CERF Studentship Report – Project Update July 2016

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Working Papers

'Nonparametric State-Price Density Estimation using High Frequency Data'

Link: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2718938

Abstract:
" This paper studies the use of irregularly spaced high frequency data to estimate the state-price density (SPD) implicit in option prices. Their large sample size allows estimation of the conditional SPD at any time point of interest, which can be directly used for model-free pricing, hedging, and conditional risk measurement. We develop asymptotic theory for a time-varying kernel estimator when the trading times and strike prices are modelled by marked point processes whose intensity goes to infinity. The pricing errors, strike prices, and trading times are weakly dependent on each other and can be nonstationary to capture intraday trading patterns. Unlike realized volatility estimation, the market microstructural noise in recorded option prices is averaged out and there is no need to subsample the data. We apply the estimator to S&P 500 E-mini European call and put option mid quotes using an iterated plug-in bandwidth, and document the intraday dynamics of the SPD and derived quantities."

Awards:

Cambridge Finance Best Student Paper Award 2015
GResearch PhD Prize 2016 for the best doctoral research in quantitative finance

Non-technical Summary:

Option prices contain detailed information about the risk perception of market participants. My research concerns developing new methods to analyse this information as summarized in the state-price density, which tells us how much value investors attach to payoffs in different states of the economy. My expected contribution is to use high-frequency data to model the dynamic behaviour of the state-price density. This tells us how the perception of market risk of investors changes over time, that is, in reaction to market events.

The availability of high-frequency data has dramatically increased the number of observed prices of European options and their underlying. This makes it feasible to estimate the state-price density at different time point within a day, as opposed to say only at the close of the trading day. This can be done by 'smoothing' transaction data over time using a time-varying nonparametric regression function.

The clear advantages of the large sample sizes in high-frequency data also come with some methodological challenges. In particular, trading times do not occur at a regular frequency, such as at the end of a trading day or month, but instead should be treated as random variables themselves. For example, transactions may cluster together in time in reaction to some particular news, or there can be time-of-the-day effects such as a lunch breaks. These contribute to the random behaviour of the nonparametric estimator and hence cannot be ignored. Another motivation to model trading times explicitly is that they may be related to the outcomes of the traded assets themselves, for example via a large stock price change which triggers new transactions.

I have focused on the theoretical part of this project, i.e. the econometric theory of random sampling times and a dynamic nonparametric estimator. I mainly worked on deriving the mean square error of the estimator and the practical issue of choosing bandwidths. This comes effectively down to choosing how many data points to include for the estimated state-price density at a specific point in time. The faster the state-price density changes, the less data points we can use without introducing large biases.

The dynamic model for the state-price density can also be used to analyse the costs of static models, such as the common practice to 'pool' together data points during a specific trading period. Also other specifications of option pricing models can be tested, such as the commonly used 'homogeneity' assumption of the stock and strike price.

To summarize, the main scientific aims of my first working paper are to

- Develop econometric methods to model time-variation in the state-price density, i.e. incorporate random observation times within a nonparametric time series regression
- Apply the model to high-frequency S&P500 options data, report stylized facts on the dynamics of the state-price density and test existing option price models

Presentations:
April 2014: Econometrics Workshop, Faculty of Economics, University of Cambridge
November 2014: Econometrics Workshop, Faculty of Economics, University of Cambridge
May 2015: CERF Cavalcade, Judge Business School, University of Cambridge
June 2015: SoFiE Spring School, Belgian Central Bank, Brussels
June 2016: SoFiE Conference 'Financial Econometrics and Empirical Asset Pricing', Lancaster UK
August 2016 (scheduled): European Finance Association Annual Meeting 2016 (main programme) in Oslo

'Nonparametric Pricing Kernel Estimation and Density Forecasting'

Non-technical Summary:

The pricing kernel or stochastic discount factor approach to asset pricing states that prices equal their expected payoff after stochastically discounting over states (Cochrane, 2009}. The pricing kernel is the key object in two main strands of asset pricing. In equilibrium models, the form of the pricing kernel follows from the Euler equation for consumption and investment. By weighing state-dependent payoffs with their marginal rate of substitution, the pricing kernel links the riskiness of assets to their expected return. In no-arbitrage models, the existence of a pricing kernel is implied by the absence of opportunities to gain sure profits, while uniqueness only follows under perfect replicability and hence market completeness. The difference between the two approaches lies primarily in the specification of the variables that define the relevant states of the world. Whereas equilibrium models are usually specified in terms of consumption and other macro-variables, no arbitrage models are typically specified in terms of the risk factors that drive the assets. Nevertheless, the two can be linked together by taking conditional expectations with respect to the asset returns. This defines the `projected' pricing kernel as a possibly nonlinear function of the asset return. While the linear projection of the pricing kernel on the space of asset returns is well studied, the nonlinear projected pricing kernel provides additional information about the shape of risk aversion

The projected pricing kernel is nonparametrically identified from the payoff structure of option prices, given the conditional density of the asset returns. In particular, the cross section of option prices at varying strike prices identifies the risk neutral density of its underlying asset for a given maturity (Breeden and Litzenberger, 1978). The risk neutral density is the product of the objective density and the pricing kernel. For the purpose of pricing options and other derivatives it suffices to model the risk neutral distribution. However, its decomposition becomes important when the objective is either extracting predictive densities from option prices, or inferring the pricing kernel to understand how risk aversion affects expected returns. The former is of direct relevance for risk management, as a density estimator generalizes popular measures of risk such as the volatility, and partial moments in the tail. The additional predictive ability of option-implied forecasts over historical data is well documented for the case of implied volatility versus past volatility, despite the bias that is due to the variance risk premium (Bollerslev, 2009). Density forecasts have so far mainly been obtained by parametrically `risk adjusting' the risk neutral density. The empirical pricing kernels are typically computed as the ratio of the current option-implied density and a nonparametric historical density estimator. The critical assumption here is that the historical density estimate consistently estimates that of the market. Any inconsistency due to omitted state variable will be erroneously ascribed to the empirical pricing kernel.

Here I propose an estimation framework which treats both the pricing kernel and the objective density as unknown functions of the asset return and some exogenous state variables. The estimator will have some robustness towards the state variables by focusing on the projected pricing kernel as a function of the chosen state variables. Option prices and stock returns over the whole sampling period are used, instead of only current option prices and historical stock returns. For this, I approximate the pricing kernel by a series expansion, and combine the conditional moment restrictions for option prices and stock returns into a sieve minimum-distance problem (Ai and Chen, 2003).

Market-based predictive densities can then be extracted via the implied densities of a variant of the conditional empirical likelihood problem studied in Gagliardini et al. (2011) for a parametric pricing kernel. The difference is that we do not assume Markovianity and hence introduce an option pricing error, which contains information about variation not explained by the chosen state variables. Effectively the procedure corrects a biased forecaster by estimating the bias at similar states in the past. The estimated pricing kernel can then be used to nonparametrically risk adjust the risk neutral density implicit in current option prices for forecasting. The identifying assumption is essentially that deviations of the risk neutral density from its average at similar states is attributed to a deviation of the real world density from its average at similar states. When in reality deviations are due to both the pricing kernel and the objective density, a mixture of the two densities can be used with data-driven mixture weights.

Presentations:
February 2016: Econometrics Workshop, Faculty of Economics, University of Cambridge

Discussions (in Econometrics Workshop at Cambridge):

Gospodinov and Otsu (Journal of Econometrics 2012) - Local GMM estimation of time series models with conditional moment restrictions

Palumbo, D. (2015) – Joint Modelling of the Term Structure of Interest Rates and Credit Spreads