Report Type
Mid Term Award Report

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Project Title
Optimal investment and consumption problem under transaction costs

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Oct 03, 2016

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Oct 03, 2017

Amount Awarded
n/a

Project Abstract
This project studies a Merton-style optimal investment and consumption problem under market frictions. In the classical setup, an agent has the opportunity to invest in a riskfree money market account and an illiquid risky asset where purchase and sale of this asset will incur a proportional transaction cost. We study the problems via a primal dynamic programming approach. A number of known results in the literature are recovered. Along the way we also provide several new analytical results related to the comparative statics of the no-transaction boundaries and demonstrate a new financial phenomenon that has not been documented to date. The methodology is extended to a multi-asset economy where the agent may also invest in another liquid risky asset without any transaction cost. This is a joint work with David Hobson (Warwick) and Yeqi Zhu (Credit Suisse).

Activities and Achievement
We begin with the classical setup with a single risky asset. By adopting a novel transformation scheme, we reduce the key dynamic programming equation of interest to a simple first order differential equation which enables us to answer a number of relevant questions. Firstly, important (and known) analytical features of the problem, such as when the problem is well-posed for all level of transaction costs, when the problem is well-posed only for sufficiently large transaction costs, and when the agent might take a leverage position, can all be deduced from the shape of a simple quadratic function. Secondly, this formulation allows us to establish simple proofs for the comparative statics of the sale and purchase boundaries. Thirdly, a new financial phenomenon can be observed where for certain parameters combination the location of the sale boundary can be independent of the transaction cost on purchase. Leveraging on this machinery, we provide a comprehensive analysis to the multi-asset extension of the problem.

Dissemination
Two working papers have been produced, one of which covers the single risky asset case while the other one covers the multi-asset extension. The former is currently under R&R at Mathematical Finance while the latter has been submitted to Finance and Stochastics. The work was also presented in The Fifth Asian Quantitative Finance Conference (AQFC) in Seoul in April 2017.

Outputs
The working papers are available on arXiv. Please refer to the links below for the abstracts, bibliographical references and any other relevant information.


Major Difficulties and Any Other Issues
The number of cases to be analyzed are numerous. The source of variations originates from whether the underlying risky asset has a positive mean return, whether the problem is unconditionally well-posed, whether the problem is conditionally well-posed only for large transaction costs, etc. It is thus challenging to provide a brief and tight analysis in our write up while retaining rigorousness with our expositions.

Web Links
Please refer to the "Outputs" section.