underbrace{(0.9 \ k - b - CL_B)}_{\text{bad scenario}}$$

where CL_G and CL_B are the credit line drawn amounts to be eventually repaid in the good and in the bad scenario respectively, that is

$$\begin{cases} CL_G = CL_0 + b - b_G\\ CL_B = CL_0 + b - b_B \end{cases}$$

The initial budget constraint and the budget constraint in the good scenario are respectively

$$15 \ \$ + b = k$$

and

$$k_G = 0.3 \ k^{0.85} + 0.9 \ k - b_G$$

The optimal firm's choice is k = 20, b = 5, $b_G = 0$, $b_B = 10$, $k_G=21.83$, and the resulting firm's value is V = 10.88. Notice that the firm's value is higher than in the previous case. The firm now uses credit lines to implement conditional liquidity management, and transfers liquid funds only to the specific states where those resources are needed. In this case, conditional liquidity allows to boost investment more in good states where profitable opportunities are available. More generally, conditional liquidity or to have a larger available buffer to hedge income shortfalls in bad states and avoid engaging in costly asset fire sales. As a consequence, implementing conditional liquidity management increases firm's value in comparison with engaging in unconditional liquidity management.

If conditional liquidity management is more efficient, why do firms use cash to transfer resources unconditionally at all? The answer is that the amount of liquid funds the firm can transfer to a specific state is limited by the presence of collateral constraints. In this example, it may be valuable to give up current investment to have more liquidity that what the firm can transfer conditionally available in the good state for future investments. To see this, consider the previous example when the firm is not constrained anymore to hoard zero cash, but can transfer liquidity both conditionally and unconditionally. The firm's objective is now

$$V = \frac{1}{2} \quad \underbrace{(0.3 \ k_G^{0.85} + 0.9 \ k_G - CL_G)}_{\text{good scenario}} + \frac{1}{2} \quad \underbrace{(0.9 \ k - b + c - CL_B)}_{\text{bad scenario}}$$

and the budget constraints become

$$15\$ + b = k + c$$

and

$$k_G = 0.3 \ k^{0.85} + 0.9 \ k - b_G + c$$

The optimal firm's choice is k = 10.62, b = 2.66, $b_G = 0$, $b_B = 5.32$, c = 7.03, $k_G=18.83$, and the resulting firm's value is V = 10.93. Thus, the firm's value is larger when the firm combines conditional and unconditional liquidity management. In sum, a tradeoff between conditional liquidity (efficient but constrained), and unconditional liquidity (inefficient but unconstrained) emerges. This tradeoff endogenously generates the co-existence of cash and debt in firms' balance sheet, an empirically relevant pattern which is difficult to rationalize in standard dynamic models of investment and financing⁴.

[Insert Figure 1 Here]

 $^{^{4}}$ An exception is Gamba and Triantis (2008).

3. Dynamic liquidity model

We now embed liquidity management into a dynamic neoclassical model of corporate investment and financing. Due to costs of external financing, corporations will benefit from the availability of liquid funds, either to fund investment opportunities or to cover cash shortfalls. Firms can provide liquid funds by saving by means of cash, or by drawing on credit lines in a state-contingent manner. We assume that the availability of credit lines is restricted by collateral constraints limit firms' debt capacity.

3.1. Technology and Investment

We consider the problem of a value-maximizing firm in a perfectly competitive environment. Time is discrete. The operating profit for firm i in period t depends upon the capital stock $k_{i,t}$ and a shock $z_{i,t}$, as described by the expression

$$\Pi(k_{i,t}, z_{i,t}) = (1 - \tau)(z_{i,t}k_{i,t}^{\alpha} - f)$$
(1)

The production function exhibits decreasing returns to scale with $0 < \alpha < 1$. As in Gomes (2001), we assume there is a per-period fixed production cost $f \ge 0$. $\tau \ge 0$ is the corporate tax rate. The variable $z_{i,t}$ reflects shocks to demand, input prices, or productivity. $z_{i,t}$ is assumed to be lognormal and to follow the AR(1) process

$$\log(z_{i,t+1}) = \mu_z (1 - \rho_z) + \rho_z \log(z_{i,t}) + \sigma_z \epsilon_{i,t+1}$$
(2)

At the beginning of each period the firm is allowed to scale its operations by choosing next period capital stock $k_{i,t+1}$. This is accomplished through investment $i_{i,t}$, which is defined by the standard capital accumulation rule

$$k_{i,t+1} = k_{i,t}(1-\delta) + i_{i,t} \tag{3}$$

where δ is the depreciation rate of capital. Investment is subject to capital adjustment costs. Following Cooper and Haltiwanger (2006), we include both fixed and convex adjustment cost components. We parametrize capital adjustment costs with the following functional form:

$$\Psi(k_{i,t+1},k_{i,t}) \equiv \left(\psi_0 k_{i,t} + \frac{1}{2}\psi\left(\frac{i_{i,t}}{k_{i,t}}\right)^2 k_{i,t}\right) \mathbf{1}_{\{k_{i,t+1} \neq (1-\delta)k_{i,t}\}}$$
(4)

where $\mathbf{1}_{\{\cdot\}}$ is an indicator function, and the parameter ψ_0 governs fixed-adjustment costs of investing and disinvesting. Non-convex costs of adjustment are typically intended to capture indivisibilities in capital, increasing returns to the installation of new capital, and increasing returns to retraining and restructuring of production activity. ψ instead drives the convex component of adjustment costs. Both convex and non-convex costs are proportional to the initial capital stock $k_{i,t}$ to eliminate any size effect.

3.2. Financing and Liquidity Management

Investment and distributions to shareholders can be financed with three potential sources: internally generated cash flows, riskfree loans (net of repayments), a credit line, and external equity. In addition, firms have the option to hoard cash for future investments. Loans are a uncontingent debt instrument. On the other hand, our view of a credit line that we embed into our model emphasizes their nature as a state-contingent debt instrument: corporations can draw upon them conditional on the realization of the state. The entirety of debt instruments in our model, that is loans plus draws on credit lines, can thus be represented as state-contingent debt.

Formally, we define $(1+r)b_{i,t+1}(z(i,t+1))$ to represent the face value to be repaid at time t + 1 in the state of the world s(t + 1) corresponding to the realization of the shock z(i,t+1), where r is the one-period rate of return.⁵ In other words, the firm is borrowing from deep-pocket lenders who are willing to lend in all states and dates at the rate of return r. We provide a decomposition of these payments into a state-contingent loan and a draw on a credit line below.

⁵Because our focus in not on endogenous costs of distress, as in Hennessy and Whited (2005) we make the assumption of riskfree debt in the interest of tractability. Given the high number of decision variables and the presence of occasionally non-binding constraints and non-convex costs, solving the model is computationally intensive. The introduction of endogenous default costs would disproportionately increase the computational burden.

To simplify notation, we introduce the shorthand $b_i(s_{t+1})$ for the decision variables $b_{i,t+1}(z(i,t+1))$. The value of new debt issues at time t in state s_t is

$$E_t[b_i(s_{t+1})] - (1 + r(1 - \tau))b_i(s_t)$$
(5)

 $1 + r(1 - \tau)$ is the effective interest rate paid by the firm, after accounting for the tax shield of debt. Firms are subject to collateral constraints, that impose an upper bound on the amount of one-period state-contingent debt that a firm can issue. Assuming that future cash flows are not pledgeable, collateral constraints take the form:

$$(1 + r(1 - \tau))b_i(s_{t+1}) \le \theta(1 - \delta)k_{i,t+1} \tag{6}$$

Up to a fraction θ of the firm's tangible capital can be used as collateral for state-contingent debt at time t + 1 in state s(t + 1). We characterize risk management and the conditional corporate liquidity policy by defining *conditional liquidity* $h_i^C(s_{t+1})$ as the slacks on the state-contingent collateral contraints:

$$h_i^C(s_{t+1}) \equiv \theta(1-\delta)k_{i,t+1} - (1+r(1-\tau))b_i(s_{t+1})$$
(7)

The higher $h_i^C(s_{t+1})$, the larger the amount of debt capacity the firm is preserving for possible investment opportunities that may arise conditionally on the realization of the state s_{t+1} . This means that firms can *conditionally* manage its liquidity, that is they can preserve their ability to raise debt and support investment in states in which their cash flows are low, and they have less internally generated resources. There is a clear-cut tradeoff between *conditional liquidity* against future income shortfalls, and available funds for current investment. The amount of raised debt $E_t[b_i(s_{t+1})]$ in equation (7) is supported by the promised payments in future states. Therefore, the higher $h_i^C(s_{t+1})$, the more firms are transferring resources from today to future states, and the lower $E_t[b_i(s_{t+1})]$.

Given the tax benefits of debt, we thus view $(1 + r(1 - \tau))E_t[b_i(s_{t+1})]$ as naturally representing the state-uncontingent component of debt, or in other words, the loan. Statecontingent draws on the credit line are thus represented by the difference of state-contingent debt and the loan, so that the debt capacity provides a natural model of the credit limit. We provide a formal proof of the representation of state-contingent debt, as defined above, by means of credit lines and loans in the appendix.

Conditional liquidity is not the only way firms can transfer liquid funds. Firms can hoard cash and implement *unconditional liquidity*. Hoarding cash is equivalent to unconditionally transferring resources from today to all future states, including those in which investment can be financed by internally generated funds. As for *conditional liquidity*, there is a tradeoff between current investment and saving resources for the future. However, is preferable to unconditional liquidity because it allows to transfer resources to the future states where they are needed the most. Nevertheless, the presence of capital adjustment costs as in equation (4) makes cash hoarding optimal for smaller firms that would not otherwise be able to invest to an economically profitable scale, even if they exhaust their debt capacity. For this reason, and consistent with empirical evidence, our model predicts that firms can simultaneously hold debt and cash instead of using cash for repaying debt. This mechanism corroborates the intuition in Acharya, Almeida, and Campello (2007) that cash is not negative debt. We denote cash holdings in period t as $c_{i,t}$. Firms earn the after-tax riskfree interest rate $r(1-\tau)$ on their cash balances, but also bear costs for holding them. Previous studies motivate the costs of holding cash by agency costs, and different lending and borrowing rates. Following DeAngelo, DeAngelo, and Whited (2011), we model these costs through an "agency parameter" $0 \le \gamma \le 1$. We interpret γ as the one-period rate to which cash holdings deteriorate in value. Accordingly, the total hedging for firm i at time t+1 in state s(t+1) is the amount of resources available from both *conditional liquidity* and *unconditional liquidity*, that is:

$$h_i^T(s_{t+1}) \equiv h_i^C(s_{t+1}) + (1 + r(1 - \tau) - \gamma)c_{i,t+1}$$
(8)

Finally, the firm can raise external equity. We assume seasoned equity offers are costly, so that it is never optimal for the firm to simultaneously pay dividends and issue equity. We assume that equity flotation costs entail a proportional component. We indicate *net equity* payout at time t as $e_{i,t}$. When $e_{i,t} < 0$ the firm is raising equity, while $e_{i,t} \ge 0$ means that the firm is making distributions to shareholders. Equity issuance costs are given by:

$$(\lambda|e_{i,t}|)\mathbf{1}_{\{e_{i,t}<0\}} \tag{9}$$

The indicator function denotes that the firm faces these costs only in the region where the the net payout is negative. Accordingly, distributions to shareholders $d_{i,t}$ are the equity payout net of issuance costs:

$$d_{i,t} = e_{i,t} - (\lambda |e_{i,t}|) \mathbf{1}_{\{e_{i,t} < 0\}}$$
(10)

3.3. The Firm Problem

Managers determine investment, financing, and liquidity management to maximize the wealth of shareholders. Hence, in period t, they decide over real capital $k_{i,t+1}$, cash $c_{i,t+1}$, and statecontingent debt $b_i(s_{t+1})$, for each state s_{t+1} . As we discuss in section 3.1, the choice set for capital is compact. Collateral constraints in equation (6) imply that state contingent debt variables are bounded between 0 and $\frac{\theta(1-\delta)k_{i,t+1}}{1+r(1-\tau)}$.

