Start-up Financing, Entry and Innovation

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May 31, 2024

Abstract

In this paper, I develop a model to study the interaction of markets for start-up financing and innovation activity. The model incorporates two primary frictions: agency costs due to informational asymmetry between investors and entrepreneurs, and search and matching frictions in finding suitable investors. In this setting, I relate funding market conditions to start-up entry and outcomes via observable features of venture capital contracts. These frictions result in capital misallocation, particularly in tight funding markets, which are often associated with economic recessions or countries with less developed risk capital markets. Therefore, the results suggest that markets with lower levels of capital supply also allocate it less efficiently. I then estimate the model separately on US and UK micro data and show that it captures a rich set of empirical moments. I use the model to analyse differences in start-up entry and outcomes between the UK and US. The results suggest that the (market-size adjusted) gap in start-up entry rates in the UK can be explained by differences in the availability of funding. However, the proportion of start-ups that achieve an IPO or acquisition appears insensitive to funding market conditions, with variations in acquisition opportunities driving these differences instead.

JEL Codes: E44, G24, G34, O31

Keywords: start-ups, innovation, venture capital.

*Email: cp649@cam.ac.uk. I am immensely grateful to Vasco Carvalho for his guidance, support and feedback. I thank Charles Brendon, Bill Janeway, and Ramana Nanda for valuable discussions, as well as participants at the Cambridge Macro Workshop and the CERF Lunch Talk. This work was supported by the Economic and Social Research Council (ESRC).
1 Introduction

The premise of the analysis in this paper is that access to venture capital funding differs both between countries at any instant and at the national level over time. The emergence of the venture capital industry played a significant role in building America’s current dominance in the Tech industry, but other countries have typically been unable to replicate the American experience (Gornall and Strebulaeva, 2021). In Europe, policymakers have long been concerned with the so-called ‘European paradox’, the perception that, despite strengths in basic research, Europe struggles to translate these advances into commercial success. The availability of risk-capital is often seen as a key driver (Quas et al., 2021). At the same time, even within the US there are large fluctuations in the aggregate volume of venture capital investments; and these fluctuations are not driven solely by changes to the availability of investment opportunities (Janeway et al., 2021). Existing research suggests that changes to financing conditions affect not only the level, but also the direction of innovation. Hot markets, characterised by a high level of investment activity, are associated with start-ups pursuing more radical projects (Nanda and Rhodes-Kropf, 2013). Understanding the effect of the financing regime on the level and direction of innovation has first-order consequences for growth and for the ability of modern economies to adapt to new challenges that require fundamentally new technologies, such as the green transition.

In order to explore the implications of access to financing on start-up innovation, I develop a framework that draws closely on key insights from entrepreneurial finance and that maps readily to data on observable features of venture capital contracts, which allows for disciplining key parameters. The model incorporates two central frictions. Building on the ideas in Gompers (1995), financing start-ups is subject to agency costs, which I motivate through an informational asymmetry between investor and entrepreneur. In addition, the process of finding a suitable investor is non-trivial and, therefore, I incorporate a search and matching friction, as in Inderst and Müller (2004). The combination of these two frictions implies that staged-financing, whereby investors provide capital in tranches rather than all upfront, arises endogenously within the model.

The first contribution of the paper is theoretical. I jointly characterise the effect of idiosyncratic and market-wide factors on funding patterns, start-up outcomes and entry rates. Some results extend findings from the existing literature to a new setting. For instance, as in Nanda and Rhodes-Kropf (2017), firms pursuing more radical (high payoff, low probability) projects are more adversely affected by tight financing conditions. For these firms, greater project uncertainty translates into more severe agency frictions, raising the cost of funding. This becomes particularly binding when financing conditions are tight, because firms would like to raise more capital to insure against the need to return to the market in the future, but doing so is particularly costly for these firms.

The model also offers new insights into the market and the relationship between funding conditions and start-up innovation. Firstly, holding fixed the types of projects that start-ups pursue, the effect of tightness in the financing market on start-up outcomes is ambiguous. Intuitively, each time a firm attempts to raise capital, a looser financing market raises the likelihood that it will be able to. However, with looser financing conditions firms are less concerned about their ability to raise capital again in the future. Therefore, they find it optimal to raise less capital at each funding round and visit the market...
more frequently. This has the effect of lowering the cost of funds for the start-up by reducing their exposure to the agency friction. However, the adjustment in contracts endogenously increases the firm’s exposure to ‘financing risk’ - failure of a healthy firm due to an inability to raise follow-on funding - and dominates the direct effect when the market is sufficient tight. Secondly, capital is misallocated among firms because the information asymmetry implies that some firms that have received negative news about their prospects continue to expend funds. I show that capital misallocation is greater in economies with tighter financing markets, leading to the conclusion that markets with less VC funding also allocate it less effectively. Third, I analyse the feedback effects of adjustments to VC contracts on market equilibrium. As the market tightens, firms that have the opportunity to raise capital raise more, which leaves even less available for other firms and so amplifies the effects of shocks to funding conditions. Finally, I consider whether, given the frictions in the market, contracts in the decentralised economy are efficient. I show that, in general, the decentralised equilibrium is inefficient because private agents fail to take into account the effect of their contract choice on the capital market equilibrium. The results imply that economies in which capital is relatively scarce, firms that raise capital raise too much, so that the extent of capital misallocation is inefficiently high.

The second contribution of the paper is applied. I estimate the model separately on venture capital data from the UK and US and show that it can explain differences in start-up outcomes between the two countries. To estimate the novel parameters in the model, I use insights from the theoretical analysis to obtain a set of moments that are informative of each parameter. For instance, the theory implies that observable features of venture capital contracts, such as the number of and duration between funding rounds at the firm level, are indicative of the extent of agency frictions and of funding market conditions. The model is able to capture not only the level of various targeted moments, but also features of the distributions, such as those for the number of funding rounds, the duration between funding rounds, exit multiples, the burn rate (the rate of capital utilisation) and time-to-exit dynamics. Therefore, the paper speaks to the literature that attempts to explain features of venture capital funding patterns.

The model estimation suggests that the average VC-backed start-up in the UK and US is fairly similar, both in terms of the quality and type of projects that they develop. However, the estimation picks up substantial differences in the funding markets, especially in the late-stage, when start-ups need to raise substantial amounts of capital to scale up their operations. As a result, my estimates suggest that 16% of capital in the UK market is misallocated, compared to 11% in the US. Furthermore, my estimates support the view that start-ups might face a “second valley of death” \cite{Wilson2018} when attempting to raise greater funding to support their expansion. Indeed, in the estimated model, 11% of UK firms fail at this point, compared to just 3% of US start-ups.

In a counterfactual exercise, I show that improving access to VC funding in the UK to the level estimated for the US is able to account for the entire gap in entry between the two countries. However, the effects on outcomes are more nuanced. Improvements in the late-stage funding market close the gap between the UK and US in terms of the share of firms that are ultimately successful, which I define as completing an IPO or acquisition by a financial-acquiror. On the other hand, acquisitions by non-financial acquirors (corporations) substitute for financing in the model, such that when financing

\footnote{See, for example, \textcite{Gompers1995,Neher1999,Bergemann2009}, and \textcite{Dahiya2012}, among others.}
conditions improve, firms become less willing to accept acquisition offers. Overall, this implies that outcomes are not particularly sensitive to the financing environment because the majority of start-up exits are via way of acquisition. Acquisition opportunities are estimated to be scarcer in the UK, and this accounts for the majority of the differential in outcomes.

The exercise within the model has implications for policymakers considering how to improve the innovation output of countries with less developed venture capital markets, such as the UK. However, caution is also required in drawing policy recommendations. The model cannot distinguish between the various drivers of limited funding supply. It can be the case that funding is a serious bottleneck but that, at the same time, a policy of pumping more capital into the market proves ineffective. Government venture capital (GVC) funds might less effectively screen potential projects and impose governance on portfolio companies, raising concerns their investments derive negative returns.\footnote{Relatedly, Brander et al. (2013) point out that an often-stated aim of GVC is to support start-ups that would not otherwise have received funding.} In this spirit, Grilli and Murtinu (2014) find that GVC investments in Europe do not generate improvements in firm sales or employment growth. An alternative to direct investments is to set up a fund-of-funds, in which the government invests as a limited partner in independent venture capital firms who then make investments on their behalf. However, this approach may not be the panacea it appears to be. Anecdotally, there are limitations to the size of funds that VCs can maintain.\footnote{Nicholas (2019, p. 279) notes that Jack Meyer from the Harvard Management Company, which oversees Harvard University’s endowment, expressed frustration in 2000 over the university’s inability to allocate as much money as desired into venture capital due to the funds’ capacity limitations.} In this vein, Cumming and Dai (2011) provide evidence in support of diseconomies of scale in the VC industry and suggest that these issues might arise from limitations in human capital availability. With this in mind, policymakers should consider that the constraints might not only stem from the absolute amounts of available capital but also from factors such as human capital availability and the expertise within the VC industry. Understanding these nuances is crucial for developing effective policies aimed at fostering a more vibrant and supportive environment for start-up innovation.

**Related literature.** This paper is related to several strands of literature. Firstly, it relates to the literature on the financing of innovation. The majority of the literature focusses on the relationship between R\&D investments and financing for mature firms, considering factors such as the importance of access to public equity markets (Acharya and Xu, 2017), the role of debt financing (Mann, 2018), and the different roles played by internal and external finance (Brown et al., 2009). A growing literature considers issues more specific to start-ups, where agency frictions are likely to be even more pronounced (Hall and Lerner, 2010). This paper relates most closely to Nanda and Rhodes-Kropf (2013) and Nanda and Rhodes-Kropf (2017), which discuss the notion of ‘financing risk’, whereby otherwise healthy firms may fail to raise follow-on funding due to issues on the supply side of the capital market. As in Nanda and Rhodes-Kropf (2017), the choice of contract in this paper trades-off heightened agency costs as funding horizons lengthen with the insurance benefit of not having to return as often to the capital market. Relative to their paper, a main contribution here is to demonstrate the equilibrium implications of this trade-off and show that private agents typically trade-off these incentives in a way that is socially inefficient. I also provide estimates of the significance of financing risk for firm outcomes.

Secondly, this paper draws on the insights of a number of papers that model the characteristics of the
venture capital market. Inderst and Müller (2004) introduces a search equilibrium model of venture capital financing and assumes that contracting between the entrepreneur and investor is subject to moral hazard; the optimal contract balances the incentives of both entrepreneur and investor to exert effort, which is non-contractible. Similarly, Michelacci and Suarez (2004) also considers a search equilibrium model and analyses the role that the public equity market plays in recycling ‘informed-capital’, which is necessary to finance companies in the early stages of their development. Finally, Jovanovic and Szentes (2013) considers optimal contracts in an equilibrium model where effort is non-contractible and estimates the model using data on venture capital funding. In this paper, I model the capital market as a search equilibrium and introduce agency frictions that affect contracting between the entrepreneur and VC, but do not consider the issue of non-contractible effort. Instead, the agency frictions considered here - motivated by asymmetric information - imply that staged-financing occurs endogenously, which is not a feature of the other papers. This allows me to speak to issues discussed in the entrepreneurial finance literature (e.g. financing risk) and to map the model tightly to data on the profile of financing rounds, such as the number of and duration between funding rounds, which can be used to identify parameters of the model.

Thirdly, this paper is related to a series of papers in the macroeconomics literature on venture capital and economic growth. For instance, Opp (2019), Akcigit et al. (2022), Greenwood et al. (2022) and Ando (2024) study the role that venture capital plays in fostering economic growth. Greenwood et al. (2022) also considers a cross-country comparison, focussing on the effect of tax rates and monitoring capabilities in explaining differences between France and the US. A key departure in my paper is that funding patterns are endogenous: firms choose contracts that determine how long they can fund their development process for (their “runway”) and this decision responds to characteristics of the capital markets. This implies that the choice of how to fund the development process matters for start-up outcomes. Furthermore, from a modelling perspective, I take venture capitalists purely as providers of external funding, abstracting from any direct role that they play in determining start-up success beyond the provision of funds.

Finally, this paper is related to the literature on staged-financing. Neher (1999) shows that staging capital injections can help to overcome a commitment problem, whereby the entrepreneur has an incentive to renegotiate the contract once capital has been committed. In this environment, staging serves the purpose of building up collateral and therefore helps to overcome this commitment problem. In addition, Gompers (1995) uses agency frictions to motivate staged financing and tests the predictions of the theory on venture capital data. For instance, he shows that the duration between funding rounds increases in the ratio of tangible assets to total assets (at the industry level), in line with the predictions of agency-based theories. In this paper, I provide microfoundations for staged-financing that draw heavily on the insights of Gompers (1995). I consider a problem of asymmetric information and assume that the entrepreneur derives private benefits from operating their firm, which leads to the problem known as inefficient continuation. The main contribution here is to incorporate this microfoundation into an equilibrium search model that facilitates considering both agency costs and search frictions in

\footnote{Jovanovic and Szentes (2013) does not model the capital market as a search equilibrium, but payoffs to the entrepreneur and VC are still influenced by the relative supply of VCs.}

\footnote{Higher asset tangibility is associated with higher liquidation values, thereby reducing investors’ exposure to entrepreneurs’ decisions.}


tandem.

**Outline.** The remainder of the paper is structured as follows. In section 2, I develop a tractable equilibrium model of start-up maturation and analyse its properties. In section 3, I extend the model slightly in order to take it to data, discuss the estimation strategy, and conduct counterfactual exercises. Section 4 concludes.

## 2 Model

This section describes a baseline version of the model that is tractable; later sections generalise the model as it is mapped to data. The model incorporates two frictions that are central to the process of financing young innovative companies with little track record. I begin by motivating the choice of frictions before describing the model setup.

**Agency friction.** In practice, difficulties in screening projects can create a market-for-lemons, while the separation of control induces potential for moral hazard (Hall and Lerner, 2010). Relatedly, having obtained investment, an entrepreneur may receive a negative signal about the prospects of the project but withhold this information from the investor due to some private benefits they obtain from entrepreneurship. These private benefits can lower the termination threshold for the entrepreneur below that of the investor, leading to “inefficient continuation” (Gompers, 1995). In this spirit, I introduce an information asymmetry: once a start-up has received funding, project developments are unobserved by the VC but not the entrepreneur, who also derives some non-pecuniary private benefits. This provides incentives to stage capital infusions.

**Search and matching friction.** Venture capital firms are more than mere providers of funding. Bernstein et al. (2016) find that the increased interactions generate improvements in innovation outcomes and in the likelihood of a successful exit. Due to their role as providers of “informed capital” (Michelacci and Suarez, 2004), characteristics of the VC and start-up are likely to be important in determining investment returns. In practice, VCs are differentiated through their level of experience (Sorensen, 2007), access to networks (Hochberg et al., 2007), degree of sectoral specialisation (Gompers et al., 2009) and location. Similarly, start-ups vary along a number of dimensions. The ability to seek external funding can differ across gender and race (Ewens, 2023). Furthermore, in choosing investments, VCs are particularly concerned with the make-up of the founding team, the business model and the market (Gompers et al., 2020). Receiving an investment requires seeking out an investor that is a good fit across a number of dimensions and an ability to stand out in a crowded market. Consequently, the market takes time to clear and congestion effects may be important. In the model, I abstract from heterogeneity among start-ups and investors, instead incorporating these features of the market through a search and matching friction, as in Inderst and Müller (2004), which simplifies the analysis while capturing the essential dynamics.

The model is in continuous time, the horizon is infinite, and I focus on the steady-state equilibrium in this section. I begin by defining the environment of a single entrepreneur in possession of a risky

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6Lerner (1995) shows that VCs are more likely to invest in companies located nearby.
project and introduce an agency friction that affects the ability of the entrepreneur and VC to write a fully state-contingent contract. I then embed this setting into an equilibrium search model. Subsequent sections then analyse the properties of the model, including efficiency and equilibrium dynamics.

2.1 Technology and financing

Entrepreneurs (E) are risk neutral and discount the future at rate $\rho$. Each E has zero wealth but possesses a risky project that requires a constant flow capital investment $k$ in order to fund development. Conditional on paying $k$, the results of the development process are realised at rate $\kappa$ and are positive (success) with probability $p$ and negative (failure) with probability $1 - p$. Success results in a payoff $\pi$, whereas a failure returns zero.

A wealth-constrained E must seek external funding from a VC. The contract specifies a Poisson arrival rate rate, $\omega$, which determines the rate at which funding is exhausted. I refer to $\omega$ as the contract rate; a firm with contract rate $\omega$ stops its development process at rate $\omega$, in which case its value returns to $V_s \geq 0$. For now, $V_s$ is treated as exogenous - it is made endogenous in the next subsection. The VC has cost of funds $\rho$, equal to its discount rate, and cares about its expected capital commitment, $K(\omega)$, which depends on $\omega$ because the contract rate determines the duration of its commitment to fund the E’s development process. For instance, a lower contract rate, $\omega$, translates into a longer funding horizon and greater capital commitment. In return for this commitment, the VC receives a share $\varsigma$ of the value of the firm gross of investment costs (post money). The contract is negotiated between the E and the VC and, for simplicity, I make the assumption that the VC has no bargaining power in the negotiations, such that the VC’s share $\varsigma$ is determined by their participation constraint.

A frictionless benchmark. Suppose first that the E and VC can write any state-contingent contract and that the NPV of the investment is positive. Then, the optimal state-contingent contract would involve the VC funding the investment cost $k$ until time $T_\kappa \sim Exp(\kappa)$ and no longer. Given that funding can be conditioned to stop when a result is obtained, this optimal contract can be implemented within the framework considered here by setting $\omega = 0$. In this case, funding is never withdrawn before the realisation of the result and so the start-up secures upfront financing. In contrast, if the investment has negative NPV, then the VC provides no funding, which is equivalent to setting $\omega \to \infty$.

A frictional setting. Now suppose that there is asymmetric information and that the arrival of the result, success or failure, is observable to the E but not to the VC. In addition, the E derives some non-pecuniary benefits of entrepreneurship, $x_e > 0$, whenever the project is being funded. This could reflect social status, access to networks, or other non-pecuniary benefits to entrepreneurship.

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7 In the full model, I assume that VCs can fund at most one project at a time. In that setting, the introduction of VC bargaining power has two effects. Firstly, funding a firm entails an opportunity cost. The opportunity cost then leads the VC to want to write a contract with a higher contract rate, $\omega$, because the VC values search. On the other hand, because of staged-financing, a VC that provides funding today faces a hold-up problem with respect to future investors, because future investors will be able to extract a positive surplus. This pushes in the opposite direction. I leave a more comprehensive analysis of this issue to future work.

8 This requires that the investment be worthwhile above and beyond the outside option value, $V_s$. Specifically, it requires that $V_s < \frac{x_e - k}{p + \kappa}$.