Despite the large number of choice variables in the firm problem, the current state can be more efficiently summarized by introducing *realized net worth* as a state variable. Realized net worth at time t in the (realized) state s(t) for firm i is given by:

$$w_{i,t} \equiv \Pi(k_{i,t}, z_{i,t}) + k_{i,t}(1-\delta) - (1+r(1-\tau))b_i(s_t) + (1+r(1-\tau)-\gamma)c_{i,t} + \tau\delta k_{i,t} \quad (11)$$

As in Rampini and Viswanathan (2012a), net worth measures the amount of resources that are available to the firm in a certain state. It includes cash flows from current investment, value of capital net of depreciation, and value of cash holdings, all net of due debt payments. Intuitively, net worth is the corporate counterpart of household's wealth (Rampini and Viswanathan (2012b)). Therefore, net worth is a measure of how constrained a firm is in terms of available funds to allocate to investment, risk management, and distributions to shareholders. In our model, the presence of capital adjustment costs implies that the current stock of capital $k_{i,t}$ is also a relevant state variable. In fact, the knowledge of net worth and of the choice variables does not suffice to determine distributions to shareholders $d_{i,t}$ that appear in the objective function, because the adjustment costs $\Psi(k_{i,t+1}, k_{i,t})$ also directly depend on the current stock of capital. The current state is therefore summarized by the vector $(w_{i,t}, k_{i,t}, z_{i,t})$. The set of state variables is compact because $k_{i,t}$ and $z_{i,t}$ are bounded, and from equation (11) it is straightforward that net worth lies in a closed and bounded interval $[\Psi, \bar{W}]$.

Investment, financing, and liquidity management decisions are intimately related. They should satisfy the following budget identities between sources and uses of funds both at time t, and for each state at time t + 1:

$$w_{i,t} + E_t[b_i(s_{t+1})] = e_{i,t} + k_{i,t+1} + \Psi(k_{i,t+1}, k_{i,t}) + c_{i,t+1}$$
(12a)

$$w_i(s_{t+1}) = \Pi(k_{i,t+1}, z_{i,t+1}) + k_{i,t+1}(1 - \delta) - (1 + r(1 - \tau))b_i(s_{t+1}) + (1 + r(1 - \tau) - \gamma)c_{i,t+1} + \tau \delta k_{i,t}$$
(12b)

where $w_i(s_{t+1})$ denotes net worth at time t+1 is state s(t+1).

The firm objective function is to maximize the equity value $V(k_{i,t}, w_{i,t}, z_{i,t})$, that is the discounted value of distributions to shareholders. The equity value be computed as the solution to the dynamic programming problem

$$V(k_{i,t}, w_{i,t}, z_{i,t}) = \max\left\{0, \max_{k_{i,t+1}, c_{i,t+1}, b_i(s_{t+1})} \left\{d_{i,t} + \frac{1}{1+r} E_t[V(k_{i,t+1}, w_{i,t+1}, z_{i,t+1})]\right\}\right\}$$
(13)

subject to the constraints in (3), (4), (6), (10), and (12). In equation (13), $V(k_{i,t+1}, w_{i,t+1}, z_{i,t+1})$ denotes the continuation value for equity, which depends on the future state $(k_{i,t+1}, w_{i,t+1}, z_{i,t+1})$ and on the values of the choice variables at time t.

3.4. Model Solution

Because of the presence of occasionally non-binding collateral constraints, and because costs of equity issues and capital adjustment depend on indicator functions, the model cannot be solved numerically by interior points methods. In principle, the model could be solved on a discrete grid by value function iteration or policy function iteration. The Bellman operator in equation (13) is indeed a contraction mapping, in that Blackwell's sufficient conditions hold in this framework. Therefore, the fixed point of the functional equation (13) is well-defined. For a standard formal proof in a similar framework, we refer to Hennessy and Whited (2005). Unfortunately, there is a computational hurdle that prevents the solution of the model with standard techniques. Due to the large number of control variables (capital, cash, and one debt variable for each future state), value function iteration and policy iteration cannot be practically implemented. In particular, the maximization step is critical. Determining for each state the combination of control variables that maximizes the sum of distributions and the continuation value implies to store and maximize over a vector of $nk \times nc \times nb^{nz}$ elements, where nk, nc, nb, and nz are the number of grid points for capital, cash, debt, and the shock. As in Rust (1997), this problem is plagued by a curse of dimensionality, since the amount of control variables. As a consequence, even for modest values for nz, such a vector becomes too large even to be stored.

We overcome this difficulty by exploiting the linear programming representation of dynamic programming problems with infinite horizon (Ross (1983)), as in Trick and Zin (1993), and Trick and Zin (1997). This technique has not been historically widely used. Despite it often allows to achieve significant speed gains over iterative methods, it requires in turn to store huge matrices and arrays that make it impractical for complex enough models. Specifically, we extend the constraint generation algorithm developed by Trick and Zin (1993), and we rely on a *separation oracle*, an auxiliary mixed integer programming problem, to avoid dealing with large vectors at all. As in Trick and Zin (1993), the constrained generation algorithm converges to the fixed point faster than traditional iterative methods. Moreover, the separation oracle allows to efficiently implement the maximization step because of a remarkable feature of our model, namely the relatively small number of state variables in spite of the large number of control variables. With this method, we manage to solve the model in a reasonable time (around three minutes on an eight-core workstation). Appendix B provides details on the solution method.

3.5. Optimal Policies

3.5.1. Hedging Formulation

Lemma 1 (Hedging formulation)

The constrained optimization problem (13) is equivalent to:

$$V(k_{i,t}, w_{i,t}, z_{i,t}) = \max\left\{0, \max_{\substack{k_{i,t+1}, h_{i,t+1}^U, h_i^C(s(t+1))}}\left\{e_{i,t} - \Lambda(e_{i,t}) + \frac{1}{1+r}E_t[V(k_{i,t+1}, w_{i,t+1}, z_{i,t+1})]\right\}\right\}$$
(14)

s.t.

$$w_{i,t} \ge e_{i,t} + E_t \left[\frac{h_i^C(s(t+1))}{1+r(1-\tau)} \right] + \frac{h_{i,t+1}^U}{1+r(1-\tau)-\gamma} + Pk_{i,t+1} + \Psi(k_{i,t}, k_{i,t+1})$$
(15a)

$$w_i(s(t+1)) \le (1-\tau)\Pi(k_{i,t+1}, z_{i,t+1}) + (1-\theta)(1-\delta)k_{i,t+1} + \tau\delta k_{i,t+1} + h_i^T(s(t+1)) \quad \forall s(t+1)$$
(15b)

$$h_i^C(s(t+1)) \ge 0 \qquad \qquad \forall s(t+1)$$

(15c)

(15d)

$$h_i^C(s(t+1)) \le \theta(1-\delta)k_{i,t+1} \qquad \forall s(t+1)$$

$$h_{i,t+1}^U \ge 0 \tag{15e}$$

where $P \equiv 1 - \frac{\theta(1-\delta)}{1+r(1-\tau)}$ is the fraction of each unit of capital paid down by the firm at time $t, h_i^C(s(t+1)) \equiv \theta(1-\delta)k_{i,t+1} - (1+r(1-\tau))b_i(s(t+1))$ is conditional hedging for state $s(t+1), h_i^U(s(t+1)) \equiv h_{i,t+1}^U = (1+r(1-\tau)-\gamma)c_{i,t+1}$ is unconditional hedging for all states at time t+1, and $h_i^T(s(t+1)) \equiv h_i^C(s(t+1)) + h_{i,t+1}^U$ is total hedging.

The hedging formulation is particularly instructive because it emphasizes the role of dynamic liquidity management. The problem (14) can be equivalently interpreted as a problem where firms pledge all their collateral, and transfer resources (net worth) from t to t + 1 both conditionally, to specific states, and unconditionally, to all future states. Regarding conditional liquidity, firms decide to purchase $\frac{h_i^C(s(t+1))}{1+r(1-\tau)}$ Arrow-Debreu securities at time t in order to obtain a payoff of $h_i^C(s(t+1))$ is state s(t+1) next period. Constraints (15c) and (15d) impose bounds on the amount of conditional hedging the firm can implement. The collat-

eral constraint imposes a lower bound, that corresponds to exhausting all debt capacity. Constraint (15d) states that the maximum amount of liquid funds that a firm can transfer to state s(t+1) corresponds to its debt capacity, that is to the firm having zero debt due in state s(t + 1). Unconditional hedging instead consists of hoarding an amount of cash $\frac{h_{i,t+1}^U}{1+r(1-\tau)-\gamma}$, in order to get to obtain a payoff $h_{i,t+1}^U$ in all future states at time t+1. The hedging formulation provides a preliminary intuition on the different nature of conditional and unconditional liquidity management. Equations (15a) and (15b) hint that transferring liquid funds conditionally is more efficient than doing so unconditionally if the firm needs to transfer resources only to some states (for example to bad states). Transferring funds to future states involves subtracting resources available to be distributed to shareholders $e_{i,t}$ and to be paid down to make investment possible $Pk_{i,t+1} + \Psi(k_{i,t}, k_{i,t+1})$. If, for example, a firms needs to transfer an amount M only to the specific state s(t+1) (for example the lowest state), the amount of resources it needs at time t is $\pi(s(t), s(t+1)) \frac{M}{1+r(1-\tau)}$, where $0 \le \pi(s(t), s(t+1)) < 1$ is the transition probability from state s(t) to state s(t+1). On the contrary, implementing unconditional hedging for the same purpose would require to subtract $\frac{M}{1+r(1-\tau)-\gamma}$. So, why should firms engage in unconditional liquidity management at all? Constraint (15d) states that the maximum amount of liquid funds that a firm can transfer conditionally is bounded by its total debt capacity $\theta(1-\delta)k_{i,t+1}$. Therefore, whenever it is optimal for the firm to have total hedging greater than this amount, hoarding cash becomes necessary. As a result, endogenously, cash is not negative debt, and consistent with empirical evidence we can observe firms simultaneously holding cash and debt.⁶ As the quantitative analysis in section 4 emphasizes, capital adjustment costs $\Psi(k_{i,t}, k_{i,t+1})$ play an important role, both qualitatively and quantitatively. Specifically, they allow to differentiate between firms that are constrained in terms of net worth, and small firms, and rationalize patterns that are observed in the data. Equation (15a) points up that different current and future investment needs yield to different needs of transferring net worth to future states. This creates sharp differences in corporate liquidity policy of large and small firms. Suppose, for example, that adjustment costs are quadratic in the investment-to-capital ratio. With

 $^{^{6}}$ As DeAngelo, DeAngelo, and Whited (2011) discuss, in frameworks in which firms never optimally hold cash and debt together, it is not necessary to model them using two separate positive control variables. In our model, letting negative debt being cash by relaxing constraint (15d) would not only prevent firms from simultaneously holding cash and debt, but also assume that state-contingent cash securities exist, which is unrealistic.

decreasing returns to scale, small firms with high investment needs would be better off in spreading investment over multiple periods to avoid incurring disproportionately high adjustment costs. Therefore, they may find optimal to hedge more, by saving debt capacity in a state contingent and possibly by hoarding cash. This creates a dependence between investment and liquidity needs, and, as a consequence, between size and risk management.