9 Technically, there is also the uninteresting knife-edge case where the E is indifferent between investing in the project and not, in which case any $\omega \in [0, \infty)$ is optimal. This occurs for $V_s = \frac{x_e - k}{p + \kappa}$. 

7
In this setting, it is no longer possible to condition on the arrival of a result at time $T_κ$ because the E may find it profitable to withhold information about the status of the project. Specifically, suppose that the result has just been realised at time $T_κ$. If the E reports this information to the VC, funding is withdrawn. By choosing not to divulge this information, the E obtains the non-pecuniary benefit for an additional period $T_ω \sim \text{Exp}(ω)$, which has discounted expected value, $\frac{x_e}{ρ + ω}$. However, if the result is a success, then the choice to delay reporting implies a delay in obtaining the pecuniary benefit, which has a cost due to discounting.

Clearly, if the E learns of a negative result, they would never willingly inform the VC since $x_e > 0$. This logic also applies to any time after the realisation of the result because the environment is stationary. Determining whether the E reports a positive result to the VC is not so straightforward. The E must decide whether the value of the private benefit is sufficient to compensate for the loss in their pecuniary payoff due to discounting, which leads to an incentive compatibility constraint. Proposition A.1 in the Appendix shows that, for any set of model parameters, there always exists an upper bound $\bar{x}_e > 0$ such that if $x_e < \bar{x}_e$, then the E always reports positive results. Intuitively, if the pecuniary benefit is sufficiently low (in a relative sense), the E would never choose to delay the realisation of the value of the venture in order to continue to obtain the private benefit. Therefore, given $x_e < \bar{x}_e$, the E always reports successes to the VC, but never reports failures. In what follows, I take the limiting case where $x_e \to 0$, so that the result of inefficient continuation is preserved and the E always reports positive results to the VC. This setting appears to accord with reality.

Going forwards, it will be helpful to refer to a firm in development prior to the realisation of a result as ‘productive’, whereas a firm that has already obtained a negative result but is still receiving development funding as ‘unproductive’. Towards deriving the contract that is signed between the E and the VC, consider the value of productive development, $V_d$, gross of investment costs, which is given by the solution to the HJB equation

$$\rho V_d = κ [p \pi - V_d] + ω [V_s - V_d]$$

where $x_e \to 0$ has been imposed.

To derive the contract, we also need to know the cost to the VC. The expected capital cost to the VC of the contract $(ω, κ)$ is given by

$$K(ω) = \frac{k}{ρ + κ + ω} + \frac{κ (1 - p)}{κ + ω} \frac{k (κ + ω)}{(ρ + ω) (κ + ω + ω)} = \left(1 + \frac{κ (1 - p)}{ρ + ω}\right) \frac{k}{ρ + κ + ω}.$$  

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10This follows from the memoryless property of the exponential distribution.
11See Lemma 2 in the Appendix. The IC depends on features of the project and environment, $(π, ρ, x_e)$ but also on the share, $1 - κ$, that the E maintains in the project. This leads to the implication that the E must maintain sufficient skin-in-the-game to report truthfully.
12See, for example, the discussion in Chapter 4 of Janeway (2018), “After more than forty years in the game, I have yet to meet the entrepreneur who dailles in delivering word that ‘the product works’ or ‘the sale has closed’. When communication ceases, then the venture capitalist can expect to discover that ‘the product needs another rev’ or that ‘we lost the order’.” Janeway refers to this as his Second Law of Venture Capital, “No news is never good news”.
13Full expressions for the general case of $x_e > 0$ can be found in the proof to Proposition A.1 in the Appendix.
14See equation (30) for a full derivation.
The expected discounted capital cost, $K(\omega)$, is given by the frictionless capital cost associated with contract $(\omega, \varsigma)$ plus an additional term that reflects the cost of the agency friction. Indeed, the second term is the product of the probability of the start-up entering the unproductive development state and the value of the losses from unproductive development, appropriately discounted. This additional term - or wedge that brings the cost of capital for contract $(\omega, \varsigma)$ above that in the frictionless setting - controls the extent of the agency friction. The agency friction is more severe when projects are more ‘risky’, in the sense of having a lower probability of success, $p$. Intuitively, high risk projects are more likely to fail and, therefore, to suffer from inefficient continuation.

The agency friction provides an incentive against upfront financing: the wedge controlling the extent of the agency friction, \( \kappa(1-p)/\rho + \omega \), is decreasing in \( \omega \). Therefore, as the contract looks more and more like upfront funding, \( \omega \to 0 \), the agency friction is exacerbated. Intuitively, this is because such a commitment raises both the probability of entering the state of inefficient continuation and the expected cost, conditional on doing so. In contrast to the frictionless benchmark, this feature of the problem generates interval of parameter values for which \( \omega \in (0, \infty) \) is optimal. Specifically, the optimal contract \((\omega, \varsigma)\) solves

\[
\sup_{\{\omega \in [0, \infty), \varsigma \in [0, 1]\}} \left\{ (1 - \varsigma) V_d(\omega) \right\}
\text{s.t. } \varsigma V_d(\omega) \geq K(\omega) \tag{3}
\]

where \( V_d(\omega) \) is the solution to equation [1], \( K(\omega) \) is given by equation [2], and \( V_s \) is taken as given in the optimisation. Noting that the participation constraint of the VC will bind, given the assumption of no bargaining power, it can be substituted into the optimand. The first-order condition is given by

\[
V'_d(\omega) = K'(\omega) \tag{4}
\]

At the optimum, the E trades-off the effect of the contract rate on the value of the firm, \( V_d \), against its effect on the total capital commitment, \( K(\omega) \), which translates into a lower equity stake for the E, \( 1 - \varsigma(\omega) \), by the participation constraint of the VC. Thus, while a lower contract rate typically increases the value of the pie, it lowers the share that it is retained by the E. And importantly, the fact that the agency friction becomes increasingly severe as \( \omega \to 0 \) means that the E may find it optimal to select an interior contract rate, \( \omega \). The solution to maximisation problem [3] is given by

\[
\omega = \begin{cases} 
0 & \text{if } V_s \leq \max\{0, V_s\} \\
-\rho(pk\pi-k-(\rho+k)V_s)+k\pi(1-p)+\sqrt{\rho^2 k(1-p)(pk\pi-pk-(\rho+k)V_s)} & \text{for } V_s \in (\max\{0, V_s\}, \bar{V}_s) \\
\infty & \text{otherwise}
\end{cases}
\tag{5}
\]

where

\[
\bar{V}_s = \frac{pk\pi - pk}{\rho + \kappa} - \frac{k(1-p)(\rho + \kappa)}{\rho^2}, \quad V_s = \frac{pk\pi - k}{\rho + \kappa}
\]
In general, there are three cases to consider. Firstly, it may not be optimal for the entrepreneur and VC to agree to a contract at all. This corresponds to the case of $\omega \to \infty$ and occurs when the value of no contract, $V_s$, is sufficiently high.\footnote{On the other extreme, if the value of no contract is sufficiently low, the two parties agree to fund the contract until the project obtains a result with certainty, setting $\omega^* = 0$; this is the case of upfront funding. Finally, for intermediate values of search, $V_s$, the contract rate is positive but finite, $\omega \in (0, \infty)$, which corresponds to the solution to the first-order condition discussed above. In this region, there is a positive probability that funding will be exhausted before the project realises a result.} On the other extreme, if the value of no contract is sufficiently low, the two parties agree to fund the contract until the project obtains a result with certainty, setting $\omega^* = 0$; this is the case of upfront funding. Finally, for intermediate values of search, $V_s$, the contract rate is positive but finite, $\omega \in (0, \infty)$, which corresponds to the solution to the first-order condition discussed above. In this region, there is a positive probability that funding will be exhausted before the project realises a result.\footnote{Recall that the intermediate case, where $\omega \in (0, \infty)$, was not optimal when agents could write fully state-contingent contracts. However, the introduction of the agency friction implies that optimal contracts may have the feature that funding is exhausted before uncertainty about the project is resolved. In the next section, I embed this setting into an equilibrium search model of venture capital funding. In the model, a start-up whose funding has been exhausted returns to the capital market to seek follow-on funding, so that $V_s$ has the interpretation of a firm in search of capital. When firms have upfront funding, $\omega = 0$, they never return to the capital market to seek additional funding. However, when $\omega \in (0, \infty)$, a firm whose funding is exhausted returns to the funding market to seek more funding in return for a further dilution of the stake of existing shareholders. This is the case of staged financing - the firm may return to the capital market multiple times to complete its development - and is an endogenous feature of the model.}

Discussion. Recall that the intermediate case, where $\omega \in (0, \infty)$, was not optimal when agents could write fully state-contingent contracts. However, the introduction of the agency friction implies that optimal contracts may have the feature that funding is exhausted before uncertainty about the project is resolved. In the next section, I embed this setting into an equilibrium search model of venture capital funding. In the model, a start-up whose funding has been exhausted returns to the capital market to seek follow-on funding, so that $V_s$ has the interpretation of a firm in search of capital. When firms have upfront funding, $\omega = 0$, they never return to the capital market to seek additional funding. However, when $\omega \in (0, \infty)$, a firm whose funding is exhausted returns to the funding market to seek more funding in return for a further dilution of the stake of existing shareholders. This is the case of staged financing - the firm may return to the capital market multiple times to complete its development - and is an endogenous feature of the model.

2.2 A baseline equilibrium search model with staged-financing

In the preceding section, I presented a simple setting in which entrepreneur contracts with a venture capitalists to funds its development process. In the frictionless setting, the E and VC were able to agree to a fully state-contingent contract in which the VC committed upfront to fund the project until the resolution of uncertainty. The introduction of asymmetric information and diverging incentives led to an agency friction and the problem of inefficient continuation. In such a setting, a commitment of full upfront funding, $\omega = 0$, exposes the VC to exploitation by the E, who chooses to continue to operate a failed venture, unbeknown to the VC. To reduce the losses stemming from this agency friction, the optimal contract may feature a positive contract rate, $\omega > 0$, which puts an (albeit random) termination date of funding and, therefore, limits the VC’s potential liability. With a positive contract rate, there is a positive probability that funding is withdrawn before a result is obtained for the project, i.e. before the realisation of the uncertainty governing its prospects.

In this section, I embed this setting into an equilibrium search model of venture capital financing. In the model, an endogenous flow of start-ups enter at any instant. Start-ups that have paid the entry cost are \textit{ex ante} homogeneous and are all in possession of a project whose prospects are inherently uncertain. The project is identical to the one discussed in section 2.1. To determine whether their project can be successfully commercialised, each start-up must seek external funding from venture capitalists (VCs).\footnote{This could occur for example, if the the flow payoff from the project, $p\kappa - k$, is negative, since $V_s$ is bounded from below by zero.}
However, the process of finding a VC to fund the project is subject to a search friction and during this search process they are at risk of failure from a lack of funding. I denote the value of a start-up in search by $V_s$. Conditional on meeting with a VC, the start-up and VC bargain over a contract as in section 2.1, where $x_e \rightarrow 0$, so that the optimal contract is the solution to optimisation problem (3). However, as discussed, the agency friction implies that the VC may not commit sufficient capital to ensure the start-up realises the results of its development process before funding is exhausted. If funding runs out before the start-up has obtained this result, it must return to the financing market and renew its search for capital. The fact that the firm may fail during its search implies that ‘financing risk’, in the spirit of Nanda and Rhodes-Kropf (2017) is present in the model; an otherwise healthy firm may fail to secure follow-on funding. Furthermore, staged financing arises endogenously.

The search process. I assume a fixed measure $M$ of VCs that can fund at most one project at time. Just like start-ups, VCs are risk-neutral and discount the future at rate $\rho$. The meeting rate in the financing market depends on the measure of start-ups searching for capital, $\mu_s$, and the measure of VCs that are not currently funding a project, $\mu_{vc}$, which I call “unencumbered”. I denote by $\mu^p_d$ the measure of start-ups in productive development and by $\mu^u_d$ the measure of start-ups in unproductive development; these are start-ups that have already obtained a failed result but have not reported this to their VC. Since the total measure of VCs is $M$ and a measure $\mu^p_d + \mu^u_d$ are funding start-ups, the measure of unencumbered VCs is given by $\mu_{vc} = M - \mu^p_d - \mu^u_d$

The flow of meetings between start-ups in search and unencumbered VCs is determined by a Cobb-Douglas matching function

$$\text{flow of meetings} = m(\mu_s, \mu_{vc}) = m(\mu_s, M - \mu^p_d - \mu^u_d) = \mu_s^\alpha (M - \mu^p_d - \mu^u_d)^\beta$$  (6)

where $\alpha, \beta > 0$. Therefore, the meeting rate for a start-up in search is given by $\nu = \mu_s^{\alpha-1} \mu_{vc}^\beta$.

During the search process, the start-up may fail due to lack of funding. This occurs at rate $\lambda$, which is exogenous, and such a start-up obtains a return of zero. Together with the specification of the matching process and denoting by $V^M$ the value of a meeting with a financier (prior to any agreement), the value of search, $V_s$, solves the HJB equation

$$(\rho + \lambda) V_s = \nu [V^M - V_s].$$  (7)

The value of a match to existing shareholders, $V^M = (1 - \varsigma) V_d(\omega)$, where $\omega$ and $\varsigma$ are optimal in the

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17 The assumption that the start-up cannot simply return to its initial funder is clearly a strong one and can be relaxed. However, there are at least two reasons to think it may not be so unreasonable as it appears. Firstly, although a firm will often receive multiple rounds of funding from the same VC, VCs typically syndicate projects within and between rounds, bringing on additional investors as the firm progresses through its development. This serves the role of providing an outsider’s perspective on the performance of the firm. The fact that new investors are typically sought implies that the search friction also applies to follow-on rounds. Secondly, there are various reasons why a start-up may have to seek the bulk of its follow-on funding from entirely new investors. This could be because the initial investor is small and cannot fund ‘big-ticket’ investments, or because the initial funder faces some idiosyncratic shock that leads to limit further investments.

18 This is has parallels to the “short-run” economy of Inderst and Müller (2004). Jovanovic and Szentes (2013) also assume a fixed supply of VCs.
sense of solving optimisation problem (3). Noting that the participation constraint is binding, this can be written as

$$V^M = V_d(\omega) - K(\omega)$$  \hspace{1cm} (8)

where $\omega$ is given by equation (5).

**Entry.** An exogenous flow $\Gamma$ of potential start-ups have a one-time chance to enter, drawing entry cost $c \sim U[0, \sigma]$. New entrants begin their life in search of capital, because entrepreneurs have zero wealth. Therefore, a given start-up enters if and only if their entry cost $c \leq V_s$. Given $c \sim U[0, \sigma]$, the probability of drawing an entry cost below the threshold is $Pr(c \geq V_s) = V_s/\sigma$. Each potential entrant faces this same probability, so that the flow of new entrants that enter endogenously is given by $\Gamma (V_s/\sigma)$. It will not be possible to disentangle $\Gamma$ from $\sigma$, so I will substitute in $\tilde{\sigma} = \sigma/\Gamma$ going forwards. The endogenous flow of entry is then $V_s/\tilde{\sigma}$.

**Equilibrium conditions.** At present, the equilibrium objects $V_d$, $V_s$, $V^M$ and $(\varsigma, \omega)$ are determined for any given meeting rate, $\nu$. To close the model, I need to specify the steady state conditions describing the two state variables, $\mu_s$ and $\mu_{vc}$, which pin down the meeting rate, $\nu$, in equilibrium. Going forwards, it will be simpler to consider the three states of the firm: search, $\mu_s$, productive development, $\mu_p^d$ and unproductive development, $\mu_u^d$. The measure of unencumbered VCs, $\mu_{vc}$, can then be recovered from the definition $\mu_{vc} = M - \mu_p^d - \mu_u^d$.

In steady-state, the inflows and outflows from each state must be equal to one another. Consider first the measure of firms in productive development. A flow $m(\mu_s, M - \mu_p^d - \mu_u^d) = \nu \mu_s$ of firms enter this state from development, having signed contracts with VCs. In steady state, this inflow must equal the outflow, which originates from firms obtaining results, a flow $\kappa \mu_p^d$, and firms who’s funding is exhausted, a flow $\omega \mu_p^d$. Whether a results leads to success or to the firm moving to unproductive development is unimportant. Therefore, we have the steady state condition

$$\nu \mu_s = (\kappa + \omega) \mu_p^d$$ \hspace{1cm} (9)

Next, consider the measure of firms in unproductive development. Firms enter this state from productive development when they obtain a negative result, a flow $\kappa (1-p) \mu_p^d$. In steady state, this inflow must equate to the outflow from firms who’s funding is exhausted, a flow $\omega \mu_u^d$. Therefore, we have the steady state condition

$$\kappa (1-p) \mu_p^d = \omega \mu_u^d$$ \hspace{1cm} (10)

Finally, consider the measure of firms in search. Firms return to search from productive development when their funding is exhausted, which corresponds to an inflow $\omega \mu_u^d$. I assume that firms in unproductive development whose funding is exhausted would fail to convince another VC to invest in their start-up and therefore find it optimal to close down. This means they not return to search for more capital. Intuitively, if investors conduct due diligence before funding new ventures, they would learn of the firms weak (non-existent) prospects and refuse to offer funding. Realising this, firms would shut down immediately after their current funding is exhausted. In addition to the inflow from productive development, new entrants begin their lifecycle in search of capital, a flow $V_s/\tilde{\sigma}$. In steady state, these inflows must equate to outflows, which occur when firms fail during the search process, a flow $\lambda \mu_s$, or
meet with VCs and so begin productive development, a flow \( \nu s \). Therefore, we have the steady state condition

\[
\omega \mu_d^s + \frac{V_s}{\sigma} = (\lambda + \nu) s
\]  

(11)

The steady state conditions are therefore given by equations (8), (10) and (11). These equations can be written purely in terms of \( s \) and \( \mu_v \). A steady-state equilibrium is a tuple \((V_d, V_s, V^M, \omega, \varsigma, s, \mu_v)\) satisfying equations (8), (9), (10), (11) and the steady state conditions, where \( \varsigma \) is consistent with the participation constraint and \( K(\omega) \) is given by equation (3).

### 2.3 Funding patterns, entry and outcomes

Given a value for the meeting rate in the financing market, \( \nu \), the model omits a closed-form solution for value and policy functions. This section analyses properties of the model taking \( \nu \) as given. Throughout, I make assumptions so that firms engage in staged-financing, which occurs when \( \omega \in (0, \infty) \), unless stated otherwise. The first proposition proves existence of an equilibrium given \( \nu \) and the resulting Corollary derives the optimal contract rate, \( \omega \), as a function of model parameters.

**Proposition 2.1.** Given \( \nu \) and for \( pk \pi > k \), there is unique solution for \( \{V_d, V_s, V^M, (\omega, \varsigma)\} \) in terms of model parameters and the resulting equilibrium is characterised by a (weakly) positive and finite contract rate, \( \omega \in [0, \infty) \).

**Corollary 2.1.** Given \( \nu \) and for \( pk \pi > k \), the equilibrium contract rate is given by

\[
\omega = \begin{cases} 
0 & \text{for } p > \tilde{p} \text{ and } \nu \leq \tilde{\nu} \\
\frac{1 - (\rho + \lambda) (\rho (k \pi - k) - k (1 - p)) - \sqrt{k (1 - p) [\rho + \lambda] (k (1 - p) (\rho + \lambda + \nu) + \nu p) - k [\rho (\rho + \lambda) + k (\rho + \lambda) + \nu p]}}{\rho + \lambda} & \text{otherwise}
\end{cases}
\]

(12)

where \( \tilde{p} \in (0, 1) \) and \( p \leq \tilde{p} \implies \tilde{\nu} < 0 \).

Corollary (2.1) allows us to investigate the dynamics of the model. It provides conditions under which the optimal contract rate is zero, implying upfront financing, or positive, implying staged financing. In particular, note that if the meeting rate in the financing market, \( \nu \), is sufficiently low and the probability of success is sufficiently high, then upfront financing can still be maintained. Intuitively, when \( \nu \) is low, search is costly so shareholders want to avoid returning to the capital market. However, the ability to write an upfront financing contract depends on the degree of agency frictions, which are most extreme for projects with low frictionless success probabilities, \( p \), because these are most likely to lead to inefficient continuation. In what follows, given any two projects with properties \((p, \pi)\) and \((\tilde{p}, \tilde{\pi})\) such that \( p \times \pi = \tilde{p} \times \tilde{\pi} \), I will refer to the former project as more radical if \( p < \tilde{p} \). Given this terminology, upfront financing requires that the projects are not too radical. When these conditions are not met, the optimal contract features a positive finite contract rate, implying staged-financing.

\[\text{From equation (11), } \mu_d^s = \frac{s (1 - p)}{w} \mu_v, \text{ so that } \mu_v = M - \mu_d^s - \mu_d^v = M - \left(1 + \frac{s (1 - p)}{w}\right) \mu_d^s = M - \left(1 + \frac{s (1 - p)}{w}\right) \frac{\pi}{\kappa s} \mu_s, \text{ where the last equality follows from equation (8). Furthermore, for } \mu_d^s \text{ in equation (11) from equation (3) yields } \frac{s \nu}{w} \mu_s + \frac{s}{w} = (\lambda + \nu) \mu_s.\]
I begin by analysing the effect of various model parameters on the optimal contract, which helps to build intuition for the effect of various parameters on start-up patterns and their relationship to outcomes, which I subsequently investigate.

**Proposition 2.2.** The optimal contract rate, $\omega$, is: strictly increasing in $\nu$ and $k$; and strictly decreasing in $p$, $\pi$, $\lambda$. For $\kappa$, $\exists \bar{\kappa} > k/p\pi$ such that for $\kappa < \bar{\kappa}$, $\omega'(\kappa) < 0$ and otherwise $\omega'(\kappa) \geq 0$. Furthermore, holding $p \times \pi$ constant, $\omega$ is strictly decreasing in $\kappa$.

**Corollary 2.2.** The expected duration of productive development, $1/(\kappa + \omega)$, is: strictly decreasing in $\nu$ and $k$; strictly increasing in $p$, $\pi$ and $\lambda$; and ambiguous with respect to $\kappa$. Furthermore, holding $p \times \pi$ constant, $1/(\kappa + \omega)$ is strictly increasing in $p$.

**Corollary 2.3.** The expected discounted capital commitment, $K(\omega)$, is: strictly decreasing in $\nu$; strictly increasing in $\pi$ and $\lambda$; and ambiguous with respect to $\kappa$, $k$ and $p$, including when $p \times \pi$ is held constant.

Proposition 2.2 is key to the dynamics of the model. Consider first the parameters that govern access to financing. When the meeting rate, $\nu$, increases, there is less incentive to insure against financing risk, so the contract rate rises and firms raise less capital. Conversely, when the failure rate in search rises, there is a greater need to protect against returning to the capital market, so the optimal contract rate falls and firms raise more capital. Furthermore, the parameters that govern the project payoff affect the contract rate. When the NPV of the project if financed increases, either because the flow cost falls, $k \downarrow$, the expected payoff conditional on success rises, $\pi \uparrow$, or the friction-free success probability rises, $p \uparrow$, the optimal contract rate is lower, reflecting a longer funding horizon. Finally, the effect of $\kappa$ on $\omega$ trades-off two forces. On the one hand, an increase in $\kappa$ increases the frictionless project NPV because of maturity mismatch between the realisation of uncertainty and the funding horizon. Intuitively, if uncertainty is expected to be resolved in a matter of months, but funding has been provided for a number of years, then the VC is at risk of exploitation by the entrepreneur. This implies that as $\kappa$ continues to increase, there is a point after which further increases lead to increases in $\omega$ in order to match the maturities of uncertainty resolution and funding. This is an incarnation of the common feature of venture capital financing that capital injections are provided in order for the venture to overcome a particular milestone.

Beyond the simple comparative statics results, Proposition 2.2 is informative about the implications of financing frictions for the direction of innovation. Nanda and Rhodes-Kropf (2017) argue that start-ups with more radical innovations require hot financial markets to succeed because the real option value to investors of staged financing is more valuable for these firms. The same effect is present in this model, albeit with a slightly different interpretation. Holding fixed $p \times \pi$, and therefore comparing projects with equal flow payoffs, Proposition 2.2 shows that more radical innovations - in the sense of lower $p$ and higher $\pi$ projects - have have lower contract rates, corresponding to a shorter financing duration. Intuitively, the probability of entering the state of inefficient continuation is higher for these firms and

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20For $k$ and $p$, this does not necessarily result in a greater capital commitment, because a lower flow investment cost $k$ naturally lowers the commitment for a given funding duration, whereas a higher frictionless success probability, $p$, reduces agency costs, reducing the wedge.
so the agency friction is more severe. The VC therefore wants to limit their exposure to these firms and so the equilibrium contract has shorter expected duration. As I show in the following analysis, these firms will be required to return to the capital market more frequently and are, therefore, more exposed to financing risk than firms with identical NPV projects that are less radical.  

2.3.1  Endogenous start-up funding patterns

To build intuition into how these results map into the model, it is informative to consider a number of empirically observable statistics about the firm, before examining implications for start-up outcomes. I focus on two statistics that are readily observable from data on venture capital funding rounds: the number of rounds and the duration between rounds.  

Consider first the number of funding rounds. I focus attention on the subset of firms that receive at least one funding round in the model because this has a natural mapping to venture capital data. To derive the distribution of the number of funding rounds, note the recursive structure of the model - the probability of participating in an additional funding round is independent of the number of funding rounds the firm has already completed and depends only on the firm’s current state, search or (productive) development. This feature of the model enables derivation of the exact distribution for the number of financing rounds conducted, which I denote by \( N_f \). As mentioned, I am interested in the distribution of \( N_f \) given that a firm has secured at least one financing round, so \( N_f \in \{1, 2, 3, \ldots\} \). A firm may not raise again either due to completing its experiment in this financing round, or because its financing was withdrawn and it could not raise new capital. Putting this together, the probability that a firm in productive development does not raise more capital is given by

\[
Pr(\text{do not raise again}) = Pr(\text{result in current round}) + Pr(\text{return to search and fail})
\]

\[
= \frac{\kappa}{\kappa + \omega} + \frac{\omega}{\kappa + \omega} \frac{\lambda}{\lambda + \nu}
\]

For notational convenience, I denote this probability by \( q_d \). Then

\[
Pr(N_f = 1) = q_d, \quad Pr(N_f = 2) = (1 - q_d) q_d, \quad \ldots \quad Pr(N_f = n) = (1 - q_d)^{n-1} q_d
\]

which is the probability mass function of the Geometric distribution. This leads directly into the next proposition

**Proposition 2.3.** The distribution of the number of financing rounds, \( N_f \sim Geo(q_d) \) where \( q_d = \frac{\kappa}{\kappa + \omega} + \frac{\omega}{\kappa + \omega} \frac{\lambda}{\lambda + \nu} \) with support \( N_f \in \{1, 2, 3, \ldots\} \).

**Corollary 2.4.** The expected number of financing rounds, \( E[N_f] = 1/q_d \), is: strictly increasing in \( \nu \), and \( k \); strictly decreasing in \( p \), \( \pi \), and \( \lambda \); and ambiguous for \( \kappa \). Furthermore, holding \( p \times \pi \) constant, \( E[N_f] \) is strictly decreasing in \( p \).

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21This is exactly the point made by Nanda and Rhodes-Kropf (2017). Future versions of this paper will consider the quantitative importance of this mechanism for explaining the empirical finding that positive innovations to capital supply lead to lower success rates but, conditional on success, higher payoffs (Nanda and Rhodes-Kropf, 2013).

22The relationship between these features of funding patterns, firm characteristics, and market conditions was first explored in the seminal work of Gompers (1995).
Through Corollary 2.4, we see a tangible effect of the thickness of capital markets on staged-financing patterns. When the capital market is thin, $\nu \downarrow$, the start-ups that raise capital raise more capital to fund a longer period of development. As a result, they complete fewer funding rounds, on average. Furthermore, more radical start-ups will tend to require more rounds of funding on average relative to firms that would have identical NPV, were there no financial frictions.

Next, turn attention to the duration between funding rounds. Between two funding rounds, a firm conducts development for a period $T_d \sim \text{Exp}(\kappa + \omega)$ but sees its funding exhausted, and then searches for new capital, which has duration $T_s \sim \text{Exp}(\lambda + \nu)$. The time between two successive funding rounds is therefore $T_{br} = T_d + T_s$, the sum of two exponentially distributed random variables. Such a random variable is said to have a hypoexponential distribution, inheriting the rate parameters of the respective exponential distribution from which it derives. That is, $T_{br} \sim \text{Hypo}(\kappa + \omega, \lambda + \nu)$. This leads directly to the next proposition

**Proposition 2.4.** The distribution of the duration between successive funding rounds, $T_{br} \sim \text{Hypo}(\kappa + \omega, \lambda + \nu)$.

**Corollary 2.5.** The expected duration between successive funding rounds for a given firm, $E[T_{br}] = \frac{1}{\kappa + \omega} + \frac{1}{\lambda + \nu}$, is: strictly decreasing in $\nu$, and $k$; strictly increasing in $p$ and $\pi$; and ambiguous with respect to $\lambda$ and $\kappa$. Furthermore, holding $p \times \pi$ constant, $E[T_{br}]$ is strictly increasing in $p$.

Again, it is fairly simple to see the implications of tighter capital markets on funding patterns. When markets are tight, $\nu \downarrow$, the expected contract duration lengthens and the expected time in search for new funding lengthens. This implies a rightward shift in the distribution of the time between rounds, as shown in Figure 1. These comparative statics results, beyond building intuition for the model, will be useful when mapping to data in section 3.

![Figure 1: Distribution of the number of and time between funding rounds](image)

The figure depicts the distribution of the number of funding rounds and time between rounds for a baseline parameterisation in which $\omega \in (0, \infty)$ and a second parameterisation in which the meeting rate, $\nu$, has been lowered.

### 2.3.2 Endogenous start-up outcomes.

I now turn attention to start-up outcomes. In the model, start-ups may fail for one of two reasons: either the firm obtains a negative result or it fails to secure financing. I denote by $p_d$ and $p_s$ the
probability that firm currently in productive development or search, respectively, is successful with their project at some point. The structure of the model allows for writing these probabilities as the solution to the following recursive equations

\[ p_d = \frac{\kappa}{\kappa + \omega} p + \frac{\omega}{\kappa + \omega} p_s, \quad p_s = \frac{\nu}{\lambda + \nu} p_d. \]

Specifically, a firm currently in development can succeed in its current financing round, with probability \( \kappa \), or it transitions to search and then is successful with probability \( p_s \). Similarly, a firm currently in search transitions to development with probability \( \nu \), in which case its success probability is \( p_d \).

This representation leads directly to the next proposition

**Proposition 2.5.** The (steady-state) probabilities of success for a firm in productive development, \( p_d \), and in search, \( p_s \), are

\[ p_d = \kappa \frac{\lambda + \nu}{\omega \lambda + \kappa (\lambda + \nu)} p, \quad p_s = \frac{\nu}{\lambda + \nu} p_d \]

where \( p_d, p_s < p \) for \( \omega \in (0, \infty) \). Furthermore,

1. (i) \( \frac{dp_d}{dp} \leq 0 \). \( \exists \nu_d \geq 0 \) such that if \( \nu \geq \nu_d \), then \( \frac{dp_d}{dp} > 0 \), and otherwise \( \frac{dp_d}{dp} < 0 \).
2. (ii) \( \frac{dp_s}{dp} \leq 0 \). \( \exists \nu_s \geq 0 \) such that if \( \nu \geq \nu_s \), then \( \frac{dp_s}{dp} > 0 \), and otherwise \( \frac{dp_s}{dp} < 0 \).
3. (iii) \( \frac{dp_d}{dp} > 0 \) and \( \frac{dp_s}{dp} > 0 \).
4. (iv) Holding \( p \times \pi \) fixed, \( \frac{d}{dp} \left( \frac{p_d}{p} \right) = \frac{d}{dp} \frac{\kappa (\lambda + \nu)}{\omega \lambda + \kappa (\lambda + \nu)} > 0 \).

Proposition 2.5 analyses the effect of financing conditions, proxied by the meeting rate \( \nu \), and the friction-free success probability, \( p \), on start-up outcomes. The result is quite striking: the probability of success in the model is not everywhere increasing in the meeting rate in the financing market. Put differently, if financing becomes easier to access, \( \nu \uparrow \), this does necessarily translate into better start-up outcomes. This obtains because, in response to an improvement in capital market conditions, \( \nu \uparrow \), firms write shorter contracts, \( \omega \uparrow \). Thus, while the probability that a firm fails to secure funding any given time it visits the capital market, it visits the capital market more often and this latter effect can dominate, leading to a heightened risk of failure from financing risk.\[^{23}\]

Furthermore, Proposition 2.5 tells us that this contract adjustment effect is more likely to dominate when the financing market is already fairly tight (low \( \nu \)). This logic becomes important when considering the implications of market tightness on start-up outcomes.

The second key result in Proposition 2.5 relates to the interaction between financing and project types, or the direction of innovation. Part (iv) states that the ratio of the probability of success for a firm in development in the economy with frictions, \( p_d \), relative to the frictionless probability, \( p \), is lower for more radical innovations, holding fixed the frictionless NPV of the project (i.e holding fixed \( p \times \pi \)). In other words, the frictions worsen outcomes but the effect is not homogeneous across technologies; instead, the effect of the frictions is amplified for more novel, or radical, innovations. Intuitively, this

\[^{23}\]To build intuition, suppose that \( \nu \) is initially such that the contract specifies \( \omega^* = 0 \), but that \( \omega^*(\nu) > 0 \), so that the contract is on the threshold between full upfront financing and staged-financing. Then initially \( p_d = p \) but any increase in \( \nu \) leads to \( \omega > 0 \), so that \( p_d < p \). This logic generalises for \( \omega > 0 \) over a subset of the parameter space.
reflects more pronounced agency frictions for firms developing more radical innovations, which means that they raise less capital (Corollary 2.3) and must visit the capital market more frequently to seek funding (2.4). This increases their exposure to financing risk and means that they are more likely to fail due to a lack of funding.

2.3.3 Endogenous entry

Before moving to consider the full equilibrium model, I analyse the effect of model parameters of the flow rate of entry, \( V_s/\bar{\sigma} \).

**Proposition 2.6.** The flow rate of entry, \( V_s/\bar{\sigma} \), is: strictly increasing in \( \nu, p, \pi \) and \( \kappa \); and strictly decreasing in \( k, \lambda \) and \( \bar{\sigma} \). Furthermore, holding \( p \times \pi \) constant, \( V_s/\bar{\sigma} \) is increasing in \( p \).

Proposition 2.6 yields intuitive results that do not require much explanation. Entry is higher when: (i) financing is easier to access, \( \nu \uparrow \) or \( \lambda \downarrow \); (ii) projects have higher frictionless NPV, \( p \uparrow , \pi \uparrow , \kappa \uparrow \) and \( k \downarrow \); or (iii) entry costs fall \( \bar{\sigma} \downarrow \). In addition to these intuitive results, the model also has implications for diverging entry incentives across technologies. Indeed, holding \( p \times \pi \) fixed, an environments in which projects are less radical, in the sense of higher \( p \) and lower \( \pi \), experience higher entry rates. This follows as an implication of the preceding analysis. Proposition 2.5 shows that more radical projects are less likely to succeed, compared to their frictionless success rates. Furthermore, the wedge \( \kappa(1-p)/(\kappa+\omega) \) is decreasing in \( p \), for fixed contract rate \( \omega \), implying that these firms must give up more equity to obtain the same contract as a firm with a less radical innovation. \( ^{24} \) Taken together, this suggests that agency frictions specifically hinder entry incentives for more radical projects.

2.4 Equilibrium dynamics

The preceding analysis has taken the meeting rate in the financing market, \( \nu \), as given. In this section, I develop results related to the dynamics of the economy in equilibrium. I begin by providing some insights into the effect of the contract choice on the equilibrium meeting rate, \( \nu \). This is useful for building intuition for the equilibrium properties of the model and the results will prove useful in interpreting the efficiency results I derive in the next section. Next, I consider the effect of changes to capital supply on the market equilibrium. The supply of VC is known to be highly cyclical (Janeway et al., 2021) and these results help to shine light on the implications of these swings in capital supply on start-up outcomes. In the analysis that follows, I hold entry constant in order to obtain clean expressions that provide insights into the problem at hand. \( ^{25} \)

**The contract rate in equilibrium.** Consider what happens when the contract rate, \( \omega \), increases marginally. Firstly, the marginal firm conducting development sees its funding withdrawn; among these firms, those conducting productive development renew their search for capital. If there is congestion in the market, \( \alpha < 1 \), then this puts downwards pressure on the equilibrium meeting rate. However,

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24 This is clear from inspection of the VC’s participation constraint, which pins down their equity stake, \( \varsigma \).  
25 Future work will consider the equilibrium implications of endogenous entry in response to capital shocks.
at the same time the marginal VC is freed up to meet with another start-ups, which pushes up the meeting rate. Furthermore, for every start-up that returns to search, $1 + \frac{\kappa(1-p)}{\omega}$ VCs return to the supply side of the market, which magnifies the effect.\footnote{This follows because unproductive start-ups do not renew their search for capital} As I show below, which of these two effects dominates depends, among other things, on market tightness. For ease of reference, I will refer to this as the rebalancing effect. Secondly, an increase in the contract rate increases the share of VCs that are needed to fund the same measure of productive firms and the surplus VCs can re-enter the supply side of the market. This effect, which I refer to as the reallocation effect, increases the meeting rate unambiguously.