3.5.2. Optimal Policy

Proposition 1 (Optimality conditions)

Denote by λ^w , $\frac{\pi(s(t),s(t+1))\lambda_{s(t+1)}^w}{1+r}$, $\frac{\pi(s(t),s(t+1))\lambda_{s(t+1)}^C}{1+r}$, $\frac{\pi(s(t),s(t+1))\overline{\lambda}_{s(t+1)}^C}{1+r}$, and $\underline{\lambda}^U$ the multipliers on constraints (15a), (15b), (15c), (15d), and (15e) respectively, where $\pi(s(t),s(t+1))$ is the Markovian transition probability from state s(t) to state s(t+1). Assume that the equity cost function $\Lambda(e_{i,t})$ is differentiable in e(i,t).⁷ Then, the first order conditions for the hedging formulation (14) can be expressed as follows:

$$\lambda^w = 1 - \frac{\partial \Lambda(e_{i,t})}{\partial e_{i,t}} \tag{16a}$$

$$\lambda^{w}(P + \frac{\partial \Psi(k_{i,t}, k_{i,t+1})}{\partial k_{i,t+1}}) = \frac{1}{1+r} E_t[\lambda^{w}_{s(t+1)}V^k(s(t+1)) + \overline{\lambda}^{C}_{s(t+1)}H^k]$$
(16b)

$$\lambda^{w} \frac{1}{1 + r(1 - \tau) - \gamma} = \frac{1}{1 + r} E_{t} [\lambda^{w}_{s(t+1)}] + \underline{\lambda}^{U}$$
(16c)

$$\frac{1}{1+r(1-\tau)}\lambda^{w} = [(\underline{\lambda}_{s(t+1)}^{C} - \overline{\lambda}_{s(t+1)}^{C}) + \lambda_{s(t+1)}^{w}]\frac{1}{1+r} \qquad \forall s(t+1) \qquad (16d)$$

where

$$V^{k}(s(t+1)) = (1-\tau)\frac{\partial \Pi(k_{i,t+1}, z_{i,t+1})}{\partial k_{i,t+1}} + \tau \delta + (1-\theta)(1-\delta) \qquad \forall s(t+1)$$
(17a)

$$H^k = \theta(1 - \delta) \tag{17b}$$

⁷In our model, we choose a functional form for equity flotation costs with a fixed and a proportional component, which is non-differentiable for e(i,t) = 0 (its derivative at zero exists only in a distributional sense). This assumption is not critical for our qualitative analysis. Alternatively, one can approximate $\Lambda(e_{i,t})$ with 0.5(1 + tanh(Ne(i,t))), with N large enough, in the neighborhood of zero. A similar argument applies to the adjustment cost function $\Psi(k_{i,t}, k_{i,t+1})$ in case fixed costs are included.

The envelope conditions imply:

$$\frac{\partial V(w_{i,t}, z_{i,t})}{\partial w_{i,t}} = \lambda^w \tag{18a}$$

$$\frac{\partial V(w_{i,t+1}, z_{i,t+1})}{\partial w_{i,t+1}} = \lambda_{s(t+1)}^w \qquad \forall s(t+1)$$
(18b)

Moreover, the investment Euler equation is:

$$P + \frac{\partial \Psi(k_{i,t}, k_{i,t+1})}{\partial k_{i,t+1}} = E_t[M^w(s(t), s(t+1))V^k(s(t+1))] + E_t[M^h(s(t), s(t+1))H^k]$$
(19)

where $M^w(s(t), s(t+1)) \equiv \frac{1}{1+r} \frac{\lambda_{s(t+1)}^w}{\lambda^w}$ and $M^h(s(t), s(t+1)) \equiv \frac{1}{1+r} \frac{\overline{\lambda}_{s(t+1)}^C}{\lambda^w}$ are stochastic discount factors. In addition:

$$M^{w}(s(t), s(t+1)) = \frac{1}{1+r(1-\tau)} - \frac{1}{1+r} \frac{\underline{\lambda}_{s(t+1)}^{C} + \overline{\lambda}_{s(t+1)}^{C}}{\lambda^{w}}$$
(20)

The optimality conditions illustrate how investment, financing, liquidity and payout policies are intimately related, and shed light on the qualitative mechanism that drive firm's decisions. Moreover, they allow to understand the rationale for liquidity management, and which future states firms optimally hedge. Since the problem has no closed-form solution, the following analysis relies on the economic interpretation of the Lagrange multipliers as shadow values.

Equation (16b) relates the costs and benefits of investing an additional unit of real capital at time t + 1. The left hand side represent the marginal cost of investing. An additional unit of capital requires that the firm puts P money down and pays capital adjustment costs. The cost of doing so is $\left(P + \frac{\partial \Psi(k_{i,t},k_{i,t+1})}{\partial k_{i,t+1}}\right)\lambda^w$. The multiplier λ^w accounts for the shadow loss in firm value of relaxing the resource constraint (15a) at time t (resource constraints are always binding). The right hand side is the marginal benefit of an additional unit of investment, discounted back to time t by the shareholders' discount factor $\frac{1}{1+r}$. The benefits correspond to the two terms on the right hand side. First, the expected value of the additional investment $V^k(s(t+1))$ across all future possible states, that consists of marginal changes in profits, of tax benefits, and of the liquidation value of the share of capital not pledged to lenders. Second, the expected increase in debt capacity available for conditional hedging H^k in all states. The multipliers $\lambda_{s(t+1)}^w$ and $\overline{\lambda}_{s(t+1)}^C$ instead account respectively for the additional future net worth (constraint (15b)), and for the additional debt capacity (constraint (15d)) available to transfer conditional liquidity to state s(t+1) because of the additional unit capital installed (in case this constraint is binding).

$$\underbrace{\lambda^{w}(P + \frac{\partial\Psi(k_{i,t}, k_{i,t+1})}{\partial k_{i,t+1}})}_{\lambda^{w}(P + \frac{\partial\Psi(k_{i,t}, k_{i,t+1})}{\partial k_{i,t+1}})} = \underbrace{\frac{1}{1+r} E_{t}[\underbrace{\lambda^{w}_{s(t+1)}V^{k}(s(t+1))}_{\text{Net worth}} + \underbrace{\overline{\lambda}^{C}_{s(t+1)}H^{k}]}_{\text{Debt capacity}}}$$
(21)

Equation (16c) describes the unconditional liquidity policy of the firm. Similar to equation (16b), the left-hand side $\lambda^w \frac{1}{1+r(1-\tau)-\gamma}$ is the cost of allocating a unit of current net worth to cash hoarding, in order to transfer one unit of cash to all future states at t + 1. The right-hand side is the value of this additional unit of net worth available in all states $\frac{1}{1+r}E_t[\lambda^w_{s(t+1)}]$. In addition, the term $\underline{\lambda}^U$ accounts for the possibility that the constraint on positive cash is binding.⁸

$$\underbrace{\lambda^{w} \frac{1}{1 + r(1 - \tau) - \gamma}}_{\text{Net worth}} = \underbrace{\underbrace{\frac{1}{1 + r} E_{t}[\lambda^{w}_{s(t+1)}]}_{\text{Net worth}} + \underbrace{\underline{\lambda}^{U}}_{\text{Positive cash holdings}} (22)$$

Equation (16d) describes the conditional liquidity policy of the firm. As for unconditional liquidity management, the marginal cost of allocating one unit of net worth to risk management is $\lambda^w \frac{1}{1+r(1-\tau)}$ (the agency parameter $\gamma > 0$ makes it more costly for unconditional hedging). It is however more interesting to examine the right-hand side, and to compare it to the optimality conditions for unconditional liquidity management in equation (16c). As for cash hoarding, the benefits are discounted to time t through the manager's discount factor $\frac{1}{1+r}$. However, the value of additional net worth potentially available for the state s(t + 1) is $\lambda^w_{s(t+1)}$. In equation equation (16c), the value of the net worth transferred to

⁸This term is more meaningful in case we interpret the first-order condition on unconditional hedging for a reduction of one unit. In this case, the marginal benefit is the additional amount $\lambda^w \frac{1}{1+r(1-\tau)-\gamma}$ available at time t for investment, distributions, and conditional hedging, and the marginal cost is the sum of the value of one less unit of net worth available in all states, and of the shadow value of being able to reduce further cash if constraint (15e) binds.

state s(t+1) is only $\pi(s(t), s(t+1))\lambda_{s(t+1)}^w$.⁹ This supports the statement in section 3.5.1 that conditional liquidity management is preferable to unconditional liquidity management because with the same amount of net worth at time t it allows to transfer more resources to a specific state s(t+1). The term $\underline{\lambda}_{s(t+1)}^C - \overline{\lambda}_{s(t+1)}^C$ instead illustrates why firms may be interested in managing its liquidity both conditionally and unconditionally at the same time. Specifically, since in our model conditional hedging can be implemented only saving debt capacity in a state contingent way, the amount of conditional liquidity is limited by the constraints (15c) and (15d). The term $\underline{\lambda}_{s(t+1)}^{C}$ accounts for the presence of occasionally binding state-contingent collateral constraints, that may become active and limit the amount of state-contingent debt that a firm can hold given the amount of pledgeable capital $k_{i,t+1}$. Symmetrically, the multiplier $\overline{\lambda}_{s(t+1)}^C$ is different from zero in case the firm would like to transfer more resources conditionally, but its amount is limited because the firm has already zero debt due in state s(t+1). The limited amount of implementable conditional hedging through liquidity management implies that firms can simultaneously hold cash and debt. To see this, suppose that the firm is interested in hedging a specific state, such as the lowest state \underline{s} , as much as possible. Ceteris paribus, the maximum amount of resources that the firm can transfer to s corresponds to exhausting all debt capacity in all states except s. This implies that no debt is due in state s. Moreover, the firm can transfer the net worth raised by the state-contingent debt issues in all states excluding \underline{s} , to all future states, including \underline{s} , by hoarding cash. As a result, the firm would hold cash and debt together.

$$\underbrace{\frac{1}{1+r(1-\tau)}\lambda^{w}}_{\text{Limited conditional liquidity}} = \underbrace{\underbrace{\left[\underbrace{\lambda_{s(t+1)}^{C} - \overline{\lambda}_{s(t+1)}^{C}\right]}_{\text{Limited conditional liquidity}} + \underbrace{\lambda_{s(t+1)}^{w}}_{\text{Net worth}}\right] \frac{1}{1+r}}_{\text{Net worth}} \quad \forall s(t+1)$$
(23)

The payout policy instead balances the marginal cost of allocating a unit of net worth to dividend distributions or, vice versa, to issue equity to increment net worth by one unit.

⁹To better see this, notice that the expectation in equation (16c) is $\sum_{s=1}^{S} \pi(s(t), s) \lambda_s^w$, where S is the total number of states.

In case of equity issues, there is not a one-to-one correspondence between raised equity and increased net worth because of equity flotation costs.