To explore this issue more formally, consider differentiating the meeting rate, $\nu = \mu_s^{\alpha-1}\mu_{vc}^\beta$ with respect to $\omega$, noting that the measures $\mu_s$ and $\mu_{vc}$ depend on $\omega$ via the steady-state conditions.

$$\frac{d\nu}{d\omega} = (\alpha - 1) \frac{\nu}{\mu_s} \frac{d\mu_s}{d\omega} - \beta \frac{\nu}{\mu_{vc}} \frac{d\mu_{vc}}{d\omega}$$

In order to analyse this expression, it will be helpful to isolate the two separate effects discussed above. To this end, recall the steady-state conditions, where I denote the fixed flow rate of entry by $\Lambda$

$$\mu_d^p : \quad \mu_s^\alpha \mu_{vc}^\beta = (\kappa + \omega) \mu_d^p$$
$$\mu_u^p : \quad \kappa (1-p) \mu_u^p = \omega \mu_d^u$$
$$\mu_s : \quad \omega \mu_d^p + \Lambda = (\lambda + \mu_s^{\alpha-1}\mu_{vc}^\beta) \mu_s$$

where I have substituted for $\nu$, and $\mu_{vc} = M - \mu_d^p - \mu_u^p$. The rebalancing effect shows up in the appearance $\omega$ in the steady-state conditions for $\mu_d^p$ and $\mu_s$, because these expressions determine whether start-ups that are yet to obtain a result are searching for capital or conducting development. The reallocation effect shows up in the steady-stage condition for $\mu_u^p$, because this determines the share of encumbered VCs that are funding unproductive firms. The next proposition characterises the effect of the contract rate, $\omega$, on the meeting rate, $\nu$, by use of the implicit function theorem to obtain the expressions in equation (13), and decomposes the effect into the rebalancing and reallocation terms.

**Proposition 2.7.** In the economy with a fixed flow of entry, $\Lambda$, the effect of the contract rate, $\omega$, on the meeting rate, $\nu$, can be decomposed as

$$\frac{d\nu}{d\omega} = \text{rebalancing effect} + \text{reallocation effect}$$

where $\text{sign(rebalancing effect)} = \text{sign}\left(\beta \theta \lambda \left[1 + \frac{\kappa(1-p)}{\omega}\right] - (1 - \alpha)\kappa\right)$, the reallocation effect is strictly positive, and $\theta = \mu_s/\mu_{vc}$ is market tightness. In general, the sign of $\frac{d\nu}{d\omega}$ is ambiguous.

When $\omega$ increases, the rebalancing effect increases both the measure of firms searching for capital and the measure of VCs able to supply it. In principle, this can put upward or downward pressure on the meeting rate, $\nu$. As can be seen from Proposition 2.7, increases in market tightness, $\theta$, push in the positive direction because the flow of matches becomes more sensitive to the supply side of the market.
market. This effect becomes more powerful as the elasticity of the matching function with respect to the measure of VCs, \( \beta \), increases. Furthermore, the contract rate shows up because a marginal change in \( \omega \) reintroduces \( 1 + \frac{\kappa(1-p)}{\omega} \) VCs to the supply side of the market for every start-up that renews its search.\(^{27}\)

Bringing these insights together, the effect of the contract rate is more likely to be positive in cold, rather than in hot, financing markets. Cold markets, associated with high \( \theta \), have a direct effect as described, but also imply that the equilibrium meeting rate, \( \nu \), is likely to be low. In turn, this implies a low contract rate, \( \omega \), by Proposition 2.2, which increases \( 1 + \frac{\kappa(1-p)}{\omega} \). In the next section, I discuss the implications of these results for the efficiency of contracts.

**The supply of VCs in equilibrium.** I now study how the economy responds to an exogenous shift in the supply of VCs, \( M \). I focus on comparison of steady-states, which keeps the problem tractable. Consider an exogenous decrease in the measure of VCs, \( M \). The partial effect of \( M \) on the meeting rate, \( \nu \), is strictly positive, \( \frac{\partial \nu}{\partial M} = \beta\mu_\alpha\mu_\alpha^{-1}(M - \mu_\beta\mu_\beta^{-1})^\beta > 0 \). Then, by Proposition 2.2, we should expect the fall in \( M \) to lead to a contract adjustment, in the direction of lower \( \omega \), or equivalently, longer duration contracts that involve greater capital commitments.

At first inspection, this conclusion seems somewhat paradoxical: firms raise more capital when less is seemingly available. This apparent contradiction is squared through consideration of equilibrium effects. Indeed, the adjustment in contracts feeds back into the capital market equilibrium, affecting not only the meeting rate, \( \nu \) (Proposition 2.7), but also the measures of firms in search and development. The implications of this feedback are described in the next proposition.

**Proposition 2.8.** In the economy with a fixed flow of entrants, \( \Lambda \), the effect of a change in the supply of VCs, \( M \), on the flow of matches, \( m(\mu_s, \mu_{vc}) \) is amplified by the endogenous response of contracts.

\[
\frac{dm(\mu_s, \mu_{vc})}{dM} > \frac{dm(\mu_s, \mu_{vc})}{dM} \bigg|_{d \omega = 0} > 0
\]

Proposition 2.8 conducts the following exercise. Consider a steady-state fall in the supply of VCs and compare two settings: (i) contracts are held fixed at their initial level, or (ii) contracts are allowed to adjust endogenously. In both cases, there will naturally be an decrease in the flow of matches, the model equivalent to the number of deals, because the direct effect of a fall in \( M \) is to lower the meeting rate, \( \nu \). However, in the second case, contracts also adjust and, specifically, firms raise more capital (write longer contracts, \( \omega \downarrow \)) each time they visit the capital market. This puts downward pressure on the steady-state measure of VCs and implies that the flow of new deals is *even lower* than it would be in the counterfactual economy with no contract adjustment.

These sort of equilibrium effects appear to be relevant to the functioning of the venture capital market. Nanda and Rhodes-Kropf (2017) state that “when venture capital investing dried up in 2009, anecdotal evidence suggests that investors told some firms that they would only invest if the firm took an extra large amount of money relative to their annual capital need to make sure they would not have to

\[^{27}\]The VCs that are funding unproductive start-ups return to the supply side of the market, but the unproductive start-ups do not begin to search for capital.
come back to the financial markets for an extended period. Thus, while many firms were finding it
impossible to raise money others were being asked to take enough for multiple years”. In the model,
this is exactly what occurs. On the intensive margin, firms that raise capital raise more of it in order
to insure themselves against the need to revisit the capital market in the near future. However, this
intensive margin adjustment provokes an adjustment on the extensive margin: fewer firms gain the
opportunity to raise capital at all. In this next section, I analyse the efficiency properties of the model
and discuss whether these dynamics are efficient in welfare sense.

2.5 Efficiency

In this section, I discuss the efficiency properties of the model. The economy is clearly inefficient
because the frictionless contract is infeasible and inefficient continuation implies that capital is wasted.
I begin by discussing factors that affect the extent of capital misallocation in the economy. However, in
light of the frictions, a degree of misallocation is inevitable. Therefore, I study next whether there are
potential welfare gains associated with changes in the contracts that agents choose in the decentralised
equilibrium, taking the existence of financing frictions as given.

Capital misallocation. Relative to an economy without frictions, capital is misallocated in the model
because start-ups that have received negative news about their prospects - the “unproductive” start-ups
- do not inform their investors and so continue to invest in projects with no prospects. The capital
would be better invested in a different, productive start-up. Nevertheless, a high degree of capital
misallocation is indicative of issues in access to funding. To see this, note that equation (10)
allows us to write the share of firms in unproductive development in the baseline model as

$$\frac{\mu_{ud}^u}{\mu_{ud}^u + \mu_{ud}^p} = \frac{\kappa(1-p)}{\kappa(1-p) + \omega}$$ (15)

so that capital misallocation, when understood as the share of firms receiving funding that are un-
productive, is decreasing in $\omega$. This leads to the following Corollary (of Proposition 2.2), which
characterises factors that affect the extent of capital misallocation in the model.

Corollary 2.6. Capital misallocation is: strictly increasing in $\pi$ and $\lambda$; strictly decreasing in $\nu$ and $k$;
and ambiguous with respect to $\kappa$ and $p$.

In short, factors that push up the contract rate, $\omega$, such as looser financing conditions, $\nu \uparrow$, reduce capital
misallocation, whereas factors that lead to a lower contract rate typically increase misallocation.29
Therefore, Corollary 2.6 implies that in economies with tight funding markets, $\nu \downarrow$, a greater share of
invested capital is misallocated.

Efficiency of contracts. From equation (15), it is clear that some capital misallocation is inevitable

28 In the baseline model where every funding round involves the same amount of capital raised, this is equivalent to the
share of capital that is misallocated.

29 The effect of $p$ is ambiguous because on the one hand, $\omega'(p) < 0$, so that increases in $p$ lead to greater misallocation,
but there is also the direct effect of $p$, which reduces the likelihood of entering the state of inefficient continuation.
in the frictional economy unless \( \omega \to \infty \), which is only optimal when there are no matching frictions, \( \nu \to \infty \). In light of this, it is natural to wonder whether the extent of capital misallocation in the decentralised equilibrium is optimal, given the frictions that agents face.

To analyse this question, I define social welfare, \( H \), as the discounted value of aggregate income flows of start-ups net of investment and entry costs. In the decentralised equilibrium, agents face a trade-off when choosing the contract rate, \( \omega \). Securing financing for a longer duration insures against the need to seek follow-on financing, limiting their exposure to financing risk. However, as the funding duration rises, the effective capital cost rises through the wedge, \( \frac{\kappa (1-p)}{\rho + \omega} \), making securing funding increasingly costly for the start-up; they must give up more equity per unit of expected funding horizon. The natural question arises: would a planner, subject to the same frictions, trade-off these two frictions in the same way?

To study this question, I consider how adjusting a time-invariant contract rate, \( \omega \), affects the social optimum. I do so assuming that entry is time-invariant and is chosen optimally by the planner. This implies that I ignore the effect of the contract rate on equilibrium entry, which is a potentially important determinant of the socially optimal contract. Albeit reductive, this approach lets me focus on the first round effects on market equilibrium and greatly clarifies the exposition.

To this end, denote by \( H(\mu_{s,t}, \mu_{d,t}^p, \mu_{d,t}^u; \omega, \Lambda) \) the social welfare function of the planner, where the contract rate, \( \omega \), and the flow rate of entry, \( \Lambda \), are taken as given. The current value Hamiltonian is given by

\[
H = \mu_{d,t}^p \left[ \kappa p \pi - k \right] - \mu_{d,t}^u k - \frac{\sigma}{2} \Lambda^2 + \gamma_{d,t}^p \dot{\mu}_{d,t}^p + \gamma_{d,t}^u \dot{\mu}_{d,t}^u + \gamma_{s,t} \dot{s}_{s,t}
\]

(16)

where

\[
\dot{\mu}_{d,t}^p = m(\mu_{s,t}, \Lambda - \mu_{d,t}^p - \mu_{d,t}^u) - (\kappa + \omega) \mu_{d,t}^p
\]

\[
\dot{\mu}_{d,t}^u = \kappa (1-p) \mu_{d,t}^p - \omega \mu_{d,t}^u
\]

\[
\dot{s}_{s,t} = \omega \mu_{d,t}^p + \Lambda - \lambda \mu_{s,t} - m(\mu_{s,t}, \Lambda - \mu_{d,t}^p - \mu_{d,t}^u)
\]

In equation (16), \( \gamma_{d,t}^p \), \( \gamma_{d,t}^u \), and \( \gamma_{s,t} \) are the co-state variables associated with each state variable. The first three terms reflect, respectively, the flow payoff deriving from firms in productive development, the capital cost associated with firms in unproductive development, and the flow entry cost.

To save on notation, going forwards I drop \( t \)-subscripts. Given that \( \Lambda \) is assumed to be chosen optimally, to understand the effect of the contract rate, \( \omega \), on social welfare it is sufficient to partially differentiate

\[\frac{\partial H}{\partial \omega} = 0\]

(17)

Focus on the time-invariant solution incurs some loss of generality because out of steady state, the distribution of contract rates is important for understanding equilibrium dynamics. However, the fact that I focus on the steady-state equilibrium implies that little is lost via this approach.

More formally, understanding the total effect of the contract rate on social welfare would entail considering \( \frac{\partial H}{\partial \omega} = \frac{\partial H}{\partial \Lambda} \frac{\partial \Lambda}{\partial \omega} \), where \( \Lambda \) is the flow of entry. In this section, I assume that entry is chosen optimally so that \( \frac{\partial H}{\partial \Lambda} = 0 \). In section A of the Appendix, I show that entry in the decentralised equilibrium is generally inefficient, so that \( \frac{\partial H}{\partial \Lambda} = 0 \) is not satisfied. An equivalent interpretation is that I consider a counterfactual economy with exogenous entry, so that \( \frac{\partial H}{\partial \Lambda} = 0 \).

Recall that entry is specified by a cutoff rule. If the flow of entry is \( \Lambda \), then the cutoff must be \( \tilde{c} = \tilde{\sigma} \Lambda \). Then, total flow entry costs are calculated as \( \int_0^\tilde{c} c/\tilde{\sigma} \, dc = \frac{\tilde{\sigma}}{2} \Lambda^2 \), which follows from the distribution of \( c \).
equation (16) with respect to $\omega$ to obtain

$$\frac{\partial H}{\partial \omega} = \mu_s^p (\gamma_s - \gamma_d^p) - \gamma_d^u \mu_u^d.$$  

(17)

The expression makes it clear the welfare effect of a marginal increase in the contract rate, $\omega$. As the contract rate rises, the marginal firm is shifted from productive development to search (the first term). This is welfare reducing because firms are more ‘valuable’ in productive development than in search. At the same time, a higher contract rate moves the marginal firm out of unproductive development, which is beneficial because fewer resources are wasted and VCs are freed up to finance other firms. The next Proposition provides a condition under which $\frac{\partial H}{\partial \omega} = 0$, such that there are no welfare gains associated with a marginal increase in the contract rate.

Proposition 2.9. In steady state and given that entry is chosen optimally by the planner or, equivalently, that entry is set exogenously, there are no welfare gains associated with marginal distortions to the contract rate if the following condition is met

$$\kappa (1 - p) \frac{k}{(\rho + \omega)^2} = \frac{X_1}{X_2 + \left( (\rho + \kappa) (\rho + \lambda + \nu) + (\rho + \lambda) \omega \right)}$$  

(18)

where

$$X_1 = \frac{\omega}{\rho + \omega} \left[ 1 - \frac{\kappa (1 - p)}{\rho + \omega} \frac{1}{\omega} \beta \nu \theta \right]$$

$$X_2 = \nu \left[ \beta \theta (\rho + \lambda) \left( 1 + \frac{\kappa (1 - p)}{\rho + \omega} \right) - (1 - \alpha) (\rho + \kappa) \right]$$

and $\theta = \mu_s / \mu_{vc}$ is an index of market tightness.

To understand the implications of Proposition 2.9, it is helpful to derive an analogous expression for the decentralised equilibrium. To this end, given an interior solution, $\omega$ is determined by equation (4). To obtain an expression purely in terms of $\omega$ and model parameters, note that $V_s$ can be written as a function of $\omega$ by combining equations (7) and (8). Substituting this in for $V_s$ in equation (4) and some algebra obtains the expression

$$\frac{\kappa (1 - p)}{(\rho + \omega)^2} \frac{k}{(\rho + \omega)^2} = \frac{(\rho + \lambda) \left[ \kappa \rho - \left( 1 + \frac{\kappa (1 - p)}{\rho + \omega} \right) k \right]}{(\rho + \kappa) (\rho + \lambda + \nu) + (\rho + \lambda) \omega}$$  

(19)

which determines $\omega$ in the decentralised equilibrium when the optimal value is interior.

Comparing equations (18) and (19), the right-hand side of the optimality condition does not contain the terms $X_1$ and $X_2$. Consider first the term $X_1$, which is strictly less than one and so unambiguously pushes $\omega$ in the decentralised equilibrium below the level that would induce $\frac{\partial H}{\partial \omega} = 0$. In steady-state, $\mu_d^u = \kappa (1 - p) \mu_d^p$ so that the share of firms receiving funding that are unproductive is decreasing in $\omega$: a higher contract rate helps to reduce capital misallocation. Agents in the decentralised equilibrium internalise the direct effect of this because, as $\omega$ increases, the likelihood of inefficient continuation falls and so the expected capital cost falls as well. This is exactly what motivates staged financing in the
model. However, agents fail to internalise the indirect effect on the market equilibrium - by funding an unproductive start-up, a VC is unable to meet with a productive start-up and so the meeting rate in the market falls. This effect is strong when: (i) there are more unproductive start-ups receiving funding, \( \frac{(1-p)}{\omega} \uparrow \); (ii) these start-ups see their funding withdrawn at a low rate, \( \frac{1}{\rho+\omega} \uparrow \); or (iii) the sensitivity of meetings to the measure of VCs is high, \( \frac{\partial m}{\partial \mu_{vc}} = \beta \nu \theta \uparrow \). Therefore, via this channel alone, the planner would prefer a higher meeting rate than private agents.

In addition, there is a second effect, which is less clear cut. \( X_2 \) has ambiguous sign and could, therefore, push the decentralised contract rate above or below the social optimum. Note that setting \( \rho = 0 \) in \( X_2 \) delivers an expression analogous to that Proposition 2.7. \( X_2 \) captures the rebalancing effect. Therefore, up to an approximation in \( \rho = 0 \), the sign of \( X_2 \) is equal to the sign of \( \frac{\partial \nu}{\partial \omega} \) when \( \frac{\partial \nu}{\partial \omega} \) is computed holding fixed the extent of capital misallocation (i.e. there is not reallocation effect). When this is positive, the effect of \( X_2 \) goes in the same direction as \( X_1 \), suggesting that contracts feature a contract rate, \( \omega \), that is inefficiently low.

**Discussion.** Recalling the discussion in Section 2.4, \( X_2 \) is more likely to be positive when funding markets are tight. The implication is that in tight funding markets, firms are more likely to sign contracts that endow with too much capital, relative to the efficient benchmark. The market, therefore, serves too few firms.

Another way to make this point is to refer back to the analysis of equilibrium effects, where I show that adjustments in contracts amplify the direct effect of changing capital market conditions. As the funding market tightens, the firms that raise capital raise more to insure against financing risk, but this means less capital is available for other firms, amplifying the initial effect on deal counts. The analysis of this section suggests that such a response is likely to be inefficient. Indeed, as the capital market is tightens, \( X_2 \) is more likely to be positive, suggesting that firms write contracts that raise too much capital, relative to the efficient benchmark. Thus, the gains that these firms obtain through more secure funding contracts are likely to be dominated by the negative spillover effects on other firms in the market.

**3 The UK funding gap**

In the preceding section, I present a theory of the market for venture capital and derive a set of predictions. The model relates capital supply to start-up outcomes and entry via observable characteristics of funding patterns. The natural mapping to data implies that the model can be used as a lens through which to analyse funding market conditions in the real world.

Therefore, in this section, I put the model to work to analyse the extent to which start-up outcomes and activity in the UK are adversely affected by the degree of development of the venture capital market, especially when compared to the US. As is well documented, there is less venture capital activity in the UK relative to the US (including when adjusting for market size) and this has led to concerns that

\[33\] It is fairly simple to show that, in an economy with fixed entry, the equilibrium flow of meetings (or deals), \( m(\mu_s, \mu_{vc}) \), is increasing in \( \omega \).
UK start-ups face a ‘funding gap’. However, in practice, the level of VC activity is the outcome of an equilibrium between the supply of capital from VCs and the demand for capital from high quality projects. To tackle this identification challenge, I leverage the fact that the model introduced in the previous section offers various explanations for differing degrees of VC activity across markets. For instance, Proposition 2.6 demonstrates that entry is increasing in capital supply (proxied by $\nu$), but also in factors related to project quality, $(p, \pi)$, whereas it is decreasing in $\tilde{\sigma}$, which relates to entry costs and the supply of entrepreneurial projects. To understand the contribution of financing to the observed differences between the UK and US, I estimate the model separately for the UK and US and conduct a counterfactual exercise in which I increase access to funding for UK start-ups to the level of their US counterparts to consider the effect on outcomes and entry rates. At present, I do this in the partial equilibrium version of the model, treating the meeting rate $\nu$ as a parameter to estimate.\footnote{Future versions of the paper will consider this exercise in equilibrium.}

This section proceeds as follows. I begin by providing an overview of the dataset that I will use to estimate the model. I then consider the UK and US start-up scenes, focussing on the differences in activity levels, outcomes and funding patterns. Next, I discuss two modifications to the baseline model that are necessary to map the model to data before discussing model identification. Finally, I estimate the model and conduct counterfactual exercises.

### 3.1 Data

My sample includes all firms in the Thomson Reuters venture capital dataset from the UK or US that received their first round of venture capital funding between 2005 and 2015 inclusive. A typical problem in analysing young firms is that their outcomes may be censored because insufficient time has passed since their inception to see their final outcomes in the data. The choice of sample period involves a trade-off: the desire to have an up-to-date sample of firms is at odds with the problem of censoring. I settle on the sample of firms receiving their first funding round between 2005 and 2015 so that I observe at least seven years of data for each firm. In order to maintain consistency across the data, I exclude any observations on firms that occur more than seven years after their first funding round. This echoes the approach used in Ewens and Farre-Mensa\cite{ewens2020}, who also consider the status of venture-backed firms seven years after their first funding round.

I apply a number of screens to the data. Financing rounds in the raw data are labelled as one of “seed”, “early stage”, “expansion”, “later stage” or “bridge loan”. I disregard bridge loans because their objective is to tie firms over while they raise their next funding round, rather than for conducting major development steps.\footnote{Despite their labelling, “bridge loans” reported in Thomson Reuters are equity investments.} Next, I label seed and early-stage rounds as “Early” and expansion and later-stage rounds as “Late” and exclude the minority of firms that only have a “Late” funding rounds to maintain consistency with the model.\footnote{I provide definitions for the round types that I include in section B.1 of the Appendix.} Finally, I collapse any two deals for a firm if they are coded...
As I discuss in more detail below, a key modification that I make to the model in order to map to data is to introduce acquisitions as a distinct form of exit from ‘successes’. A success requires that a firm complete its development, whereas an acquisition allows a firm to sell a partially-developed project. In order to map exit outcomes to the model, I define an “acquisition” in the data as an M&A transaction by a non-financial buyer and a “success” as an IPO or an M&A transaction by a financial (including SPAC) buyer. The choice to distinguish between financial and non-financial buyer M&A transactions is motivated by the idea that a non-financial buyer typically has the capabilities to acquire start-ups that have not yet succeeded in commercialising their innovations, whereas a financial buyer is less likely to have such capabilities. This echoes the distinction in the model, in which a “success” requires that the firm reach full development.

<table>
<thead>
<tr>
<th>Country</th>
<th># firms</th>
<th># rounds</th>
<th># successes</th>
<th># acquisitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>11,376</td>
<td>34,360</td>
<td>416</td>
<td>2,111</td>
</tr>
<tr>
<td>UK</td>
<td>1,130</td>
<td>2,419</td>
<td>33</td>
<td>109</td>
</tr>
</tbody>
</table>

Table 1: Data summary

I face two distinct issues of missing data. Firstly, in approximately 10% of rounds for the US (slightly higher for the UK), the investment amount is not reported, which has implications for the calculation of some moments that I use to estimate the model (namely the burn rate and exit multiples). Secondly, deal value is missing in approximately 50% of M&A transactions. To deal with the first problem, I follow a slightly abridged version of the method used in Jagannathan et al. (2022) to impute capital injections. Specifically, I regress funding amount on a funding stage dummy (i.e. “Seed”, “Early Stage”, etc), the previous amount invested and a number of fixed effects. This process is described in more detail in section B.2 of the Appendix. With the estimates from this regression, I impute missing capital injections using fitted values. To deal with the second issue, I first string-match acquisitions to Crunchbase and use the deal value reported their if available. When this is also missing - the majority case - I assume that deal value implies an exit multiple of 1.5 times capital invested, following Kerr et al. (2014). A summary of the final data is provided in Table 1.

3.2 A comparison of UK and US start-up activity

In this section, I provide a brief overview of key differences between start-up activity, outcomes and funding patterns for UK and US start-ups, as shown in Figure 2. All figures plot data on firms who had their first funding round between 2005-2015, except for the series for the number of first funding rounds per year, which I plot up to 2022.

The data shows a clear picture. Firstly, the level of innovative entry in the US, proxied by the number of first funding rounds per year and adjusted for differences in population, is approximately twice the UK level throughout the sample period. Secondly, US start-ups experience better outcomes as measured by

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37 For instance, on some occasions Thomson Reuters reports IPOs that were announced by never occurred.
the share of firms that have a successful exit within seven years of their first funding round. Bringing these insights together, more start-ups have access to venture capital funding in the US compared to the UK and these firms perform better. Finally, the model offers predictions for funding patterns and we observe marked differences across the two countries. The second row of Figure 2 shows the distribution of the duration between the first and second funding round for UK and US firms and the distribution of the number of funding rounds. On average, UK firms experience a longer duration between their funding rounds and conduct fewer rounds in total. Later in this section, these results form part of the identification strategy for the model’s parameters.

Figure 2: UK and US start-up activity, outcomes and funding patterns

The plot shows various statistics for UK and US venture-backed start-ups. All data is truncated at seven years following the first funding round for the firm. Success: IPO or financial buyer acquisition (incl. SPAC). Acquisition: non-financial buyer acquisition. Firm status recorded 7 years following first funding round. Source: Thomson Reuters.

3.3 A more general model

In order to map the model to data, I need to make two modifications to the baseline framework. Firstly, I introduce acquisitions as a distinct form of ‘exit’ for start-ups. In practice, firms do not need to fully develop their technologies in order to profit from their efforts; there exists a market in which firms with partially-developed innovations can sell to incumbent firms who are better placed to commercialise the innovation. This is the predominant form of exit for start-ups. Furthermore, acquisitions substitute for financing: a start-up who is unable to attract funding may choose to sell out to an incumbent firm who can further develop the project. Therefore, to properly capture the effect of financing on start-up activity and outcomes, it is important to allow firms to choose between being acquired and

---

38The patterns are the same if we consider the distribution of the duration between any two rounds, but these observations are not uniformly truncated, so for exposition I focus on the first funding round only here, where the truncation is at seven years for all observations.
self-developing their project.

Secondly, I extend the model of section 2 to include two stages. In the first (or “early”) stage, the firm attempts to overcome some early-stage uncertainty surrounding the project. This could include developing a minimum viable product, conducting first-stage clinical trials, or overcoming some sort of technological uncertainty about their product or service. Having successfully navigated this early-stage uncertainty, the firm begins to search for funding to complete “late-stage” development, where it attempts to commercialise (or scale-up) its project. It is also in this late-stage where the firm meets with potential acquirors and may choose to sell its project 39. In the baseline version of the model, the hazard rate for success is approximately constant in the time since first funding due to the exponential arrival time of a result 40. In the data, the hazard rate is first increasing and then decreasing and the introduction of two stages allows the model to capture this feature of the data. This is particularly important because the model is estimated on censored data; events that occur more than seven years after a firm’s first funding round are excluded (or unavailable). Therefore, to ensure that the choice of censoring horizon does not impact the results, it is crucial that the model is able to capture the shape of the ‘time-to-exit’ distribution.

For clarity, Figure 3 depicts the potential paths that a start-up may take through this process. I now discuss in more detail how these features are included in the model but leave the full model specification to Section C of the Appendix.

![Diagram of start-up lifecycle](image)

**Figure 3: Start-up lifecycle**

**Details of model adjustments.** Full details are provided in Section C of the Appendix; here, I provide an overview and the key equations. In simple terms, the model that I take to the data merely appends an additional stage in the spirit of the model in Section 2 and allows for acquisitions to occur in this second stage. It then becomes necessary to take a stance on which parameters change across stages and which remain constant. This choice reflects a balance between internal consistency, realism and the ability to identify separately various parameters with the available data.

39 This setup mirrors the environment of Arora et al. (2024), where a start-up must overcome both technology risk and commercialisation risk but may sell out to an acquiror before attempting to commercialise its innovation.

40 This statement becomes exact in the case of upfront financing, $\omega = 0$. 

28
To this end, it is clearly necessary to assume that the frictionless success probability, $p$, differs between the early and late stages. I maintain the convention that $p_e$ gives the probability that an early-stage firm would successfully commercialise its innovation absent any frictions and $p_l > p_e$ gives the same probability for a late-stage firm. Then, for these definitions to be consistent, an early-stage firm that resolves its early-stage uncertainty progresses to the late-stage with probability $\hat{p}_e = p_e / p_l$. With regards to the flow capital cost, $k$, Gompers (1995) shows that later-stage firms raise larger amounts of capital, suggesting that “the need for investment in plant and working capital accelerates as the scale of the project expands.” To allow the model to capture this feature of the data, I distinguish between $k_e$ and $k_l$ and anticipate $k_l > k_e$ in the estimation (although I do not impose this). Furthermore, I permit the meeting rate $\nu$ to differ across stages. Together with increasing capital requirements as the firm progresses ($k_l > k_e$), permitting differing meeting rates allows the model to capture parsimoniously the idea that VC funds in certain national markets may be incapable of funding a large number of ‘big ticket’ investments. Such a market would have a relatively low meeting rate in the late-stage funding market, $\nu_l$, because start-ups find it difficult to secure large capital commitments. Finally, I assume that the rate of uncertainty resolution and of failure in search, $\kappa$ and $\lambda$ respectively, do not differ across stages.

With this in mind, the early-stage is exactly as described in Section 2, with two small modifications. Firstly, an early stage firm that has realised a result progresses to the late stage with probability $\hat{p}_e$. Secondly, rather than obtain the payoff $\pi$, the firm obtains the value of entering late-stage search, $V_{s,l}$.

Next, consider the late-stage. The key difference here is that firms have the opportunity to be acquired. I conceptualise acquisitions as the sale of a partially-developed project and, therefore, acquisitions may occur whether the firm is in search or development. Specifically, I assume that for firms in the late stage potential acquirors arrive at rate $\phi$, whether the firm is in search or development. This rate is exogenous and the same for all firms. I suppose that the acquiror values the fully-developed project possessed by the start-up at $\epsilon \cdot \pi$, where $\epsilon \in [0, \infty)$ is a random variable with CDF $F(\cdot)$ (typically exponential) and that $\epsilon$ is match-specific, common knowledge and i.i.d. across matches. However, because the acquiror purchases the start-up before it has reached full development and there is inherent uncertainty as to the start-ups prospects, the acquiror has expected valuation $p_l \cdot (\epsilon \cdot \pi)$. The assumptions imply that this is known to the start-up. The match-specific term $\epsilon$ has a variety of potential interpretations, for instance as a match-specific synergy, or as a reflection of the costs that the acquiror anticipates incurring in developing the project fully. Finally, I assume that the start-up is permitted to make a take-it-or-leave it offer to the potential acquiror.

Given these assumptions, the start-up will set the acquisition price at the maximum of its current value, $V_{i,l}$ for $i \in \{s, d\}$, and the valuation of the potential acquiror. If the former exceeds the latter, the potential acquiror will naturally choose not to acquire the start-up. Bringing these insights together,

41 See, for instance, Quas et al. (2021) for a discussion in the case of Europe as a whole.

42 One could conceive of an alternative scenario in which an entrepreneur does not fully comprehend the potential commercial applications of its innovation and that the arrival of an acquiror provides a signal to the entrepreneur, leading them to update their priors about the expected value of the project. I do not consider such mechanisms in this paper.
the probability that an acquisition occurs conditional on a meeting is given by

In development: \( \Pr(p_l \cdot (\epsilon \cdot \pi) \geq V_{d,l}) = 1 - F(V_{d,l}/(p_l\pi)) \)

In search: \( \Pr(p_l \cdot (\epsilon \cdot \pi) \geq V_{s,l}) = 1 - F(V_{s,l}/(p_l\pi)) \)

which permits defining an effective arrival rate of acquisitions \( \hat{\phi}_i = \phi \cdot [1 - F(V_{i,l}/(p_l\pi))] \) for \( i \in \{s,d\} \). The effective arrival rate of acquisitions modifies the arrival rate of potential acquisitions by the acceptance rules outlined above and so is endogenous. With this in mind, we can write down the HJB equations for the value of a firm in late-stage search and development as follows

\[
\text{Development: } \rho V_{d,l} = \kappa [p_l\pi - V_{d,l}] + \omega_l [V_{s,l} - V_{d,l}] + \hat{\phi}_d \cdot [p_l\pi E[\epsilon|\epsilon > V_{d,l}/(p_l\pi)] - V_{d,l}]
\]

\[
\text{Search: } (\rho + \lambda_l)V_{s,l} = \nu_l [V_{i,l}^M - V_{s,l}] + \hat{\phi}_s \cdot [p_l\pi E[\epsilon|\epsilon > V_{s,l}/(p_l\pi)] - V_{s,l}]
\]

where \( \omega_l \) is the contract rate for late-stage contracts and \( V_{i,l}^M \) is the value of a meeting in the late stage.\(^{43}\) Relative to the baseline model, there is the additional flow value deriving from acquisitions and the conditional expectation obtains because only potential acquirors with sufficiently high synergies result in acquisitions.

**Discussion.** Three points are worth noting here. Firstly, \( V_{s,l} \) now appears in the value function for early-stage development, rather than \( \pi \). Referring back to the comparative statics results of Section 2.3, the results now carry through where \( \pi \) is replaced by \( V_{s,l} \). That is, anything that increases the value of late-stage search, \( V_{s,l} \), such as improvement in late-stage funding conditions, filters back into the value of overcoming early-stage uncertainty and, therefore, affects early-stage funding patterns.\(^{43}\) Secondly, with regards to acquisitions, to the extent that the value of a firm in development exceeds that in search, \( V_{d,l} > V_{s,l} \), the effective arrival rate of acquisitions (i.e. the hazard rate) will be higher in search than in development. Intuitively, a firm in search has worse outside options than a firm in development and, therefore, should be more willing to accept acquisition offers; acquisitions substitute for financing. Finally, firms may write a contract that guarantees sufficient capital to overcome their stage-specific uncertainty, but I do not permit firms in the early-stage to write a contract that guarantees them sufficient funding to reach full development. A firm that overcomes early-stage uncertainty must renew its search for capital in the late stage, which implies that financing risk associated with transitioning to the late-stage is non-insurable. While alternative modelling assumptions are certainly feasible, the choice to model the market in this way reflects a degree of segmentation between the early and late-stage capital markets in terms of the types of investors, which means that start-ups typically have to seek funding from new investors as they attempt to achieve significant scale. This relates to the concept of the “second valley of death” (Wilson et al., 2018), and I am able to generate model-implied estimates for the share of firms that fail to secure funding at this stage.