The Euler condition (19) clarifies the important matter of the firm's rationale for liquidity management, and of which states it is optimal to hedge. The Euler equation can be interpreted as a pricing relationship. The left-hand side can be seen as the valuation of the paid down share $P + \frac{\partial \Psi(k_{i,t}, k_{i,t+1})}{\partial k_{i,t+1}}$ per unit of capital. The right-hand hand side shows that this value is supported by two terms. The term $E_t[M^w(s(t), s(t+1))V^k(s(t+1))]$ is the stochastically discounted valuation of the benefits $V^k(s(t+1))$ of investing an additional unit. $M^w(s(t), s(t+1))$ is the firm's stochastic discount factor, and is equal to $\frac{1}{1+r} \frac{\lambda_{s(t+1)}^{\lambda}}{\lambda^w}$. The concavity properties of the value function imply that the marginal value of a certain level of net worth is higher in bad times, so that the stochastic discount factor puts more weight on bad states through the Lagrange multipliers. Indeed, envelope conditions (18a) and (18b) show how Langrange multipliers are related to the shape of the value function, so that $\lambda_{s(t+1)}^w$ is decreasing in $w_i(s(t+1))$. In a valuation perspective, since a larger share of P is supported by those states, the firm behaves as if it were risk-averse. This provides incentives to implement liquidity management by preserving net worth for investments and distributions for bad future states, where internally generated cash flows and *future realized* net worth are, other conditions equal, lower. vice versa, the payoff from investments $V^k(s(t+1))$ suggests that the firm may want to hedge good states as well. If the law of motion of shocks to capital productivity $z_{i,t}$ is persistent enough, the payoff of investing in good (bad) times is higher (lower) because the firm expects a sequence of good (bad) shock realizations. The firm will therefore save resources for good states and boost investment in good times. If this is the case, the marginal value of net worth is not necessarily lower in bad states anymore. An instructive benchmark case is the case with independent productivity shocks. In such a scenario, the expected productivity of capital $\frac{\partial \Pi(k_{i,t+1}, z_{i,t+1})}{\partial k_{i,t+1}}$ is independent of the current state. As a consequence, firms only hedge bad states because of the properties of the discount factor $M^{w}(s(t), s(t+1))$. In practice, however, the productivity process in quite persistent. Therefore, the matter of whether firms hedge good or bad states (or both), and how much, is a purely quantitative question. Also, it is a quantitative question whether firms hedge at all. As in Rampini and Viswanathan (2012a), firms that are particularly constrained may not hedge, and prefer to allocate their scarce resources to current investment and distributions. The second term on the right-hand side instead H^k reflects that capital is valuable also because it serves as collateral, it increases debt capacity and, as a consequence, the amount of conditional liquidity management implementable in all states. The stochastic discount factor $M^h(s(t), s(t + 1))$ depends on the multiplier $\overline{\lambda}_{s(t+1)}^C$. Therefore, the value of increased debt capacity is higher is states where firms hold no debt because conditional liquidity is more valuable.

$$\underbrace{P + \frac{\partial \Psi(k_{i,t}, k_{i,t+1})}{\partial k_{i,t+1}}}_{k_{i,t+1}} = \underbrace{E_t[M^w(s(t), s(t+1))V^k(s(t+1))]}_{E_t[M^w(s(t), s(t+1))V^k(s(t+1))]} + \underbrace{E_t[M^h(s(t), s(t+1))H^k]}_{E_t[M^h(s(t), s(t+1))H^k]}$$
(25)

Finally, equation (20) explicitly relates the stochastic discount factor $M^w(s(t), s(t+1))$, which appears in the investment Euler equation, to the hedging policy of the firm. The multipliers $\underline{\lambda}_{s(t+1)}^C$, and $\overline{\lambda}_{s(t+1)}^C$ differ from zero respectively when the firm exhausts all its debt capacity in state s(t+1), and when the firm has zero debt in state s(t+1). These multipliers enter the Euler equation because of market incompleteness. Given the stochastic nature of the model, firms anticipate that collateral and debt positivity constraints may bind in the future, and this affect their investment and liquidity management policy. By transferring liquid funds conditionally, the firm can therefore influence the relative importance of different states for determining the value of paid-down capital. For example, if a company borrows constrained in the low state \underline{s} and saves all its debt capacity for future investment in the high state \overline{s} , the stochastic discount factor puts more weight on the high state, namely $\frac{1}{1+r(1-\tau)} + \frac{1}{1+r} \frac{\overline{\lambda}_w^{\overline{x}}}{\lambda_w}$ versus $\frac{1}{1+r(1-\tau)} - \frac{1}{1+r} \frac{\overline{\lambda}_w^{\overline{x}}}{\lambda_w}$.

$$M^{w}(s(t), s(t+1)) = \underbrace{\frac{1}{1+r(1-\tau)}}_{\text{Unconditional component}} - \underbrace{\frac{1}{1+r} \underbrace{\frac{\lambda_{s(t+1)}^{C}}{\lambda_{s(t+1)}^{C}} + \frac{\lambda_{s(t+1)}^{C}}{\lambda_{s(t+1)}^{C}}}_{\text{State-contingent component}}$$
(26)

3.5.3. Numerical Illustration

We provide numerical examples to illustrate the analytical analysis in section 3.5.2, and to better understand the qualitative importance of different types of capital adjustment costs for corporate investment and liquidity policy. In the interest of clarity, in all the examples we solve the model with three possible states and in absence of equity issues, and report the policy for the middle state. The details of the parametrizations are reported in the captions of figures 2 to 5.

Figure 2 refers to the case with no adjustment costs and independent investment opportunities. Specifically, Markovian transition probabilities are uniform (equal to one third for each pair of states), so that the expected capital productivity is the same for every state at time t. Panels A and B depict investment and payout as a function of current net worth. Similar to Rampini and Viswanathan (2012a), there exist a threshold of net worth below which investment is increasing, and dividends are zero. Above the threshold investment is constant and dividends are linear. Panel C shows that the value function is weakly concave in net worth. This is an important property, because the firm's stochastic discount factor in equation (25) is equal to $\frac{1}{1+r} \frac{\lambda_{s(t+1)}^{w}}{\lambda^{w}}$. As a consequence, the firm behaves as if risk averse with respect to net worth. Such a behavior is clearly visible in panel F. As we pointed out in the previous section, with independent productivity, the firm implements downstate liquidity management. In this example, it saves all its debt capacity for the low state for almost all levels of net worth. The dashed line (conditional hedging for the low state), and the thin line (available debt capacity) are indeed very close. The amount of liquidity decreases for the middle states (solid line), and is equal to zero for the high state (dashed-dotted line). Panel E shows the cash policy of the firm. When hedging needs exceed the available debt capacity, that is the amount of implementable conditional liquidity, and the firm is unconstrained enough in terms of net worth, it implements unconditional liquidity too. This way, additional resources are transferred to the low state. As a consequence, as panel D depicts, cash is not negative debt, and it is optimal for the firm to simultaneously hold them.

[Insert Figure 2 Here]

Figure 3 removes the assumption of independent investment opportunities, and introduces some persistence. In particular, the firm has now a probability of one half to stay in the current state, and of one quarter to move to another state. The policy is generally similar to that in figure 2, except for conditional liquidity management. The dashed-dotted line in Panel F is no longer equal to zero, meaning that the firm hedges upstate as well. Intuitively, with independent investment opportunities, the firm has no incentive to hedge the state where the marginal value of future net worth is lower. However, as equation (25) states, if there is a high probability that periods of high profits are followed by periods of high profits, expected future productivity is higher in good states. Therefore, the firm may rationally save resources for future investments in states where investment opportunities are likely to remain good.

[Insert Figure 3 Here]

Figures 4 to 5 emphasize the importance of capital adjustment costs to disentangle net worth from capital. We consider, one at a time, the types of adjustment costs in the general functional form (4), namely convex investment costs and fixed investment costs. This approach allows to see how the firm implements conditional and unconditional liquidity management for investment and disinvestment motives. Moreover, we can assess how the investment, liquidity, and risk management policy differs if we consider either fixed or smooth costs.

Figure 4 illustrates investment and liquidity management in presence of smooth investment costs. Panels A to C show how, for some values of the current capital stock, the policy is similar to the case with no adjustment costs. *Conditional on capital*, unconstrained firms transfer more liquidity, both conditionally and unconditionally. However, Panels D to F depict how the level of current capital now influences investment and hedging decisions, *conditional on net worth*. Panel D reports the optimal investment-to-capital ratio as a function of firm's size. Because of decreasing returns to scale in the production function, capital installment is relatively more profitable for small firms, which have higher investment needs. Because adjustment costs are quadratically increasing in the investment-to-capital ratio, smaller firms cannot instantaneously adjust to the desired capital level. Partial adjustment is hence optimal, and small firms transfer net worth for (costly) investment to future states, both conditionally (panel F), and unconditionally (panel E). This behavior results in small firms having more cash.

[Insert Figure 4 Here]

Figure 5 shows instead the case of liquidity management for investment in presence of fixed capital adjustment costs. As panel D clearly shows, the firm has a standard (S,s) policy as a function of current capital.¹⁰ In the figure, k^* denotes the "frictionless" level of capital in absence of investment adjustment costs, defined as in Caballero, Engel, and Haltiwanger (1995), and Caballero and Engel (1999). Intuitively, the more the firm deviates from the "target" level, the higher the cost it bears. As a consequence, when the disequilibrium $|k_{i,t} - k^*|$ is large, it is optimal to pay the fixed cost and to re-adjust the capital level to k^* . This policy determines an inaction region bounded by the low barrier k^D , and by the high barrier k^U . In this region, optimal investment is zero. Panels E and F emphasize how firms transfers conditional and unconditional liquid funds precisely in the inaction region. Intuitively, since they are not currently investing, they transfer some net worth to future states, instead of paying it off as dividends.

 $^{^{10}}$ For an exhaustive treatment of models with fixed costs we refer to Stokey (2008).

4. Structural estimation

We now proceed to formally estimate the parameters of the model by means of a simulationbased estimator, namely the simulated method of moments (SMM). We start by describing our data in some more detail. We present then the estimation method and provide a discussion on identification. Finally, we present our results.

4.1. Data

Estimating the dynamic corporate liquidity model requires merging data from different sources. In particular, we obtain financial statements data from Compustat annual files and credit lines data from Capital IQ. We remove all regulated (SIC 4900-4999) and financial firms (SIC 6000-6999). Observations with missing total assets, market value, gross capital stock, cash, long-term debt, debt in current liabilities, credit line limit, drawn portion of the credit line and SIC code are excluded from the final sample. We obtain a panel dataset with 19,796 observations for 3424 firms for the period of 2002 to 2011 at the annual frequency.

4.2. Estimation

We estimate most of the model parameters using simulated method of moments (SMM). However, we estimate some of the model parameters separately. For example, we set the risk-free interest rate, r, equal the average over the sample period of the one-year Treasury rate. We set the depreciation of capital, δ , equal to 12%, which is the average depreciation rate in the Compustat dataset. Finally, we set the agency cost of holding cash, γ , equal to 0.77%, which corresponds to the estimate from DeAngelo, DeAngelo, and Whited (2011).

We then estimate 8 parameters using the simulated method of moments: the curvature of the profit function, α ; the fixed production cost, f; the serial correlation of ln(z), ρ_z ; the standard deviation of the innovation of ln(z), σ_z ; the fixed capital adjustment cost, ψ_0 ; the variable adjustment cost, ψ ; the debt capacity, θ ; and is the equity flotation cost, λ . The simulation method of moments, although computationally intensive, is conceptually simple. We solve the model using numerical techniques and generate simulated data. Then, we compute interesting moments using both the simulated and the observed data. The SMM estimator selects the parameters such that the distance between the simulated and the actual data is minimized. We next describe the estimation procedure in some more detail.