\(^{43}\)Details of the contract problem in the late-stage are provided in Section 3 of the Appendix. The key deviation is that the expected capital cost now depends on the propensity of acquisitions and acquisition offers. While productive, the firm is acquired at endogenous rate \( \hat{\phi}_d \), which reduces the expected duration of funding. When unproductive, I assume that the VC is aware of potential acquisition offers and that the due-diligence that an acquiror would conduct is sufficient to make public (to the VC) the status of the project, so that an unproductive firm sees its funding withdrawn at rate \( \omega_l + \phi \).\(^{44}\) This leads, for instance, to the conclusion that while improvements in early-stage funding conditions (\( \nu_e \uparrow \)) should reduce the duration between early-stage funding rounds, improvements in late-stage funding conditions (\( \nu_l \uparrow \)) have the opposite effect.
3.4 Model to data

I begin by discussing the estimation strategy. Table 2 lists the partial equilibrium parameters along with their estimated values for the UK and US. I fix the discount rate $\rho = 0.08$ for both countries, which is similar to the value used in Jovanovic and Szentes (2013). The remaining 12 parameters are estimated in two steps. Firstly, denoting by $\Theta$ the set of all parameters excluding $\tilde{\sigma}$, I select a set of moments $m_j$ and minimise the objective function

$$C(\Theta) = \sum_{j \in \Theta} \left( \frac{\tilde{m}_j - m_j(\Theta)}{\frac{1}{2} |\tilde{m}_j| + \frac{1}{2} |m_j(\Theta)|} \right)^2$$  \hspace{1cm} (22)$$

where $\tilde{m}_j$ is the empirical moment and $m_j(\Theta)$ its model counterpart. Then, given these estimates, I am able to recover $\tilde{\sigma}$ as follows. The measure of firms that have paid the entry cost but not yet engaged in a funding round evolves according to

$$\dot{\mu}_{s,e,t} = \frac{V_{s,e,t}}{\tilde{\sigma}} - (\lambda + \nu) \mu_{s,e,t}^0$$

which implies the steady-state condition

$$\tilde{\sigma} = \left( \frac{1}{\lambda + \nu_e} \times \frac{1}{\mu_{s}^0} \right) V_{s,e}$$

where $\lambda, \nu_e$ and $V_{s,e}$ are known given the estimates from the previous step.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>UK</th>
<th>US</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.08</td>
<td>0.08</td>
<td>Discount rate</td>
</tr>
<tr>
<td>$p_e$</td>
<td>0.14</td>
<td>0.13</td>
<td>Early-stage development level</td>
</tr>
<tr>
<td>$k_e$</td>
<td>1.89</td>
<td>2.45</td>
<td>Early-stage flow investment cost</td>
</tr>
<tr>
<td>$\nu_e$</td>
<td>1.78</td>
<td>2.94</td>
<td>Early-stage meeting rate</td>
</tr>
<tr>
<td>$p_l$</td>
<td>0.17</td>
<td>0.21</td>
<td>Late-stage development level</td>
</tr>
<tr>
<td>$k_l$</td>
<td>12.57</td>
<td>18.40</td>
<td>Late-stage flow investment cost</td>
</tr>
<tr>
<td>$\nu_l$</td>
<td>2.68</td>
<td>5.31</td>
<td>Late-stage meeting rate</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.29</td>
<td>0.31</td>
<td>Result arrival rate</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.88</td>
<td>0.35</td>
<td>Failure rate in search</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.19</td>
<td>2.64</td>
<td>Acquisition arrival rate</td>
</tr>
<tr>
<td>$\pi$</td>
<td>359.09</td>
<td>436.96</td>
<td>Payoff in case of success</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.59</td>
<td>0.18</td>
<td>Acquisition price distribution parameter ($\epsilon \sim Exp(1/\xi)$)</td>
</tr>
<tr>
<td>$\tilde{\sigma}$</td>
<td>1.05</td>
<td>4.67</td>
<td>Entry cost distribution parameter</td>
</tr>
</tbody>
</table>

Table 2: Model parameters

I do not observe $\mu_{s}^0$ directly in the data, because my data only includes firms that do actually receive VC funding. However, I do have an empirical counterpart to $\nu_e \mu_{s,e}^0$, the flow of first-time funding rounds; indeed, this is shown in the first row of Figure 2 for the US and UK. Using this insight, I can recover a model-implied estimate for the entry cost distribution as
\[\tilde{\sigma} = \left( \frac{\nu_e}{\lambda + \nu_e} \times \frac{1}{\text{flow first-time rounds}} \right) V_{s,e} \]  

(23)

where \( \nu_e \) has now entered in the numerator as a result of the substitution. In this exercise, I normalise the flow rate of first funding rounds in the US to one. For the UK, I calculate the number of first funding rounds each year from 2005 to 2015 and then adjust it upwards by the ratio of the US to UK population in order to net out size differences between economics, before finally adjusting based on the normalised value for the US.\(^5\)

Although the parameters are identified jointly by all moments that enter equation (22), specific moments are particularly informative about certain parameters. The moments are presented with their model and data values in Table B.3. Consider first the meeting rates, \( \nu_e \) and \( \nu_l \). In a typical search environment, search is observed and so the meeting rates can be computed directed from the data.\(^4\) In the setting considered here, search is latent and this creates an identification challenge. Proposition 2.4 provides a route forwards because it enables direct estimation of \( \lambda + \nu_e \) and \( \lambda + \nu_l + \hat{\phi}_s \) from the data on the duration between funding rounds. In short, the model characterises the distribution of the duration between funding rounds as hypoexponential and the parameters of this distribution can be estimated directly from the data via MLE.\(^4\) I provide more details on this approach in Section B.3 of the Appendix and include these estimates as targetted moments with the aim of providing information on \( \nu_e \) and \( \nu_l \).\(^4\)

Relatedly, Corollary 2.5 states that the duration between rounds is increasing in \( p_i \); since most funding rounds are early-stage rounds, the mean duration between funding rounds is particularly informative of \( p_i \). To illicit information on \( p_l \), I take the insights of Proposition 2.3 and target the average number of late-stage funding rounds conditional on reaching the late-stage, which I expect to be decreasing in \( p_l \). The conditional, rather than unconditional, average removes the effect of \( p_l \) on the probability of reaching the late-stage, which is ambiguous.\(^6\) To pin down \( k_e \), I target the median burn rate, where the burn rate is defined as the rate of cash utilisation between funding rounds.\(^4\) Targeting the median allows for limiting the effect of some incredibly high burn rates in the true data that are not representative and is a statistic more closely related to early-stage rounds, since most rounds are early-stage. For reasonable parameter values, the median burn rate is increasing in \( k_e \).\(^4\) To obtain information on \( k_l \), I target the share of capital that is raised by late-stage firms, which is typically not representative and is a statistic more closely related to early-stage rounds, since most rounds are early-stage. For reasonable parameter values, the median burn rate is increasing in \( k_e \).\(^4\)

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45I choose to normalise by population rather than GDP because I think of \( \tilde{\sigma} \) as capturing the supply side of the market for entrepreneurs, which feels more naturally related to population than output.

46For instance, in a frictional labour market setting, those searching for work are tagged as unemployed. Unemployment durations then provide direction information on the meeting rate.

47In the late stage, the distribution of the time between funding rounds is \( T_{br} \sim Hypo(\kappa + \omega_l + \hat{\phi}_s, \lambda + \nu_l + \hat{\phi}_e) \).

48When obtaining these values from the model, I do not estimate the values from simulated data but simply take the sum of the relevant parameter values.

49As \( p_i \) increases, \( \mu_i \) falls, so firms that realise a result in the early-stage are less likely to progress. Furthermore, this puts upward pressure on \( \omega_e \) by Proposition 2.2. However, at the same time \( V_{s,d} \) rises, which is equivalent to an increase in \( \pi \) from the perspective of Proposition 2.2, and so puts downward pressure on \( \omega_e \). Overall, the effect on \( \omega_e \) is ambiguous and this translates to an ambiguous effect of \( p_i \) on the share of firms that reach late-stage development.

50For example, if a firm raises $10Mn today and raises capital again in two years, then its burn rate is $5Mn/year. In the model, I compute the burn rate for a given round by dividing the total amount of capital raised, \( K_i(\omega_i) \) for \( i \in \{s,d\} \), by the duration between successive rounds.\(^5\)

51Increasing \( k_e \) reduces the time between rounds, pushing up the burn rate. However, Corollary 2.3 shows that \( K_e(\omega_e) \) is ambiguous with respect to \( k_e \). Specifically, \( K_e(\omega_e) \) becomes decreasing in \( k_e \) as \( \omega_e \) becomes very large, but for parameter values consistent with the duration between funding rounds, this effect is dominated.
increasing in $k_1$.

<table>
<thead>
<tr>
<th>Panel A: Targeted Moments</th>
<th>UK Data</th>
<th>Model</th>
<th>US Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate of $\lambda + \nu$</td>
<td>2.70</td>
<td>2.65</td>
<td>3.28</td>
<td>3.29</td>
</tr>
<tr>
<td>Estimate of $\lambda + \nu + \hat{\phi}$</td>
<td>3.58</td>
<td>3.69</td>
<td>5.96</td>
<td>6.05</td>
</tr>
<tr>
<td>Mean duration b/w rounds</td>
<td>1.39</td>
<td>1.42</td>
<td>1.16</td>
<td>1.18</td>
</tr>
<tr>
<td>Mean # late rounds</td>
<td>1.99</td>
<td>1.89</td>
<td>2.48</td>
<td>2.42</td>
</tr>
<tr>
<td>Median burn rate</td>
<td>2.86</td>
<td>2.89</td>
<td>4.50</td>
<td>4.55</td>
</tr>
<tr>
<td>Share of capital in late stage</td>
<td>0.62</td>
<td>0.63</td>
<td>0.61</td>
<td>0.62</td>
</tr>
<tr>
<td>Share funding after year 5</td>
<td>0.12</td>
<td>0.12</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>Acquisition-to-success ratio</td>
<td>3.30</td>
<td>3.28</td>
<td>5.07</td>
<td>5.04</td>
</tr>
<tr>
<td>Share acquisition multiples $&gt; 10X$</td>
<td>0.14</td>
<td>0.14</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>Mean # funding rounds</td>
<td>2.14</td>
<td>2.29</td>
<td>3.02</td>
<td>3.13</td>
</tr>
<tr>
<td>Share of firms to late-stage</td>
<td>0.34</td>
<td>0.34</td>
<td>0.42</td>
<td>0.41</td>
</tr>
<tr>
<td>Mean exit multiple</td>
<td>8.91</td>
<td>8.92</td>
<td>6.22</td>
<td>6.15</td>
</tr>
<tr>
<td># first funding rounds</td>
<td>0.49</td>
<td>0.49</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Untargeted Moments</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Share successful</td>
<td>0.029</td>
</tr>
<tr>
<td>Share acquired</td>
<td>0.10</td>
</tr>
<tr>
<td>Share late rounds</td>
<td>0.32</td>
</tr>
<tr>
<td>Mean time-to-success</td>
<td>3.81</td>
</tr>
<tr>
<td>Mean time-to-acquisition</td>
<td>3.64</td>
</tr>
<tr>
<td># funding rounds</td>
<td>2.48</td>
</tr>
<tr>
<td># funding rounds</td>
<td>2.31</td>
</tr>
<tr>
<td># funding rounds</td>
<td>2.11</td>
</tr>
</tbody>
</table>

Table 3: Empirical and Model Moments

The table provides various targeted moments (Panel A) and untargeted moments (Panel B) for data and model estimates for the UK and US. Estimation is carried out separately for each country. Moments in both cases are computed based on censored data, where observations on firms that occur more than seven years after their first funding round are excluded.

Consider next $\lambda$, which I would ideally identify using information on failures. However, such data is generally unreliable as many firms remain “living-dead” \cite{Kerr2014}. With this in mind, I target the share of firms that undergo a funding round more than five years after their first. Intuitively, as $\lambda$ increases, firms that have to search for follow-on funding are more likely to fail from a lack of capital and so not survive to the five-year mark. Furthermore, Proposition \ref{prop:success} shows that the expected number of funding rounds is decreasing in $\lambda$, so that they are less likely to have a funding round in any given period. For $\phi$, I target the ratio of the number of firms who are acquired relative to those that have a success, as I expect a larger set of potential acquirors (higher $\phi$) to make acquisitions are more common exit strategy. For $\xi$, which is the mean of the exponentially-distributed ‘synergy’ distribution, $\epsilon$, I target the tail of the acquisition exit multiple distribution, where the exit multiple is defined as the sale price divided by the total capital invested. Specifically, I target the share of acquisitions that have an exit multiple in excess of 10X, meaning that their sale price was more than ten times larger than the total capital invested. As the average synergy rises, we should expect this share to rise. Next, for $\pi$, I target the average number of funding rounds, which Proposition \ref{prop:success} suggests is decreasing in $\pi$. 

33
Identification of $\kappa$ is challenging because of the two opposing forces outlined in Proposition 2.2, leading to an ambiguous relationship between $\kappa$ and $\omega$. The theory, therefore, offers only limited guidance in choosing a relevant target moment. In the estimation exercise, I use the share of firms that reach the late-stage to provide information on $\kappa$. The argument is as follows. Differentiating $p_d$ from Proposition 2.5 with respect to $\kappa$, we find that

$$\text{sign}\left(\frac{dp_d}{d\kappa}\right) = \text{sign}(\omega - \kappa \omega'(\kappa))$$

For small $\kappa$, $\omega'(\kappa) < 0$ so this is always positive. Furthermore, in the limit as $\kappa \to \infty$, $\omega \to \infty$ but $\omega'(\kappa) \to 0$, so again the right-hand side is positive. It is only for intermediate values of $\kappa$ that $\omega - \kappa \omega'(\kappa) < 0$ is possible and, depending on the parameterisation, it may always be positive. Moving to the full model, $p_d$ is analogous to the share of firms that are successful in navigating the early-stage. Furthermore, $\kappa$ also enters the late stage and Proposition 2.6 suggests that $V_{s,l}$ is increasing in $\kappa$, so firms will write longer contracts to increase their likelihood of navigating the early stage. This intuition is confirmed through exploration of the parameter space, so that the claim that the share of firms reaching the late stage is increasing in $\kappa$ is fairly robust.

Finally, I also include the average exit multiple, conditional on an exit, as a targeted moment, implying that the estimation is over-identified. While not completely necessary, the inclusion of the exit multiple aids the model in differentiating between $\pi$ and $p$, which are only distinguished in the baseline model through the wedge in $K(\omega)$. The inclusion of the exit multiple, which is much more closely related to $\pi$ than to $p_e$ in the full model, provides increased precision in terms of the estimates of these parameters.

**Model validation.** As Panel A of Table 3 shows, the model is able to capture the targeted features of venture capital data well. In Panel B, I include a series of moments that are left untargeted in the estimation. Four points are worth noting. Firstly, the model is able to capture empirical exit rates and correctly ranks exit outcomes between the UK and US: UK firms perform relatively poorly in both the true data and in the model. Secondly, the model replicates the fact that a higher share of US funding rounds are late-stage, relative to the UK. Thirdly, the model is broadly consistent with the data in terms of the mean time-to-exit, whether from a success or acquisition. This is the case even though the time-to-exit distribution is left fully untargeted. Furthermore, the model correctly replicates the fact that the mean time-to-success exceeds the mean time-to-acquisition for both countries. Finally, the observation that successful start-ups typically undergo more rounds of venture capital financing than unsuccessful ones (Gompers, 1995), and broadly consistent in the data here, is borne out in the model.

Beyond simple untargeted moments, Figure 4 plots the distribution of various outcomes in the data and model and shows that the model is able to capture well the entire distribution of various statistics. One concern with the estimation approach is that the choice of censoring date, set at seven years, may materially affect the results. Specifically, large discrepancies in the timing of funding rounds and in

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52Put differently, if the wedge were taken to be fully exogenous, then funding patterns would be identical in any two parameterisations with the same value of $p \times \pi$.

53For the US, it seems that firms that are acquired actually have marginally fewer rounds than those with no exit in the first seven years following their first funding round. However, the difference is small and firms marked as ‘successes’ clearly undergo more financing rounds.
the distribution of time-to-exit between the model and data would raise concerns that small changes to the censoring time could generate significant divergence in the model and empirical moments. When re-estimating the model, the estimation would likely pick out a markedly different set of parameter values. However, as Figure 4 shows, the model does a good job of capturing the timing of funding rounds, as indicated by the plot of the share of firms still receiving funding after each year, and the time-to-exit distribution. This is reassuring, especially given that only the share of firms receiving funding more than five years after their first round is targeted, whereas the time-to-exit distribution is left fully untargeted. Another concern is that the approach used to provide information on the meeting rates, $\nu_e$ and $\nu_l$, imposes an unreasonable level of structure on the underlying data. Again, the close mapping between simulated data from the model and the true distribution of the duration between funding rounds depicted in Figure 4 is supportive of the approach taken. Finally, the figure shows that the model aligns very closely with the data in terms of the distribution of the number of funding rounds, the burn rate distribution and the distribution of exit multiples.

**Discussion.** Here, I provide a brief discussion of the estimated parameter values, as shown in Table 2, focusing predominately on parameters related to financing conditions, $\nu$ and $\lambda$, and to project type and quality, $p$ and $\pi$.

The estimated meeting rates in the financing market, $\nu_e$ and $\nu_l$, are significantly higher in the US estimation. It takes longer for firms to secure follow-on financing in the UK than the US and the differences are large: absent acquisitions or failure due to a lack of funding, an early stage firm in the UK would expect to secure financing after 6.7 months, compared to 4.1 months in the US. In the late-stage, the differences are even more striking: 4.48 months versus 2.26 months, or roughly twice as long.

Focusing next on the failure rates in search, $\lambda$, the UK rate is more than twice the US estimate. The failure rate in search reflects a firm’s ability to survive without a large capital injection. There are two natural explanations for cross-country differences along this margin. Firstly, it could simply be that a higher share of US VC-backed firms are cash-flow positive, reducing the urgency of raising additional equity financing. Unfortunately, data limitations prevent cross-country comparison of firm-level cash flows. However, for firms that eventually completed an IPO, it is generally possible to see whether they were profitable in the lead up to going public. Given that these firms are typically the most successful, the profitability rates among these firms should provide an upper bound for the general population of firms. Considering US VC-backed start-ups that completed an IPO between 2005-2022, just 18% were profitable on completion of their IPO, suggesting that differences in the share of firms that are cash-flow positive is unlikely to account for differences in estimates of $\lambda$. Secondly, US VC-backed firms may have access to additional financial instruments not readily available to UK start-ups. For instance, González-Uribe and Mann (2024) argue that venture debt is a tool used by firms to “extend the runway” between equity financing rounds. According to data from Statista, the total size of the

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54 Recall that exit multiples that were missing are coded as 1.5X, following Kerr et al. (2014). This explains the large peak in the distribution in the underlying data.

55 These are the unconditional wait times, computed simply by inverting the meeting rates. The conditional probabilities would take into account the diverging failure and acquisition rates. I discuss this in the next section.

56 Data sourced from Jay Ritter’s website (https://site.warrington.ufl.edu/ritter/ipo-data/). Figures based on Table 4d of https://site.warrington.ufl.edu/ritter/files/IPOs-VC-backed.pdf.
Figure 4: Model validation

The figure depicts a series of outcomes in the data and model for the UK (Panel A) and the US (Panel B). All data is censored seven years after a firm’s first funding round. Data on the burn rate is truncated at the 95th percentile to improve visualisation.
UK venture debt market averaged below $1Bn USD between 2017-2022, compared to a level in excess of $25Bn USD in the US. Estimated differences in $\lambda$ are consistent with US start-ups having access to a source of funding that can bridge-the-gap between equity financing rounds, a tool not readily available to UK VC-backed firms.

Interestingly, the estimated differences in access to capital appear more significant than differences in projects, as indicated by the estimates for $p$, $\pi$ and $\kappa$. Differences in $p$ and $\pi$ can reflect differences in the project types - the radicalness of innovations. However, at the cross-country level, they also likely reflect differences in the underlying environment, including factors such as regulation and the supply of skilled labour. Furthermore, exit opportunities are related to the performance of the local stock market and it is well-known that the FTSE has under-performed international rivals since the turn of the millennium. The estimates for $p_e$ and $p_l$ do not differ too much between estimations, but the payoff to start-ups conditional on success, $\pi$, is slightly higher in the US. Nevertheless, any differences are marginal and it would seem at first inspection that observed differences in outcomes and activity in the UK are more closely related to differences in access to financing, and possibly acquisition opportunities, rather than differences in project types. In the next section, I perform counterfactual exercises to explore this further.

### 3.4.1 Analysis and counterfactual exercises

In this section, I explore properties of the estimation. I begin by discussing estimates of the extent of capital misallocation across firms before discussing the role of financing risk in determining firm outcomes. Finally, I conduct a counterfactual exercise where I reduce the extent of the search friction in the UK estimation to the level estimated for the US.

**Capital misallocation.** Table 4 displays various estimates for the extent of capital misallocation based on the output of the model. The first two rows break down misallocation by stage and use variants of the formula in equation (15), adjusted to be appropriate to the general model. In the early stage, the extent of capital misallocation is estimated to be higher in the US, even though the financing market is more liquid ($\nu^USe > \nu^UKe$) and the early-stage contract rate in the US exceeds that for the UK. Instead, the fact that US start-ups are estimated to overcome more uncertainty in the early-stage implies that more of them fail in their attempt, conditional on obtaining a result, and so enter the state of inefficient continuation. In the late-stage, it is the UK market that experiences greater capital misallocation. This is related to the tighter financing conditions in the UK late-stage market, where the estimated meeting rate is significantly lower than the in the US, which pushes down the contract rate.

In the aggregate (i.e. across stages), capital misallocation can be computed based on the share of unproductive firms receiving capital or the share of capital allocated to unproductive firms. Both figures are provided in Table 4. On pure firm counts, capital misallocation is greater in the US because

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58 $\omega^US_e = 0.53$, whereas $\omega^UK_e = 0.46$.
59 See estimates of $p_e$ and $p_l$ from Table 2.
the majority of firms are early stage. However, it is in the UK where more capital is misallocated when weighting by investment amounts, which reflects the fact that later stage firms need more capital to develop \((k_l > k_e)\). This leads to the interesting conclusion that no only is capital relatively more scarce in the UK, as indicated by estimates of the meeting rates, but a higher share of it is misallocated, a result that is implied theoretically by Corollary 2.6.

<table>
<thead>
<tr>
<th></th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early-stage</td>
<td>0.12</td>
<td>0.18</td>
</tr>
<tr>
<td>Late-stage</td>
<td>0.18</td>
<td>0.07</td>
</tr>
<tr>
<td>Aggregate: share of firms</td>
<td>0.13</td>
<td>0.16</td>
</tr>
<tr>
<td>Aggregate: share of capital</td>
<td>0.16</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Table 4: Capital misallocation

The Table provides model-implied estimates of the extent of capital misallocation in the funding market. For the early-stage, this is simply equation (15), where \(p\) is replaced by \(\hat{p}_e\) and \(\omega\) is replaced by \(\omega_e\). For the late-stage, \(p\) is replaced by \(p_l\) and \(\omega\) is replaced by \(\omega_l + \phi\), where \(\phi\) enters due to the assumption that the arrival of a potential acquiror leads funding to be withdrawn for an unproductive firm (see discussion in Section C of the Appendix). In the aggregate, figures are reported based on counts of firms in each stage (share of firms) or the share of firms in each stage weighted by the flow capital cost, \(k_e\) or \(k_l\) (share of capital).

**Counterfactual exercise.** I now conduct a counterfactual exercise in which I increase the meeting rates in the UK funding market to the level estimated for the US. I do this in partial equilibrium, so the analysis does not account for feedback into the meeting rates generated by the endogenous entry and contract responses.

Table 5 displays the results of this exercise. Columns (1) and (2) display figures for reference from the estimations, as in Tables 2 and 3. The remaining columns report figures from counterfactual exercises. In column (3), the meeting rate in the UK estimation is set to the level of the US estimation. Column (4) does the same thing but for the late-stage estimates, whereas column (5) increases both the early and late-stage meeting rates in the UK estimation to their US levels. All figures account for outcomes that occur within the first seven years following a firm’s first funding round in order to maintain consistency across all results presented in this section.

Panel A of Table 5 focusses on start-up outcomes and entry, where entry is proxied by the number of first funding rounds. Notice first that improvements in early-stage funding conditions do little to affect success rates. This is an application of Proposition 2.5. On the one hand, as funding conditions improve, firms find it easier to access capital and so should be more likely to succeed. On the other hand, contracts adjust and firms raise less capital each round, visiting the funding market more frequently, which endogenously increases their exposure to failure. In the UK estimation, both of these effects approximately cancel out in the early-stage. In line with Propositions 2.3 and 2.4, Panel B shows that the average duration between rounds falls, while the average number of funding rounds increases.

Changes in late-stage funding market conditions have a more subtle effect on success rates. From the perspective of an early-stage firm, an improvement in late-stage funding market conditions increases...
Table 5: Counterfactual exercises

The US and UK baseline columns replicate the figures from Table 3 for the baseline model estimation. The final three columns conduct different counterfactual exercises: (i) UK C. Early sets $\nu_{UK}^e = \nu_{US}^e$; (ii) UK C. Early sets $\nu_{UK}^l = \nu_{US}^l$; and (iii) UK C. Both sets $\nu_{UK}^e = \nu_{US}^e$ and $\nu_{UK}^l = \nu_{US}^l$. All observations on firms that occur more than seven years following the first funding round are dropped to maintain consistency with the figures shown in Table 3. In Panel B, “share late rounds” gives the share of funding rounds that are categorised as late-stage. Panel C provides figures for the share of firms that fail in either early or late-stage search. Among firms that fail in late-stage search, it also provides the share of firms (among all firms) that fail when seeking their first late-stage funding round. In all cases, only firms that have at least one funding round are included. Finally, Panel D provides information on the average equity stake retained by founders conditional on a success or acquisition.

The continuation value associated with successfully overcoming early-stage uncertainty and, therefore, leads the firm to raise more early-stage capital each round. Panel C provides figures on the share of firms that fail while searching for funding. As a result of the contract adjustment in the early-stage, the share of firms that fail when searching for early-stage follow-on funding falls from 32% to 24% of all entrants, while a higher share of firms complete early-stage development. In addition to the effect on the early-stage, firms that overcome early-stage development are more likely to successfully meet with a late-stage VC. In the UK baseline, the model-implied estimates suggest that 11% of firms fail when attempting to secure their first late-stage funding round. When late-stage market conditions improve, this falls to 7%. This is important because the risk of failing to secure late-stage funding for the first time is fully uninsurable; thereafter, this risk can be insured against by setting $\omega_l = 0$. Therefore, ensuring that firms are able to navigate this “second equity gap” (Wilson et al., 2018) is key to their outcomes.

Next, notice that improvements in funding market conditions do little to affect the share of exits via acquisition. This reflects the fact that acquisitions and late-stage funding are substitutes in the model. Indeed, when funding market conditions are tight and so the value of a firm in search falls,
firms become more willing to accept lower acquisition prices and so the hazard rate of acquisitions rises. Therefore, improvements to funding market conditions should reduce the acquisition rate via this channel. However, with improved market conditions, fewer firms fail due to a lack of funding (Panel C) and some of these firms are acquired. In practice, these two opposing effects cancel out, so that funding market conditions do little to affect the share of acquisitions.

Combining these insights, funding market conditions have only a minor effect on start-up outcomes: even when both the early and late-stage market conditions in the UK are set to their US levels, 12.5% of firms have a successful exit, relative to over 20% in the baseline US estimation. Any effect that funding does have on exit rates goes via successes, rather than acquisitions. Therefore, innate differences in the market for firms, reflected in estimates of $\phi$, rather than financing, appears to be the key driver of differences in start-up outcomes between the US and UK.

Nevertheless, Panel A demonstrates that funding market conditions are significant in affecting entry, proxied by the number of firms that have access to VC funding. Indeed, when both early and late-stage market conditions improve, the (population adjusted) entry rate in the UK increases above the US level, providing evidence that difficulties in accessing funding is a key driver of the UK’s lagging start-up activity. Entry increases because the improvement in funding market conditions significantly raises the value of projects, moving them closer to the frictionless NPV. This is clear from inspection of Panel C: firms are much less likely to fail from a lack of funding. An implication of this is that entrepreneurs need not give up as much of their companies to raise capital. Panel D provides model-implied estimates of the average share retained by the entrepreneur upon a successful exit (success or acquisition). When conditions improve in both the early and late-stages, there is almost a 50% increase in the share retained by the entrepreneur. With improved financing conditions, the incentives for business creation are greatly enhanced because entrepreneurs retain a more significant share of the payoffs of their innovations.

4 Conclusion

In this paper, I explore the implications of the supply of VC funding on start-up innovation activity, both theoretically and quantitatively. To this end, I develop a model with a rich funding market and show that access to financing has implications for start-up entry and outcomes. I also develop a number of insights into the market equilibrium and efficiency of venture capital contracts.

Using insights from the theoretical analysis, I then estimate the model separately to the US and UK using data on venture capital funding rounds. I show that the model is able to replicate a rich set of features of the data. Through the lens of the model, differences in observable features of venture capital contracts between the UK and US are consistent with a tighter funding market in the UK but only minor differences in the quality and type of projects undertaken by venture backed start-ups in each country. I then perform counterfactual exercise, which suggests that differences in start-up entry rates between the two countries can be traced to the supply side of the capital market.

In future work, the paper will consider more seriously the impact of capital supply on the direction
of start-up innovation. Such an analysis requires a tight interpretation of parameters such as $p$ and $\pi$, which is more difficult in a cross-country setting. Indeed, $p$ could refer to something innate to the project, but also to features of labour markets that make it more difficult to succeed in certain countries. The ideal setting to study this question, therefore, is at the country level, where such factors can be held constant. Future versions of the paper will consider quantitatively how fluctuations in the supply of capital within countries affect the direction of start-up innovation activity over the funding cycle.

Furthermore, I plan to study the question of government intervention more formally. A naive interpretation of the results of Section 3 is that the government should simply intervene to increase the supply of capital in the market. However, one might question the government’s ability to supply “informed capital”, due to human capital deficiencies and the pursuit of alternative objectives. Studying this question in a quantitative environment will aid policymakers with their interventions into the market.
References


Appendix

A Derivations and proofs

The expected discounted capital commitment

Suppose the contract is signed at date $t = 0$. Until time $T = \min\{T_\kappa, T_\omega\} \sim \text{Exp}(\kappa + \omega)$, the E has not obtained a result and funding has not been withdrawn, so the VC pays $k$. Once time $T$ arrives, there are three potential outcomes: (i) $T = T_\omega$, with probability $\omega/(\kappa + \omega)$; (ii) $T = T_\kappa$ and the result is a success, with probability $\kappa p/(\kappa + \omega)$; or (iii) $T = T_\kappa$ and the result is a failure, with probability $\kappa (1-p)/(\kappa + \omega)$. In cases (i) and (ii), funding ceases. In case (iii), the VC continues to pay $k$ until time $T + T_\omega = T_\kappa + T_\omega$. Therefore, from time $t = 0$ to $T$, the flow cost to the E is $e^{-\rho t} k$ from the perspective of time $t = 0$. At time $T$, the probability that the VC continues to pay $k$ is $\kappa (1-p)/(\kappa + \omega)$, in which case they pay $k$ from date $T$ to $T + T_\omega$. The expected capital cost is then computed by taking expectations over $T_\omega \sim \text{Exp}(\omega)$ and $T \sim \text{Exp}(\kappa + \omega)$

$$K(\omega) = \int_0^\infty \left( \int_0^T e^{-\rho t} k \, dt \right. + \frac{\kappa (1-p)}{\kappa + \omega} \int_0^\infty \left( \int_T^{T + T_\omega} e^{-\rho s} k \, ds \right) \omega e^{-\omega T_\omega} \, dT_\omega \left.) \right) (\kappa + \omega) e^{-(\kappa + \omega) T} \, dT$$

$$= \left( 1 + \frac{\kappa (1-p)}{\rho + \omega} \right) \frac{k}{\rho + \kappa + \omega} \quad (24)$$

Contract sufficient conditions

Consider the environment of section 2.1. As discussed in the main text, the E will never report positive results. The contract in the body of the paper makes assumptions such that the E always reports positive results. Specifically, the private benefit is taken to zero in the limit, $x_e \to 0$. The proposition below demonstrates that this is sufficient for the E to always report positive results and never report negative results.

Proposition A.1. For any set of parameters $(\rho > 0, k > 0, \kappa > 0, 0 < p < 1, \pi > 0, V_s \geq 0)$, there exists an $\bar{x}_e$ such that if $x_e < \bar{x}_e$, then the E reports positive results but does not report negative results.

Before stating the proof, I give a brief outline of the argument. In part 1, I derive the payoffs to the E given a contract $(\varsigma, \omega)$ under two cases: (i) the E reports never reports results and (ii) the E reports only positive results. Then, I show that $(\varsigma, \omega)$, the E always prefers the payoff associated with case (ii). In part 2, I derive a condition under which case (ii) is not incentive compatible; that is, a condition under which a commitment to report all results is not credible. In part 3, I demonstrate that for any set of parameters, case (ii) can always be made incentive compatible by choosing a suitably small
Denote by \( V \) is delayed until time \( t = 0 \). Suppose a positive result arrives at time \( t \). Part 3.

Consider the contract problem when the \( E \) commits to report positive results, Part 2.

Lemma 2. \( E \) is to delay the pecuniary payoff, of which they obtain share \( 1 \).

Lemma 1. The value of the firm in development, \( V \) \( V \) ends at date \( T \), regardless of results.

Part 1. Without loss of generality, assume \( x_e < k \). The \( E \) never reports negative results, since \( x_e > 0 \). Suppose a positive result arrives at time \( t \) and call the decision to report a positive result \( \chi = 1 \), where \( \chi = 0 \) reflects the choice not to report a positive result. \( \chi = 0 \) implies that the payoff, \( \pi \), from success is delayed until time \( t + T_\omega \), for \( T_\omega \sim \text{Exp}(\omega) \), so has time- \( t \) expected value \( \frac{\omega}{\rho + \omega} \). If \( \chi = 0 \), the contract ends at date \( T_\omega \), regardless of results.

The value of the firm in development, \( V_d \), is determined analogously to equation (1)

\[
\rho V_d = \begin{cases} 
\kappa [p \pi - V_d] + \omega [V_s - V_d] & \text{if } \chi = 1 \\
\kappa [p \pi - \frac{\omega}{\rho + \omega}] + \omega [V_s - V_d] & \text{if } \chi = 0
\end{cases}
\]  

(25)

Denote by \( V^{np}_d(\chi) \) the non-pecuniary value of the contract to the \( E \). The value to the \( E \) of contract \( (\zeta, \omega) \) is \( V_E(\omega, \zeta, \chi) = (1 - \zeta) V_d(\omega, \chi) + V^{np}_d(\omega, \chi) \), where

\[
V^{np}_d = \begin{cases} 
(1 + \frac{\kappa (1-p)}{\rho + \omega}) \frac{x_e}{\rho + \kappa + \omega} & \text{if } \chi = 1 \\
\frac{x_e}{\rho + \omega} & \text{if } \chi = 0
\end{cases}
\]  

(26)

where the expression for \( V^{np}_d(\omega, \chi) \) is analogous to \( K(\omega, \chi = 1) \) in equation (2). Similarly, \( K(\omega, \chi = 0) = \frac{k}{\rho + \omega} \). Given the binding PC, \( \zeta V_d(\omega, \chi) = K(\omega, \chi) \). The value of the contract to the \( E \) is then

\[
V_E(\omega, \chi) = \begin{cases} 
\frac{\kappa p \pi + \omega V_s}{\rho + \kappa + \omega} - \left(1 + \frac{\kappa (1-p)}{\rho + \omega}\right) \frac{k - x_e}{\rho + \kappa + \omega} & \text{for } \chi = 1 \\
\frac{\kappa p \pi + \omega V_s}{\rho + \kappa + \omega} - \left(1 + \frac{\kappa (1-p)}{\rho + \omega}\right) \frac{k - x_e}{\rho + \omega} & \text{for } \chi = 0
\end{cases}
\]  

(27)

Lemma 1. Given \( 0 < x_e < k \) and for any contract \( (\omega, \zeta) \), \( V_E(\omega, \chi = 1) > V_E(\omega, \chi = 0) \).

Part 2. The benefit to the \( E \) of delay, \( \chi = 0 \), upon observing a positive result is \( \frac{x_e}{\rho + \omega} \). The cost to the \( E \) is to delay the pecuniary payoff, of which they obtain share \( 1 - \zeta \). The delay therefore costs the \( E \) \( E[(1 - e^{-\rho T_\omega}) (1 - \zeta) \pi] = \frac{\rho}{\rho + \omega} (1 - \zeta) \pi \), where expectations are taken over \( T_\omega \).

Lemma 2. \( \chi = 1 \) is incentive compatible if \( 1 - \zeta \geq \frac{x_e}{\rho \pi} \).

Part 3. Consider the contract problem when the \( E \) commits to report positive results, \( \chi = 1 \)

\[
\sup_{\{\omega \in [0, \infty), \zeta \in [0,1]\}} \left\{ \frac{\kappa p \pi + \omega V_s}{\rho + \kappa + \omega} - \left(1 + \frac{\kappa (1-p)}{\rho + \omega}\right) \frac{k - x_e}{\rho + \kappa + \omega} \right\}
\]

s.t. \( 1 - \zeta \geq \frac{x_e}{\rho \pi}, \quad \zeta \frac{\kappa p \pi + \omega V_s}{\rho + \kappa + \omega} = \left(1 + \frac{\kappa (1-p)}{\rho + \omega}\right) \frac{k}{\rho + \kappa + \omega} \)
where $V_s$ is taken as given and the IC constraint from Lemma 2 is imposed. Define $\bar{V}_s = \frac{pκπ-(k-x_e)}{\rho+κ}$. For $V_s ≥ \bar{V}_s$, the objective function is everywhere increasing in $\omega ∈ [0, \infty)$ given $x_e < k$, so the solution is $\omega → \infty$. Restrict attention to $V_s ∈ [0, \bar{V}_s)$. The first-order condition for an interior optimum is

$$\frac{(k-x_e)(1-p)}{(ρ+κ)} + \frac{(k-x_e-κπ)p + V_s(ρ+κ)}{(ρ+κ+ω)^2} = 0$$

which delivers two candidate solutions for $\omega$

$$\omega = \frac{ρκπ - ρV_s(ρ+κ) - (k-x_e)/(κ(1-p)+ρ) ± \sqrt{κ^2(k-x_e)(1-p)(ρκπ-p(k-x_e)-V_s(κ+p))}}{k-x_e-κπρ + V_s(ρ+κ)}$$

The ‘positive’ solution is strictly negative on $V_s ∈ [0, \bar{V}_s)$. Define $\bar{p} = \frac{(k-x_e)(ρ+κ)^2}{κ(πρ^2+(k-x_e)(κ+2ρ))}$ and $V_s = \frac{pκπ-p(k-x_e)}{ρ+κ}$ - $\frac{(k-x_e)(1-p)(ρ+κ)}{ρ^2}$. When $p ≤ \bar{p}$, the ‘negative’ solution is positive for $V_s ∈ [0, \bar{V}_s)$. When $p > \bar{p}$, the solution is positive for $V_s ∈ (\bar{V}_s, V_s)$. The SOC is satisfied in both cases. Conversely, when $p > \bar{p}$, the solution is negative for $V_s ∈ [0, V_s]$. In this case, the optimand is decreasing on $\omega ∈ [0, \infty)$. The solution is then $\omega = 0$.

It remains to check the feasibility and incentive compatibility constraints. To this end, note the following lemma.

**Lemma 3.** If there are parameters such that $c \in (0, 1)$ (i.e. strictly), then $\exists x_e$ such that $\forall x_e ∈ [0, \bar{x}_e)$, $1 - c ≥ \frac{x_e}{ρπ}$.