One important aspect of the SMM estimation is the choice of weighting matrix. A natural candidate for this choice is the identity matrix. While intuitive, this choice comes with a significant drawback. Indeed, the identity matrix allocates more weight to the moments that are larger in absolute value. This property does not have any valid economic interpretation. As a consequence, we choose to use the optimal weighting matrix that is the inverse of the covariance matrix of the moments. Intuitively, this method allocates more weight on the moments that are measured with greater precision. To compute the covariance matrix of the moments, we follow the influence function approach of Ericson and Whited (2000).

Finally, one last aspect of the estimation relates to unobserved heterogeneity. Indeed, our model generates predictions for a representative firm. However, for the estimation, we use data from Compustat and Capital IQ, i.e. a panel data set. To have consistency between the simulated and the observed data, we need to either add heterogeneity to the simulated data or remove heterogeneity from the observed data. We select the second approach. To do so, we use firm and year fixed effects when we estimate variances, covariances, and regression coefficients.

4.3. Identification

Before proceeding with the estimation of the model, it is important to understand how we can identify the model parameters in the data. A sufficient condition for identification is a one-to-one mapping between the structural parameters and a set of data moments of the same dimension. It is however difficult to obtain such a closed-form mapping in any economic model. As a consequence, to achieve identification, we select a set of moments such that every estimated parameter has a differential impact on this set of moments. Heuristically, a moment h is informative about an unknown parameter β if that moment is sensitive to changes in the parameter and the sensitivity differs across parameters. Formally, local

identification requires the Jacobian determinant, $det(\partial h/\partial \beta)$, to be nonzero. To aid in the intuition of the identification of the model parameters, we compute elasticities of the model-implied moments with respect to the parameters, $(\partial h/\partial \beta)/(\beta/h)$. Inspection of these elasticities reveals that condition $det(\partial h/\partial \beta) \neq 0$ holds and that we can separately identify the parameters of the model.

We further proceed to describing the details of the identification and the choice of moments. In particular select 18 moments that relate to the distributions of cash, credit lines, leverage, operating income, investment, equity issance, and Tobin's q. Average profitability primarily identifies the curvature of the profit function α . Next, the variance and autocorrelation of profits directly identify the parameters σ_z and ρ_z . The fixed and variable capital adjustment costs directly affect the pace and size of investment changes and are identified by the variance and autocorrelation of investment. The fixed production cost f increases the need for liquidity management and is identified by average cash and undrawn debt capacity. Average credit line limit helps identify the debt capacity parameter θ . Higher θ implies higher debt capacity and thus a larger limit on the credit line. Average equity issuance and cash help identify equity flotation costs. Finally, all parameters affect Tobin's q.

4.4. Results

Table 1 presents the main results of the structural estimation. Panel A reports simulated and actual moments. Panel B reports structural parameter estimates and their corresponding standard errors.

The most important result in Panel A is that our dynamic liquidity model fits the data reasonably well. The model performs well in matching average cash, credit line limit, investment, operating income, and Tobin's q. The model predicts a higher average undrawn credit. While the model performs well at matching operating income variance, it overestimates variances of cash and undrawn credit. The simulated variances of leverage and investment are in the same order of magnitude as the actual ones. The simulated autocorrelations of most of the moments match their empirical counterparts reasonably well. Panel B shows that all model parameters are statistically significant. In addition, the estimates of the technological and investment parameters are in line with those reported in the previous literature. The debt capacity parameter θ is estimated at 56%. This implies that the average firm can use up to half of its assets as collateral to seek financing.

4.5. Empirical Implications

In this section, we examine some empirical implications of our model for corporate liquidity. Subsection 4.5.1 discusses the relationship between asset tangibility and liquidity. Subsection 4.5.2 considers the different roles of the state variables of the model, namely firms' net worth, capital, and profitability as determinants of liquidity. Subsection 4.5.3 presents stylized evidence related to the joint dynamic behavior of corporate liquidity, investment, and external financing.

4.5.1. Tangibility and Corporate Liquidity

In our framework, as in Albuquerque and Hopenhayn (2004), and in the Arrow-Debreu Limited Enforcement economies in Rampini and Viswanathan (2010) and Rampini and Viswanathan (2013), conditional liquidity is limited by a fraction of the physical capital that the firm can pledge as collateral. Thus, *ceteris paribus*, firms with more tangible assets to pledge will be able to fulfill their liquidity needs primarily conditionally. Table 2 provides suggestive stylized empirical evidence of the aforementioned economic tradeoff. The table reports panel regressions of total liquidity, defined as the ratio of total liquidity (cash plus undrawn credit) to total assets (Panel A), of the fraction of conditional-to-total liquidity (Panel B), and of the fraction of unconditional-to-total liquidity (Panel C), on two proxies of tangibility (Berger, Ofek, and Swary (1996), and fixed-to-total-asset ratio), and two proxies of intangibility (the fraction of organization capital measured with the perpetual inventory method applied to SG&A expenses, and the fraction of knowledge capital measured with the perpetual inventory method applied to R&D expenses) employed in the empirical literature. The results in Panels B and C emphasize that firms with more pledgeable assets fulfill their liquidity needs mostly conditionally. The results in Panel A suggest instead that firms with more intangible assets tend to transfer more liquidity in total, but mostly unconditionally using cash. This result is consistent with Falato, Kadyrzhanova, and Sim (2013), who find that high-growth firms with a large fraction of intangible assets hold disproportionately more cash.

[Insert Table 2 Here]

4.5.2. Determinants of Corporate Liquidity

The panel regressions in Table 3 report suggestive evidence on how the state variables of our model, namely net worth, capital, and profitability, affect observed corporate liquidity choices, in terms of both amount transferred and of mix between conditional and unconditional liquidity. The dependent variable in columns (1) and (2) is the total amount of liquidity firms transfer both conditionally using undrawn credit and unconditionally using cash (scaled by total assets). The dependent variable in columns (3) and (4) is the fraction of corporate liquidity that firms choose to preserve conditionally. The specifications in columns (1) and (3) are based on the sample we describe in Section 4.1. The specifications in columns (2) and (4) are averages across 100 simulated panels of 1000 firms for 20 years under the baseline model estimation in Table 1.

The regressions in columns (1) and (2) show that the total amount of corporate liquidity is positively related to net worth and negatively to capital.¹¹ In our model, as in Rampini and Viswanathan (2013), there is an intertemporal tradeoff between net worth allocated to current investment and future liquidity. As we discuss in Section 3.5, the marginal value of current net worth is lower for unconstrained firms. As a result, they allocate more resources to liquidity management. On the contrary, consistent with the predictions in Froot, Scharfstein, and Stein (1993), small firms have more growth options and larger total liquidity needs than large firms conditional on their level of net worth. Finally, more profitable firms need to transfer less liquidity since they internally generate resources for future needs.

The estimates in columns (3) and (4) reflect the economic tradeoff between conditional and unconditional liquidity in our model. Firms with more physical capital have more collateral to pledge, and prefer to use efficient conditional liquidity to fulfill their needs for

¹¹The magnitude of the coefficients in the model and in the data are not directly comparable since the numerical values of net worth and capital in the model depend on the choice of the numerical grid.

future resources. In practice, the slack they keep on their lines of credit is a large part of their total liquidity. Given their capital stock, unconstrained firms have a lower cost on transferring liquidity, and they do so hoarding cash. Profitability does not appear to significantly affect firms' liquidity mix.

Overall, the evidence in Table 3 highlights the importance of distinguishing between small (low capital) and constrained (low net worth) firms to interpret corporate liquidity choices. While net worth and capital are positively correlated and are occasionally both used as proxies of the severity of firms' financial constraints, they play a different role as determinants of corporate liquidity. Our model emphasizes their distinct function, in that net worth and capital are separate endogenous state variables.

[Insert Table 3 Here]

4.5.3. Co-Movements between Corporate Liquidity, Investment, and External Financing

Table 4 reports the average across firms of time-series correlations among variables that describe firms' investment, financing, and liquidity policies. The first column considers the sample we describe in Section 4.1, while the second column refers to a simulated sample of 1000 firms for 100 years under the estimated parameter values in Table 1. These correlations not only capture the dynamic nature of the model by quantifying co-movements among relevant variables, but also serve, along with the regressions in Table 3, as an out-of-sample test of the model.

We consider correlations among several variables that describe firms' sources and uses of funds, namely internally generated operating income, credit line draws, equity issuances, changes in cash balances, and investment expenses. Table 4 shows that the model is broadly consistent with co-movements observed in our sample. For example, equity issuances are positively correlated not only to investment expenses (0.110 in the data, 0.102 in the model) but also to changes in cash (0.070 in the data, 0.114 in the model). This suggests that firms use a part of the proceeds of equity issuances to replenish their cash reserves. In addition, firms appear to issue equity when internally generated profits are low and may not suffice to cover their liquidity needs. Indeed, the two variables are negatively correlated, with a coefficient of -0.265 in the data, and -0.223 in the model.

[Insert Table 4 Here]

5. Conclusion

In the presence of capital market imperfections expectations of future investment opportunities or cash shortfalls provide a rationale for dynamic liquidity management. We develop and estimate a quantitative model to examine the cross-sectional and time-series determinants of corporations' liquidity management. The result is a quantitative theory of optimal liquidity management based on the trade-off between conditional liquidity subject to collateral constraints and unconditional, unconstrained liquidity. To estimate our model structurally by means of the simulated method of moments (SMM), we develop a novel and efficient approach to solving high-dimensional dynamic programming problems based on linear programming.

Our model identifies unconditional liquidity management using cash and conditional liquidity management by means of drawing on credit lines as important instruments of corporate policy. In particular, our model predicts substantial cross-sectional variation in the relative usage of these instruments for liquidity purposes across firms, for which we find strong empirical support. Similarly, the model successfully rationalizes time-series patterns in corporations' liquidity management. Overall, the model thus provides a quantitatively and empirically successful framework explaining corporate investment, financing and liquidity policies and the joint occurrence of cash, debt and credit lines in the presence of capital market imperfections.

A large literature has recently attempted to rationalize the apparent secular trend in firms' cash holdings. It has been widely documented that in the US, firms' cash-to-asset ratios have increased dramatically since the 1970's. While in this paper we focus on stationary properties of firms' liquidity policies, we think it would be interesting to examine the possible determinants of this trend through the lens of our model. We leave this important question for future research.

6. Appendix

6.1. Definition of Credit Lines

Proposition 2 (Implementation with Credit Lines) State-contingent debt $b_i(s(t+1))$ can be implemented by the following combination of securities: state-uncontingent debt with face value $D_{i,t+1} \ge 0$, and a line of credit with undrawn credit $C_i^U(s(t+1)) \ge 0$, drawn part $C_i^D(s(t+1)) \ge$ 0, interest rate r, and limit $\overline{C_i^L(s(t+1))} \ge 0$. The firm uses the credit line to draw

$$\Delta CL^{D}(s(t+1)) = \max(0, (1+r(1-\tau))(E_{t}[b_{i}(s(t+1))] - b_{i}(s(t+1))))$$

or restore

$$\Delta CL^{R}(s(t+1)) = \max(0, (1+r(1-\tau))(b_{i}(s(t+1)) - E_{t}[b_{i}(s(t+1))]))$$

in each state $s(t+1) \in S$

such that

$$C_{i}^{D}(s(t+1)) = C_{i,t}^{D} + \Delta CL^{D}(s(t+1)) - \Delta CL^{R}(s(t+1))$$

from the credit line in each state $s(t+1) \in S$, and arranges a state-uncontingent loan $L_{i,t}$ at time t of size

where the face value $D_{i,t+1}$ is given by

$$D_{i,t+1} = (1 + r(1 - \tau)) \cdot L_{i,t}$$
(28)

The dynamics of undrawn credit, credit line limit, and drawn part in state $s(t+1) \in S$ are as follows:

$$C_{i}^{D}(s(t+1)) = C_{i,t}^{D} + \Delta CL(s(t+1))$$

$$\overline{C_{i}^{L}(s(t+1))} = C_{i,t}^{D} + \Delta CL(s(t+1)) + D_{i,t+1}$$

$$C_{i}^{U}(s(t+1)) = \overline{C_{i}^{L}(s(t+1))} - C_{i}^{D}(s(t+1))$$
(29)

Proof of Proposition 6.1. To prove the claim, we proceed in two steps. First, we show that the payoff $b_i(s(t+1))$ can be replicated with the combination of securities described above. Second, we verify that the recursive problem with the new securities is equivalent to the original one in terms of constraints. First, in the recursive problem, at time t+1 in each state s(t+1) the firm pays back $D_{i,t+1} - C_i^D(s(t+1))$. Therefore, using (28) and (29) we obtain

$$D_{i,t+1} - C_i^D(s(t+1)) = (1 + r(1-\tau))b_i(s(t+1))$$
(30)

from which the replication result follows:

$$b_i(s(t+1)) = \frac{D_{i,t+1} - C_i^D(s(t+1))}{1 + r(1-\tau)}$$
(31)

Because state-contingent debt can be directly expressed as the combination in (34) of state-uncontingent debt and the credit line, the replicating strategy is trivially budget feasible at time t + 1. The resource constraint at time t is also unchanged, because

$$w_{i,t} + L_{i,t} \ge e_{i,t} + k_{i,t+1} + c_{i,t+1} + \Psi(k_{i,t}, k_{i,t+1})$$

can be rewritten as

$$w_{i,t} + E_t[b_i(s(t+1))] \ge e_{i,t} + k_{i,t+1} + c_{i,t+1} + \Psi(k_{i,t}, k_{i,t+1})$$

using (27) and (34). Finally, we shall show that the limits for the feasible set for $b_i(s(t+1))$ implied by collateral and debt positivity constraints are preserved by the replicating portfolio of debt and lines of credit, namely that:

$$0 \le D_{i,t+1} - C_i^D(s(t)) \le \theta(1-\delta)k_{i,t+1}$$

The debt positivity constraints can be rewritten as

$$C_i^D(s(t+1)) \le (1 + r(1-\tau))E_t[b(s(t+1))]$$

implying that the firm draws at most an amount equal to $D_{i,t+1}$ and preserves its entire debt capacity $\theta(1-\delta)k_{i,t+1}$ as conditional liquidity in state s(t+1), and effectively repays $b_i(s(t+1)) = 0$. The collateral constraints can be rewritten as

$$-C_i^D(s(t+1)) \le \theta(1-\delta)k_{i,t+1} - D_{i,t+1}$$

implying that the firm sets the credit line balance $C_i^B(s(t+1))$ to $\theta(1-\delta)k_{i,t+1}$ in state s(t+1), and effectively repays $b_i(s(t+1)) = \theta(1-\delta)k_{i,t+1}$.

6.2. Proofs of Propositions

Proof of Lemma 1. From the definition of $h_i^C(s(t+1))$ we obtain:

$$b_i(s(t+1)) = \frac{\theta(1-\delta)k_{i,t+1} - h_i^C(s(t+1))}{1 + r(1-\tau)}$$
(32)

Substituting (32) and the definition of $h_{i,t+1}^U$ into the original problem yields the result.

Proof of Proposition 1. Denote the total number of states by S. The Lagrangian function for the constrained optimization problem is:

$$\mathcal{L}(e_{i,t}, k_{i,t+1}, h_{i,t+1}^{U}, \{h_{i}^{C}(s(t+1))\}, \{w_{i}(s(t+1))\}, \lambda^{w}, \{\frac{\pi(s(t), s(t+1))\lambda_{s(t+1)}^{w}}{1+r}\}, \{\frac{\pi(s(t), s(t+1))\lambda_{s(t+1)}^{C}}{1+r}\}, \{\frac{\pi(s(t), s(t+1))\lambda_{s(t+1)}^{C}}{1+r}\}, \{\frac{\pi(s(t), s(t+1))\lambda_{s(t+1)}^{C}}{1+r}\}, \lambda^{U}) \\ e_{i,t} - \Lambda(e_{i,t}) + \frac{1}{1+r}E_{t}[V(w_{i,t+1}, z_{i,t+1})] + \lambda^{w}(w_{i,t} - e_{i,t} - E_{t}\left[\frac{h_{i}^{C}(s(t+1))}{1+r(1-\tau)}\right] - \frac{h_{i,t+1}^{U}}{1+r(1-\tau)-\gamma} - Pk_{i,t+1} - \Psi(k_{i,t}, k_{i,t+1})) + \\ + \sum_{s=1}^{S} \frac{\pi(s(t), s)\lambda_{s}^{w}}{1+r}((1-\tau)\Pi(k_{i,t+1}, z_{i,s}) + (1-\theta)(1-\delta)k_{i,t+1} + \tau\delta k_{i,t+1} + h_{i}^{T}(s) - w_{i}(s)) + \\ + \sum_{s=1}^{S} \frac{\pi(s(t), s)\lambda_{s}^{C}}{1+r}(h_{i}^{C}(s)) + \sum_{s=1}^{S} \frac{\pi(s(t), s)\overline{\lambda}_{s}^{C}}{1+r}(\theta(1-\delta)k_{i,t+1} - h_{i}^{C}(s)) + \underline{\lambda}^{U}(h_{i,t+1}^{U}) \\ \text{Differentiating the Lagrangian with rispect to } e_{i,t}, k_{i,t+1}, h_{i,t+1}^{U}, \{h_{i}^{C}(s(t+1))\}, \text{ and } \{w_{i}(s(t+1))\} \text{ yields}$$

equations (16a), (16b), (16c), (16d), (18b) after some algebraic manipulation. Because the Slater condition holds, the envelope theorem can expressed as:

$$\begin{split} \frac{\partial V(w_{i,t},z_{i,t})}{\partial w_{i,t}} &= \frac{\partial e_{i,t} - \Lambda(e_{i,t})}{\partial w_{i,t}} + \lambda^w \frac{\partial (w_{i,t} - e_{i,t} - E_t \left[\frac{h_i^C(s(t+1))}{1 + r(1 - \tau)}\right] - \frac{h_{i,t+1}^O}{1 + r(1 - \tau) - \gamma} - Pk_{i,t+1} - \Psi(k_{i,t},k_{i,t+1}))}{\partial w_{i,t}} + \\ &+ \sum_{s=1}^S \frac{\pi(s(t),s)\lambda_s^w}{1 + r} \frac{\partial ((1 - \tau)\Pi(k_{i,t+1},z_{i,s}) + (1 - \theta)(1 - \delta)k_{i,t+1} + \tau\delta k_{i,t+1} + h_i^T(s) - w_i(s))}{\partial w_{i,t}} + \sum_{s=1}^S \frac{\pi(s(t),s)\overline{\lambda}_s^C}{1 + r} \frac{\partial (\theta(1 - \delta)k_{i,t+1} - h_i^C(s))}{\partial w_{i,t}} + \underline{\lambda}^U \frac{\partial (h_{i,t+1}^U)}{\partial w_{i,t}} \end{split}$$

which immediately yields (18a). The Euler equation (19) can be simply obtained, by dividing both sides of (16b) by λ^w . The division is well-defined because the resource constraint at time t is always binding. Finally, equation (20) can be derived by substituting $\lambda^w_{s(t+1)}$ from (16d) into the definition of $M^w(s(t), s(t+1))$.

Proof of Proposition 6.1. To prove the claim, we proceed in two steps. First, we show that the payoff $b_i(s(t+1))$ can be replicated with the combination of securities described above. Second, we verify

that the recursive problem with the new securities is equivalent to the original one in terms of constraints. First, in the recursive problem, at time t + 1 in each state s(t + 1) the firm pays back $D_{i,t+1} - C_i^D(s(t + 1))$. Therefore, using (28) and (29) we obtain

$$D_{i,t+1} - C_i^D(s(t+1)) = (1 + r(1-\tau))b_i(s(t+1))$$
(33)

from which the replication result follows:

$$b_i(s(t+1)) = \frac{D_{i,t+1} - C_i^D(s(t+1))}{1 + r(1-\tau)}$$
(34)

Because state-contingent debt can be directly expressed as the combination in (34) of state-uncontingent debt and the credit line, the replicating strategy is trivially budget feasible at time t + 1. The resource constraint at time t is also unchanged, because

$$w_{i,t} + L_{i,t} \ge e_{i,t} + k_{i,t+1} + c_{i,t+1} + \Psi(k_{i,t}, k_{i,t+1})$$

can be rewritten as

$$w_{i,t} + E_t[b_i(s(t+1))] \ge e_{i,t} + k_{i,t+1} + c_{i,t+1} + \Psi(k_{i,t}, k_{i,t+1})$$

using (27) and (34). Finally, we shall show that the limits for the feasible set for $b_i(s(t+1))$ implied by collateral and debt positivity constraints are preserved by the replicating portfolio of debt and lines of credit, namely that:

$$0 \le D_{i,t+1} - C_i^D(s(t)) \le \theta(1-\delta)k_{i,t+1}$$

The debt positivity constraints can be rewritten as

$$C_i^D(s(t+1)) \le (1+r(1-\tau))E_t[b(s(t+1))]$$

implying that the firm draws at most an amount equal to $D_{i,t+1}$ and preserves its entire debt capacity $\theta(1-\delta)k_{i,t+1}$ as conditional liquidity in state s(t+1), and effectively repays $b_i(s(t+1)) = 0$. The collateral constraints can be rewritten as

$$-C_i^D(s(t+1)) \le \theta(1-\delta)k_{i,t+1} - D_{i,t+1}$$

implying that the firm sets the credit line balance $C_i^B(s(t+1))$ to $\theta(1-\delta)k_{i,t+1}$ in state s(t+1), and effectively repays $b_i(s(t+1)) = \theta(1-\delta)k_{i,t+1}$.

6.3. Solution by Mixed-Integer Programming

Any finite dynamic programming problem with infinite horizon can be equivalently formulated as a linear programming problem (LP), where for each grid point on the state space, every feasible decision corresponds to a constraint in the LP. Specifically, our model can be formulated as an LP as follows:

$$\min_{\substack{v_{k,w,z}\\v_{k,w,z}}} \sum_{k=1}^{nk} \sum_{w=1}^{nw} \sum_{z=1}^{nz} v_{k,w,z}$$
s.t.
(35)

$$v_{k,w,z} \geq d_{k,w,z,a} + \sum_{z'=1}^{nz} \pi(z'|z) \frac{1}{1+r} v_{k'(a),w'(a),z'} \qquad \forall k, w, z, a$$
(36)

where nk, nw, and nz are the number of grid points on the grids for $k_{i,t}$, $w_{i,t}$, and $z_{i,t}$ respectively, $v_{k,w,z}$ is the value function on the grid point indexed by k, w and z, a is an index for an action on the grid for both future capital, cash, and state-contingent debt repayments, and $d_{k,w,z,a}$ denotes the payout corresponding to the action a starting from the state indexed by k, w and z. k'(a) and w'(a) denote the future values for the state variables given the current firm's decisions. For a formal proof, we refer to Ross (1983).

As Trick and Zin (1993) discuss, solving the LP above would require to store a huge matrix, because the number of constraints in the problem is very large. Computational requirements would therefore be enormous. Thus, we implement constraint generation, a standard method in operation research to solve problems with a large number of constraints. First we solve a relaxed problem with the same objective. Second, we identify the remaining constraints in the problem that are violated by the current solution. Third, we add a subset of the violated constraints to the relaxed problem according to a selection rule. We iterate the procedure is iterated until all constraints are satisfied. The following constraint generation algorithm converges to the unique fixed point of our Bellman problem.

1. solve the problem in 35 with an initial random subset of constraints for each state (k, w, z); 2. if all constraints $a \in \Gamma^n(k, w, z)$, for all (k, w, z), are satisfied, terminate the algorithm (where $\Gamma^n(k, w, z)$ is the set of feasible actions at iteration n); 3. for each state (k, w, z), add the constraint $a \in \Gamma^n(k, w, z)$ that generates the highest violation in (36) with respect to the current solution $v^n(k, w, z)$;

4. solve the problem with the current set of constraints;

5. go back to step 2.

To practically implement the above procedure, another issue must be addressed. The selection of the most violated constraint in the third step searching over an extremely large vector of grid points for all the decision variables. The computational burden would still be excessive for a model with many controls variables. We therefore use a separation oracle in the third step. A separation oracle is an auxiliary mixed-integer programming problem that identifies the most violated constraint.¹² We specify the separation oracle for this problem below:

 $^{^{12}}$ Separation oracles are standard tools in operation research. See for example Vielma and Nemhauser (2011), Schrijver (1998), and Cook, Cunningham, Pulleyblank, and Schrijver (2011)

Separation Oracle

$$\max_{a=\{k',c',b(z')\}} d_{k,w,z,a} - \Lambda(d_{k,w,z,a}) + \sum_{z'=1}^{nz} \pi(z'|z) \frac{1}{1+r} v_{k'(a),w'(a),z'} - v_{k,w,z}$$
(37)

s.t.

$$0 \le b(z') \le \frac{\theta k'(1-\delta)}{1+r} \quad \forall z'$$
(38)

$$0 \le c' \le C \qquad \qquad \forall z' \tag{39}$$

$$0 \le p(i_k) \le 1 \qquad \qquad \forall i_k = 1, \dots, n_k \tag{40}$$

$$\sum_{i_k=1}^{n} p(i_k) = 1$$
(41)

$$k' = \sum_{i_k=1}^{n_k} p(i_k) k^G(i_k)$$
(42)

$$d_{k,w,z,a} = w - k' - \Psi(k,k') - c' + \sum_{z'=1}^{nz} \pi(z'|z) \frac{1}{1+r} b(z')$$
(43)

$$d_{w,s,a} \ge 0 \tag{44}$$

$$f(k') = \sum_{i_k=1}^{n_k} p(i_k) (k^G(i_k))^{\alpha}$$
(45)

$$w(z') = (1 - \tau)(z'f(k') - f) + k'(1 - \delta) - (1 + r(1 - \tau))b(z')$$
(46)

$$+(1+r(1-\tau)-\gamma)c'+\tau\delta k' \qquad \forall z'=1...nz$$
(47)

Equations (39) and (38) define the bounds for cash and debt, Equations (40) and (41) define the variables $p(i_k)$ that have the role to select a grid point for capital on the grid $k^G(i_k)$ and linearize the term k'^{α} in the production function, Equation (42) picks the grid point for the chosen capital stock from $k^G(i_k)$, Equation (43) defines dividends, Equation (45) computes the nonlinear term in capital in the production function, and Equation (47) defines future net worth in each state z'. The computation of the law of motion for future net worth is obtained by interpolation with the logarithmic formulation of Vielma and Nemhauser (2011). Capital adjustment costs $\Psi(k, k')$ and equity flotation costs $\Lambda(d_{k,w,z,a})$ are instead incorporated using a Big-M formulation. The solutions of the separation oracle for cash and state-contingent debt are continuous variables and are interpolated to the nearest point on the corresponding grid. We implement the codes with Matlab[®], and the solver for the mixed-integer programming problems is CPLEX[®]. The two applications are interfaced through the CPLEX Class API[®]. Our workstation has a CPU with 8 cores and 64GB of RAM. The model is solved with seven grid points for the idiosyncratic shock, 21 grid points for capital, 17 grid points for current net worth, and 500 points for cash and each state-contingent debt variable. Following McGrattan (1997), the grids for net worth and capital are not evenly spaced, but more points are collocated in the low net worth region, where the curvature of value function is more relevant.

6.4. Estimation Procedure

We provide a brief discussion of the estimation procedure¹³. Let x_i be an *i.i.d.* data vector, i = 1, ..., n, and let $y_{is}(\beta)$ be an *i.i.d.* simulated vector from simulation s, i = 1, ..., n, and s = 1, ..., S. Here, n is the length of the simulated sample, and S is the number of times the model is simulated. We pick n = 20,000 and S = 10, following Michealides and Ng (2000), who find that good finite-sample performance of a simulation estimator requires a simulated sample that is approximately ten times as large as the actual data sample.

The simulated data vector, $y_{is}(\beta)$, depends on a vector of structural parameters, β . In our application $\beta \equiv (\alpha, f, \rho_z, \sigma_z, \psi_0, \psi, \theta, \lambda)$. The goal is to estimate β by matching a set of *simulated moments*, denoted as $h(y_{ik}(\beta))$, with the corresponding set of actual *data moments*, denoted as $h(x_i)$. The candidates for the moments to be matched include for example simple summary statistics or OLS regression coefficients. Define

$$g_n(\beta) = n^{-1} \sum_{i=1}^n \left[h(x_i) - S^{-1} \sum_{s=1}^S h(y_{is}(b)) \right].$$

The simulated moments estimator of β is then defined as the solution to the minimization of

$$\hat{\beta} = \arg\min_{\beta} g_n \left(\beta\right)' \hat{W}_n g_n \left(\beta\right),\,$$

¹³The exposition closely follows Nikolov and Whited (2014).

in which \hat{W}_n is a positive definite matrix that converges in probability to a deterministic positive definite matrix W. In our application, we use the inverse of the sample covariance matrix of the moments, which we calculate using the influence function approach in Erickson and Whited (2000).

The simulated moments estimator is asymptotically normal for fixed S. The asymptotic distribution of β is given by

$$\sqrt{n}\left(\hat{\beta}-\beta\right) \stackrel{d}{\longrightarrow} \mathcal{N}\left(0,\operatorname{avar}(\hat{\beta})\right)$$

in which

$$\operatorname{avar}(\hat{\beta}) \equiv \left(1 + \frac{1}{S}\right) \left[\frac{\partial g_n\left(\beta\right)}{\partial \beta} W \frac{\partial g_n\left(\beta\right)}{\partial \beta'}\right]^{-1} \left[\frac{\partial g_n\left(\beta\right)}{\partial \beta} W \Omega W \frac{\partial g_n\left(\beta\right)}{\partial \beta'}\right] \left[\frac{\partial g_n\left(\beta\right)}{\partial \beta} W \frac{\partial g_n\left(\beta\right)}{\partial \beta'}\right]^{-1}$$
(48)

in which W is the probability limit of \hat{W}_n as $n \to \infty$, and in which Ω is the probability limit of a consistent estimator of the covariance matrix of $h(x_i)$.

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Table 1. SIMULATED MOMENTS ESTIMATION

Calculations are based on a sample of nonfinancial, unregulated firms from the annual COMPU-STAT and Capital IQ datasets. The sample period is from 2002 to 2011. The estimation is done with SMM, which chooses structural model parameters by matching the moments from a simulated panel of firms to the corresponding moments from the data. Panel A reports the simulated and the actual moments. Panel B reports the estimated structural parameters. α is the curvature of the profit function. f is the fixed production cost. ρ_z is the serial correlation of ln(z). σ_z is the standard deviation of the innovation of ln(z). ψ_0 is the fixed capital adjustment cost. ψ is the variable adjustment cost. θ is the debt capacity. λ is the equity flotation cost. Standard errors are in parenthesis under the parameter estimates.

Panel A: Moments		
	Actual moments	Simulated moments
Average cash	0.1444	0.1115
Variance of cash	0.0048	0.0178
Autocorrelation of cash	0.6992	0.7525
Average credit line limit	0.1785	0.1651
Average undrawn credit	0.7959	0.8836
Variance of undrawn credit	0.0366	0.0520
Autocorrelation of undrawn credit	0.2710	0.2608
Average leverage	0.3548	0.2705
Variance of leverage	0.0107	0.0191
Autocorrelation of leverage	0.5453	0.7159
Average operating income	0.1161	0.1252
Variance of operating income	0.0039	0.0047
Autocorrelation of operating income	0.6662	0.8289
Average investment	0.1124	0.1266
Variance of investment	0.0031	0.0137
Autocorrelation of investment	0.4309	0.3813
Average Tobin's q	1.8106	1.7196
Average equity issuance	0.0342	0.0137

Panel B. Paramete	r estimates
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α	f	$ ho_z$	σ_z	ψ_0	ψ	θ	λ
0.6011	0.1192	0.8698	0.1365	0.0219	0.4944	0.5552	0.0937
(0.0582)	(0.0062)	(0.0684)	(0.0157)	(0.0052)	(0.1995)	(0.0738)	(0.0179)

Table 2. Tangibi	LITY AND) Corpo	RATE LI	(QUIDIT)	2							
The table reports erent proxies of a 2000-2011. Under t as the undrawn an iquidity L^T is give:	estimates sset tangil he implem nount from a by	from lin bility and nentation n firms' l	d intangil with crec lines of c	l regress bility. Γ lit lines ε redit, an	ions of to bata are and cash of d uncond $L^T = L$	otal, col obtained described litional 1 $C + L^U$	nditional l by men d in the : liquidity	, and un rging Cor manuscrif L^U is me	conditiona npustat aı ət, conditic assured as	l corpor nd Capit onal liqui firms' c	ate liquid tallQ for idity L^C i cash holdi	ity on dif- the period s measured ngs. Total
Total liquidity measured as in B is measured as in B cotal assets; followi the perpetual inver- organization capita reganization capita 36,262 firm-year ob standard errors are	sures are science. Of e. derger, Of e. ng Falato, ntory meth 1 measured 3 6000-699 servations clustered	caled by f k, and Sv k Kadyrzh hod appli d with th 99), and f s. All var at the fir	firms' tota wary (1990 hanova, al ied to sel ne perpetu firms fron iables are m level. 7	ul assets. 6) and A nd Sim (ling, gen 1al inven 1 other r winsoriz Γ-statisti	The prox lmeida ar 2013), 'Ir eral, and tory meth egulated ced at the cs are in	ies for te nd Camp ntangibil adminis hod appl industri parenthe parenthe	ungibility bello (200 ity 1' is strative ϵ lied to R es (SIC ξ sut level. sses.	and intar 7); 'Tang the fracti expenses, &D expen greater th All speci	ibility 2' is ibility 2' is on of knov and 'Intal nses. We (an 9000). ifications i	e the foll s the rati vledge cs ngibility exclude f The fin nclude fi	lowing: 'T to of fixed apital mea 2' is the financials al sample rm fixed	angibility1' assets over usured with fraction of (SIC 4900- consists of effects, and
	r (1)	A) Total (2)	$\frac{\text{Liquidity}}{(3)}$	(4)	$\frac{B) \text{ Cond}}{(5)}$	litional-t (6)	$\frac{\text{o-Total I}}{(7)}$	Liquidity (8)	C) Uncor (9)	iditional- (10)	-to-Total (11)	Liquidity (12)
Tangibility 1	-0.31 (-15 96)				0.34				-0.34			
Tangibility 2		-0.52				0.38				-0.38		
Intangibility 1			0.73 (30.30)				-0.17				0.17	
Intangibility 2				$0.50 \\ (21.55)$				-0.20 (-8.58)				0.20 (8.58)
Adj. R^2	17.76	16.04	38.52	11.48	18.27	9.91	23.29	6.19	18.27	9.91	23.29	6.19

Table 3. CAPITAL, NET WORTH, PROFITABILITY AND CORPORATE LIQUIDITY

The table reports estimates from linear panel regressions of total liquidity, and the proportion of conditional to total liquidity on the determinants of liquidity identified by the model as its state variables. The specifications denoted as Data use real data, while those denoted as Model are averages of 100 simulated panels of 1000 firms for 20 years. Data are obtained by merging Compustat and CapitalIQ for the period 2000-2011. Under the implementation with credit lines and cash described in the manuscript, conditional liquidity L^C is measured as the undrawn amount from firms' lines of credit, and unconditional liquidity L^U is measured as firms' cash holdings. Total liquidity L^T is given by

$$L^T = L^C + L^U$$

Total liquidity measures are scaled by firms' total assets. The determinants are firms' (log) net worth, (log) capital, and profitability. Following Rampini, Sufi, and Viswanathan (2014), net worth is defined as the book value of equity, measured as in Fama and French (1993). Capital is the book value of property, plant and equipment. Profitability is the ratio of firms' operating profits before depreciation and firms' capital.We exclude financials (SIC 4900-4099), utilities (SIC 6000-6999), and firms from other regulated industries (SIC greater than 9000). The final sample consists of 36,262 firm-year observations. All variables are winsorized at the 1 percent level. The specifications in columns (1) and (3) include firm fixed effects. All standard errors are clustered at the firm level. T-statistics are in parentheses.

	Total I	liquidity	Condition	al Liquidity
	(1) Data	(2) Model	(3) Data	(4) Model
Net Worth	0.046	0.502	-0.010	-0.640
	(17.063)	(45.907)	(-2.806)	(-22.528)
Capital	-0.071	-0.543	0.054	0.734
	(-21.235)	(-93.862)	(13.312)	(21.129)
Profitability	-0.002	-0.555	0.001	0.232
	(-2.941)	(-15.327)	(1.240)	(1.361)
Adj. R^2	27.16	95.50	15.36	59.40

Table 4. CORRELATIONS: LIQUIDITY, INVESTMENT, AND EXTERNAL FINANCING.

The table reports pairwise time-series correlations among equity issues, changes in cash, changes in drawn credit, investment and operating income. The column denoted as 'Data' use real data, while the one denoted as 'Model' are based on a simulated panel of 1000 firms for 100 years. Data are obtained by merging Compustat and CapitalIQ for the period 2000-2011. All correlations are equally-weighted averages of individual firms' correlations. All variables are scaled by total assets with the exception of the change in drawn credit, which is scaled by the credit line limit. We exclude financials (SIC 4900-4099), utilities (SIC 6000-6999), and firms from other regulated industries (SIC greater than 9000). The final sample consists of 36,262 firm-year observations. All variables are winsorized at the 1 percent level.

Correlation	Data	Model
Equity Issuance and Change in Cash	0.070	0.144
Equity Issuance and Change in Drawn Credit	0.008	-0.005
Equity Issuance and Investment	0.110	0.102
Equity Issuance and Operating Income	-0.265	-0.223
Drawn Credit and Change in Cash	-0.011	0.059
Drawn Credit and Investment	0.035	-0.041
Drawn Credit and Operating Income	0.017	0.032
Operating Income and Change in Cash	-0.075	0.059
Operating Income and Change in Drawn Credit	0.017	0.032
Operating Income and Investment	0.107	0.494

Figure 1. The Fundamental Economic Tradeoff: Illustration

The figure depicts the timing of events for the illustration of the tradeoff between conditional and unconditional liquidity in Section 2. Panel A refers to the case where the firm can only engage in unconditional liquidity management using cash. Panel B refers to the case where the firm can also implement conditional liquidity management using lines of credit.





Figure 2. FIRM'S POLICY WITH NO PERSISTENCE AND NO ADJUSTMENT COSTS

The figure illustrates the investment, financing, and risk management policy of the firm as a function of current net worth $w_{i,t}$. For illustrative purposes, the model is solved with a number of states equal to three, with uniform transition probabilities, and with all adjustment costs parameters set to zero. Dividends are constrained to be positive, that is no equity issues are possible. The values for the exogenous productivity shock $z_{i,t}$ are set to -0.3000 for the low state, to 0.5000 for the middle state, and to 1.7000 for the high state. Panels A through F show: the future capital stock $k_{i,t+1}$, the net equity payout $e_{i,t}$ the equity value $V_{i,t}$, the observed debt stock $E[b_i(s(t+1))]$, unconditional hedging (cash) $h_{i,t+1}^U$, and conditional hedging $h_i^C(s(t+1))$. In panel F, the solid blue line represents total debt capacity $\theta \delta k_{i,t+1}$ the solid thick line conditional hedging for the middle state, the dashed thick line conditional hedging for the low state, the dashed-dotted thick line conditional hedging for the high state. The remaining parameter values are as follows: $\alpha = 0.5000$, f = 0.0000, $\tau = 0.3500$, $\theta = 0.5000$, $\gamma = 0.0010$, r = 0.0100.



Figure 3. FIRM'S POLICY WITH PERSISTENCE AND NO ADJUSTMENT COSTS

The figure illustrates the investment, financing, and risk management policy of the firm as a function of current net worth $w_{i,t}$. For illustrative purposes, the model is solved with a number of states equal to three, and with all adjustment costs parameters set to zero. From each state, the transition matrix attaches probability 0.5 to remain in the same state, and 0.25 to move to each of the remaining two states. Dividends are constrained to be positive, that is no equity issues are possible. The values for the exogenous productivity shock $z_{i,t}$ are set to 0.2000 for the low state, to 0.5000 for the middle state, and to 0.8000 for the high state. Panels A through F show: the future capital stock $k_{i,t+1}$, the net equity payout $e_{i,t}$ the equity value $V_{i,t}$, the observed debt stock $E[b_i(s(t+1))]$, unconditional hedging (cash) $h_{i,t+1}^U$, and conditional hedging $h_i^C(s(t+1))$. In panel F, the solid blue line represents total debt capacity $\theta \delta k_{i,t+1}$ the solid thick line conditional hedging for the middle state, the dashed thick line conditional hedging for the low state, the dashed-dotted thick line conditional hedging for the high state. The remaining parameter values are as follows: $\alpha = 0.5000$, f = 0.0000, $\tau = 0.3500$, $\theta = 0.6000$, $\gamma = 0.0010$, r = 0.0100.



Figure 4. FIRM'S POLICY WITH CONVEX INVESTMENT ADJUSTMENT COSTS

The figure illustrates the investment, and risk management policy of the firm as a function of current net worth $w_{i,t}$ (Panels A-C) and current capital stock $k_{i,t}$ (Panels D-F). For illustrative purposes, the model is solved with a number of states equal to three. The convex investment adjustment cost parameter ψ^+ is set to 1.0000. All the other capital adjustment cost parameters are set to zero. From each state, the transition matrix attaches probability 0.5 to remain in the same state, and 0.25 to move to each of the remaining two states. Dividends are constrained to be positive, that is no equity issues are possible. The values for the exogenous productivity shock $z_{i,t}$ are set to 0.3000 for the low state, to 0.7000 for the middle state, and to 1.1000 for the high state. Panels A through C show: the future capital stock $k_{i,t+1}$, unconditional hedging (cash) $h_{i,t+1}^U$, and conditional hedging $h_i^C(s(t+1))$ as a function of current net worth. Panels D through F show: the investment-to-capital ratio $i_{i,t}/k_{i,t}$, unconditional hedging (cash) $h_{i,t+1}^U$, and conditional hedging $h_i^C(s(t+1))$ as a function of the current capital stock. In panels C and F, the solid blue line represents total debt capacity $\theta \delta k_{i,t+1}$ the solid thick line conditional hedging for the middle state, the dashed thick line conditional hedging for the low state, the dashed-dotted thick line conditional hedging for the high state. The remaining parameter values are as follows: $\alpha = 0.5000, f = 0.0000, \tau = 0.3500, \theta = 0.6000, \gamma = 0.0010, r = 0.0100.$



Figure 5. FIRM'S POLICY WITH FIXED INVESTMENT ADJUSTMENT COSTS

The figure illustrates the investment, and risk management policy of the firm as a function of current net worth $w_{i,t}$ (Panels A-C) and current capital stock $k_{i,t}$ (Panels D-F). For illustrative purposes, the model is solved with a number of states equal to three. The convex investment adjustment cost parameter ψ_0^+ is set to 0.0750. All the other capital adjustment cost parameters are set to zero. From each state, the transition matrix attaches probability 0.5 to remain in the same state, and 0.25 to move to each of the remaining two states. Dividends are constrained to be positive, that is no equity issues are possible. The values for the exogenous productivity shock $z_{i,t}$ are set to 0.3000 for the low state, to 0.7000 for the middle state, and to 0.9000 for the high state. Panels A through C show: the future capital stock $k_{i,t+1}$, unconditional hedging (cash) $h_{i,t+1}^U$, and conditional hedging $h_i^C(s(t+1))$ as a function of current net worth. Panels D through F show: the future capital stock $k_{i,t+1}$, unconditional hedging (cash) $h_{i,t+1}^U$, and conditional hedging $h_i^C(s(t+1))$ as a function of the current capital stock. In panel D, k^* denotes the "frictionless" level of capital with $\psi_0^+ = 0$, while k^D and k^U are the bounds of the inaction region. In panels C and F, the solid blue line represents total debt capacity $\theta \delta k_{i,t+1}$ the solid thick line conditional hedging for the middle state, the dashed thick line conditional hedging for the low state, the dashed-dotted thick line conditional hedging for the high state. The remaining parameter values are as follows: $\alpha = 0.3500, f = 0.0000, \tau = 0.3500, \theta = 0.4000, \gamma = 0.0010,$ r = 0.0100.



Figure 6. Comparative Statics - Technology.

Figure 6 depicts the relation between the curvature of the profit function, α , the fixed production cost, f, the serial correlation of ln(z), ρ_z , and the standard deviation of the innovation of ln(z), σ_z , and i) the cash to asset ratio, ii) the fraction of undrawn credit over the credit line limit, and iii) the leverage ratio.



Figure 7. Comparative Statics - Investment and Financing.

Figure 7 depicts the relation between the fixed capital adjustment cost, ψ_0 , the variable adjustment cost, ψ , debt capacity, θ , and the equity flotation cost, λ , and i) the cash to asset ratio, ii) the fraction of undrawn credit over the credit line limit, and iii) the leverage ratio.