**Proof of Lemma 3.** The PC gives $c(ω) = \frac{κ(1-p)+ρ+ω}{(p+ω)(κπρ+V_s)}$, so that $c'(ω) < 0$. Furthermore, from the solution to the contract problem, $ω'(x_e) ≤ 0$, with strict inequality when $ω > 0$ is optimal. Together, $c'(x_e) > 0$. Then, suppose the parameters are such that $c ∈ (0, 1)$ and consider reducing $x_e → 0$. The $E$’s share must increase, $(1-c) ↑$, from a positive value, whereas $x_e/(ρπ) → 0$. Therefore, there exists a region $[0, \bar{x}_e)$ such that the IC is satisfied, i.e. $1 - c ≥ \frac{x_e}{ρπ}$.

Suppose first that $ω = 0$ is optimal. Then $c(0) = \frac{κ(1-p)+ρ}{pκπ}$. Clearly, $c(0) > 0$ since $p ∈ (0, 1)$. Note that $c(0) < 1 \iff pκπρ > κ(1-p)+ρ$. Furthermore, if $ω = 0$ is optimal, then $p > \bar{p} \implies pκπρ > ρ^{-1}((k-x_e)(ρ+κ)^2 - pκ(k-x_e)(κ+2ρ))$, so a sufficient condition for $c(0) < 1$ is $pκπρ > κ(1-p)+ρ$ is $ρ^{-1}((k-x_e)(ρ+κ)^2 - pκ(k-x_e)(κ+2ρ)) > κ(1-p)+ρ$. For $x_e < \frac{κ(1-p)(ρ+κ)}{ρ^2+(1-p)(κ+2ρ)}$, this condition is satisfied, so $c(0) ∈ (0, 1)$. Then, by Lemma 3, it is possible to choose a positive $x_e$ sufficiently small such that the IC is satisfied.

Suppose next that $ω ∈ (0, \infty)$ is optimal. Note $c(ω) > 0 \forall ω ∈ (0, \infty)$. It remains to check $c(ω) < 1$, which is equivalent to requiring that the pecuniary return to the E is positive, $(1-c) V_d(ω, χ = 1) =$
\[ V_d(\omega, \chi = 1) - K(\omega, \chi = 1) > 0. \]

Evaluating this expression

\[
V_d(\chi = 1) - \left(1 + \frac{\kappa (1 - p)}{\rho + \omega}\right) \frac{k}{(\rho + \kappa + \omega)} \bigg|_{\omega = \omega^* \in (0, \infty)} = p\pi - \frac{1}{\kappa^2} \left[ \frac{(2k - x_e)\sqrt{k^2(k - x_e)(1-p)(p\kappa \pi - p(k - x_e) - V_s(\kappa + p))}}{k - x_e} \right. \\
+ \kappa \left(2kp - kV_s\rho + \frac{(1 - p)p(k - x_e)x_e\kappa}{\sqrt{k^2(k - x_e)(1-p)(p\kappa \pi - p(k - x_e) - V_s(\kappa + p))}} \right]
\]

This expression is decreasing in \( x_e \) on \( x_e > 0 \) given \( V_s < \frac{\kappa p\pi - p(k - x_e)}{\rho + \kappa} \), which is satisfied whenever \( V_s < \bar{V}_s \) since \( p \in (0, 1) \). Therefore, if \( V_d(\omega, \chi = 1) - K(\omega, \chi = 1) > 0 \) at \( x_e = 0 \), it is positive over some region \( x_e \in [0, \bar{x}_e) \) where \( \bar{x}_e > 0 \). Evaluating \( V_d(\omega, \chi = 1) - K(\omega, \chi = 1) > 0 \) at \( x_e = 0 \) obtain

\[
p\pi - \frac{1}{\kappa} \left( k(2p - 1) + V_s\rho + 2\sqrt{k(1-p)(\kappa p\pi - kV_s(\rho + \kappa))} \right)
\]

which is equivalent to

\[
\kappa p\pi - k(2p - 1) - V_s\rho > 2\sqrt{k(1-p)(\kappa p\pi - kV_s(\rho + \kappa))}
\]

Note that the left-hand side is positive if \( V_s < \frac{\kappa p\pi - k}{\rho} + \frac{2k(1-p)}{\rho} \), which is valid on \( V_s \leq \bar{V}_s \) when \( x_e = 0 \). Therefore, squaring both sides, subtracting the right-hand side from the left-hand side and some algebra obtains

\[
4kV_s(1-p)\kappa + (\kappa p\pi - k - \rho V_s)^2 > 0
\]

which is satisfied given the initial restrictions on parameters (i.e. positivity constraints and \( p \in (0, 1) \)). Therefore, there are parameters such that \( \varsigma \in (0, 1) \), so Lemma 3 applies and there exists an upper bound \( \bar{x}_e \) such that \( \forall x \in [0, \bar{x}_e) \), the IC is slack.

Suppose finally that \( \omega \to \infty \) is optimal. Then \( \lim_{\omega \to \infty} \varsigma(\omega) = 0 \) and so satisfying the IC requires only that \( x_e \leq \rho\pi \), which is easily satisfied.

\textbf{Part 4.} By Lemma 4 the E prefers to payoff associated with commitment for any contract \((\omega, \varsigma)\). Furthermore, for any contract \((\omega, \varsigma)\) and parameters \( (p > 0, k > 0, \kappa > 0, 0 < p < 1, \pi > 0, V_s \geq 0) \), it is possible to find a strictly positive upper bound on \( x_e \) such that the feasibility constraint is slack if \( x_e \) is less than this upper bound. Then, by Lemma 4, there is a (potentially different) upper bound on \( x_e \) such that the IC constraint is also slack if \( x_e \) does not exceed this upper bound. Then, there is an interval \([0, \bar{x}_e)\) such that both the feasibility and incentive compatibility constraints are slack. Since the contract with \( \chi = 1 \) is incentive compatible and for any contract \((\omega, \varsigma)\) \( \chi = 1 \) is preferred ex ante, the optimal contract involves \( \chi = 1 \) on this interval \( x_e \in [0, \bar{x}_e) \). Furthermore, the expected capital cost is given by \( K(\omega, \chi = 1) \) and the optimal contract rate is determined from part 3. ■
Proof of Proposition 2.1

An equilibrium given \( \nu \) is a tuple \((V_d, V_s, V^M, \omega, \varsigma, \mu_s, \mu_{vc})\) satisfying equations (6), (7), (8) and the steady state conditions, where \( \varsigma \) is consistent with the participation constraint and \( K(\omega) \) is given by equation (3). There are two cases: (1) \( \omega \in (0, \infty) \) and (2) \( \omega = 0 \).

Case 1. Given \( \omega \), \( V_s \) and \( V^M \) are determined by the solution to equations (7) and (8). Eliminating \( V^M \) and substituting for \( \omega \in (0, \infty) \) from equation (5) obtains an expression in \( V_s \) only

\[
\frac{k(\rho + \lambda + \nu)}{\nu} V_s = k(1 - p) + p(k\pi - k) - \rho V_s - 2\sqrt{k(1 - p)(p(k\pi - k) - (\rho + k)V_s)}
\]

This equation can be simply rearranged to be of the form

\[
a - bV_s = \sqrt{c - dV_s}
\]

(28)

where

\[
a = \frac{1}{2} \kappa(k(1 - p) + p(k\Pi - k))
\]

\[
b = \frac{1}{2\nu}(\kappa(\rho + \kappa(\rho + \lambda + \nu)))
\]

\[
c = \kappa^2 k(1 - p)p(k\Pi - k)
\]

\[
d = \kappa^2 k(1 - p)(\rho + \kappa)
\]

and \( a > 0, b > 0, c > 0, d > 0 \) given \( p\kappa \pi > k \), which is required for \( \bar{V}_s > 0 \) and so required for \( \omega > 0 \) to be maintained in equilibrium.

Equation (28) has up to two solutions but at most one is consistent with the equilibrium. Squaring both sides of equation (28) and solving for \( V_s \) obtains the candidates solutions

\[
V_s = \frac{2ab - d \pm \sqrt{4b^2c - 4abd + d^2}}{2b^2}
\]

The solution is complex if \( 4b^2c - 4abd + d^2 < 0 \). Substituting for values of \( a, b, c, d \), \( 4b^2c - 4abd + d^2 \geq 0 \) requires

\[
\kappa p\pi \geq \kappa \frac{\rho\nu + \kappa(\rho + \lambda) + \nu}{\rho\nu + \kappa(\rho + \lambda + \nu)}
\]

which is implied by \( \kappa p\pi > k \) and therefore always slack. Next, consider whether either candidate solution is extraneous, which happens when \( a - bV_s \geq 0 \). Given that the solution is real-valued, the ‘negative’ solution always satisfies this condition. However, the ‘positive’ solution only satisfies the condition if \( ad \geq bc \). In terms of the model’s parameters, this implies an upper bound on \( \kappa p\pi \)

\[
\kappa p\pi \leq \frac{k((1 - p)\nu(\rho + \kappa) + p\kappa(\rho + \lambda))}{\kappa(\rho + \lambda)}
\]

However, suppose that this condition were satisfied so that the ‘positive’ solution does indeed solve
equation (28). For this solution to be consistent with equilibrium, it must then be the case that
\[ V_s^+ < \frac{\kappa p \pi - k}{\rho + \kappa} \]
where \( V_s^+ \) is the ‘positive’ solution to equation (28). This in turn requires that
\[ \kappa p \pi \geq \frac{k(4(1-p)\nu(\rho + \kappa) + \kappa(\rho + \lambda))}{\kappa(\rho + \lambda)} \]
and it is clear by inspection that these two conditions on \( \kappa p \pi \) cannot be satisfied simultaneously. Therefore, \( V_s^+ \) can be ruled out.

Next, I consider whether \( V_s^- \) may be consistent with \( V_s \in (\max\{0, V_s\}, \bar{V}_s) \), so that \( \omega \in (0, \infty) \) is optimal. First, suppose that \( p \leq \bar{p} \) where, \( \bar{p} = \frac{k(\rho+\kappa)^2}{\kappa(\pi \rho^2 + k(\kappa + 2\rho))} \), so that \( V_s < 0 \). The condition \( \kappa p \pi > k \) is sufficient for \( V_s^- \in (0, \bar{V}_s) \). Therefore, whenever \( p \leq \bar{p} \) and \( \kappa p \pi > k \) is met, \( V_s^- \) is consistent. Next, suppose \( V_s > 0 \), which in turn implies \( p > \bar{p} \). Comparing \( V_s^- \) to \( V_s \), we obtain a condition on the meeting rate, \( \nu \), that must be satisfied
\[ \nu > -\frac{(\rho + \lambda)((\kappa p \pi - k)^2 - k(1-p)\kappa^2 - 2k(1-p)\rho \kappa)}{k(1-p)\kappa(\rho + \kappa)} \]
Call this lower bound \( \bar{\nu} \). Note that \( p > \bar{p} \) implies \( \bar{\nu} > 0 \). Therefore, we obtain the following condition that is necessary for consistency. Given \( \kappa p \pi > k \), \( \omega \in (0, \infty) \) requires \( p \leq \bar{p} \) or \( p > \bar{p} \) and \( \nu > \bar{\nu} \).

Case 2. Eliminating \( V^M \) from equations (7) and (8) and substituting \( \omega = 0 \) obtains
\[ V_s = \frac{\nu}{\rho + \lambda + \nu} \left( \frac{p \kappa \pi - pk}{\rho + \kappa} - \frac{k(1-p)}{\rho} \right) \]
For this to be consistent with \( \omega = 0 \), it must be that \( V_s < V_s^* \). This requires \( p > \bar{p} \) and \( \nu \leq \bar{\nu} \).

I have shown that given \( \kappa p \pi > k \), the solution for \( (V_s, \omega) \) is unique and characterised by (weakly) positive and finite contract rate, \( \omega \). Given \( V_s \) and \( \omega \), \( V_d \) and \( V^M \) are determined uniquely by equation (1) and (8), respectively. The share, \( \varsigma \), is pinned down uniquely by the PC. ■

**Proof of Corollary 2.1**

When \( p \leq \bar{p} \) or \( p > \bar{p} \) and \( \nu > \bar{\nu} \), the solution for \( \omega \) is positive, finite, and characterised by the first-order condition, \( V_d'(\omega) = K'(\omega) \). Furthermore, \( V_s \) is given by the solution of equations (8) and (8).\(^{64}\) Given \( \omega \), these two equations yield an expression for \( V_s \)

\(^{64}\)This is the same \( \bar{p} \) from the proof of Proposition XXX.

\(^{65}\)An alternative is to simply plug the value obtained for \( V_s \) into the first order condition directly, but this is a simple route to the solution.
\[ V_s = \frac{\nu[(\rho + \omega)(k\rho - k) - \kappa(1 - p)k]}{(\rho + \omega)((\rho + \kappa)(\lambda + \nu + \rho) + \omega(\rho + \lambda))} \]

Then, solving for \( \omega \) from the first-order condition and given this expression for \( V_s \), we obtain two candidate solutions for \( \omega \):

\[ \omega = -\frac{(\rho + \lambda)(\rho(k\rho - k) - k\kappa(1 - p)) \pm \sqrt{\kappa k(1 - p)(\rho + \lambda)(k\rho - k)(\rho + \lambda)(\kappa p(\rho + \lambda + \nu + \nu p) - k(\nu + \kappa + k(\rho + \lambda + \nu + \nu p)))}}{(\rho + \lambda)(k\rho - k)} \]

but \( k\rho > k \) implies that only the ‘negative’ solution is valid for \( \omega \in (0, \infty) \). \( \blacksquare \)

**Proof of Proposition 2.2**

The comparative static for \( \nu, k, p, \pi \) and \( \lambda \) is clear from differentiation of equation (5) given parameter restrictions. Furthermore, consider differentiating equation (3) but first setting \( p \pi = x \) for some \( x > 0 \). The resulting expression is negative.

For \( \kappa \), note first that \( \lim_{\kappa \to k/p\pi} \omega'(\kappa) = -\infty \), \( \lim_{\kappa \to \infty} \omega'(\kappa) = 0 \) and \( \lim_{\kappa \to \infty} \omega(\kappa) = \infty \), where the last limit implies that \( \omega'(\kappa) \) approaches zero from above as \( \kappa \to \infty \). Define \( f(\kappa) = p\pi\kappa^2(\lambda + \nu + \rho) - k(\nu + 2\kappa(\lambda + \nu + \rho) \left( \sqrt{\frac{1 - p(\lambda + \rho)}{\lambda + \nu + \rho}} + 1 \right)) \), then we have that \( \text{sign}(\omega'(\kappa)) = \text{sign}(f(\kappa)) \), given \( \omega > 0 \). Furthermore, \( \lim_{\kappa \to k/p\pi} \omega'(\kappa) = -\infty \Rightarrow f(k/p\pi) < 0 \), whereas the fact that \( \omega'(\kappa) \) approach zero from above as \( \kappa \to \infty \) implies that \( f(\kappa) > 0 \) for large \( \kappa \). The result then follows since \( f(\kappa) \) is quadratic in \( \kappa \), so that there can be at most one crossing of the axis on \( \kappa \geq k/p\pi \). That is, there is a unique \( \bar{k} \) such that \( f(\kappa) < 0 \) for \( \kappa \in [k/p\pi, \bar{k}] \) and \( f(\kappa) > 0 \) otherwise.

**Proof of Corollary 2.2**

For all parameters \( x \in \{\nu, k, p, \pi, \lambda\} \), \( \text{sign}\left( \frac{d}{dx} \frac{1}{\kappa + \omega} \right) = -\text{sign}(\omega'(x)) \). The result therefore follows directly in these cases. The same logic applies for the effect of \( p \) when \( p \pi \) is held constant. For \( \kappa \), \( \text{sign}\left( \frac{d}{dx} \frac{1}{\kappa + \omega} \right) = -\text{sign}(1 + \omega'(\kappa)) \), which is ambiguous.

**Proof of Corollary 2.3**

For \( x \in \{\nu, \pi, \lambda\} \), \( \frac{dK}{dx} = \frac{dK}{dx} \frac{d\omega}{dx} = \Rightarrow \text{sign}\left( \frac{dK}{dx} \right) = -\text{sign}\left( \frac{d\omega}{dx} \right) \text{ since } \frac{dK}{dx} < 0 \). The result follows immediately. For \( x \in \{k, p, \kappa\} \), \( \frac{dK}{dx} = \frac{dK}{dx} + \frac{dK}{dx} \frac{d\omega}{dx} \). For \( p \), \( \frac{dK}{dx} < 0 \) and \( \frac{d\omega}{dx} < 0 \); for \( k \), \( \frac{dK}{dx} > 0 \) and \( \frac{d\omega}{dx} > 0 \); and for \( \kappa \), \( \frac{dK}{dx} < 0 \) and the sign of \( \frac{dK}{dx} \) is ambiguous. It is not possible to sign these effects in general. The same logic applies for \( p \) when \( p \pi \) is held fixed.

**Proof of Proposition 2.3**

Follows immediately from main text.
Proof of Corollary 2.4

\[ E[N_f] = \left( \frac{\kappa}{\kappa + \omega\lambda} + \frac{\omega}{\kappa + \omega\lambda + \nu} \right)^{-1}. \]

Then

- \( \frac{d}{d\nu} E[N_f] = \frac{\kappa(\lambda + \nu)\omega'(\nu) + \omega(\nu)(\kappa' + \omega(\nu))}{(\kappa(\lambda + \nu) + \omega(\nu))^2} > 0 \) since \( \omega'(\nu) > 0 \).
- \( \frac{d}{d\nu} E[N_f] = \frac{\kappa\nu(\lambda + \nu)\omega'(\lambda) - \nu\omega(\lambda)(\kappa + \omega(\lambda))}{(\kappa(\lambda + \nu) + \omega(\lambda))^2} < 0 \) since \( \omega'(\lambda) < 0 \).

For \( x \in \{p, \pi, k\} \) and \( x = p \) given \( p \times \pi \) is constant, \( \frac{d}{d\nu} E[N_f] = \frac{\kappa\nu(\lambda + \nu)\omega'(k)}{(\kappa(\lambda + \nu) + \omega(k))^2} \Rightarrow \text{sign}\left( \frac{d}{d\nu} E[N_f] \right) = \text{sign}\left( \omega'(x) \right) \). The result follows from Proposition 2.2.

- \( \frac{d}{d\nu} E[N_f] = \frac{(\lambda + \nu)(\kappa\omega'(\nu) - \nu\omega(\nu))}{(\kappa(\lambda + \nu) + \omega(\nu))^2} \). This is ambiguous.

Proof of Proposition 2.4

Follows immediately from main text.

Proof of Corollary 2.5

For all parameters except for \( \nu \) and \( \lambda \), the result follows immediately from Corollary 2.2. Then, \( \frac{d}{d\nu} E[T_{br}] = -\frac{\omega'(\nu)}{\kappa(\lambda + \nu)} - \frac{1}{(\lambda + \nu)^2} < 0 \), where the inequality follows from Proposition 2.2 and \( \frac{d}{d\nu} E[T_{br}] = -\frac{\omega'(\lambda)}{\kappa(\lambda + \nu)} - \frac{1}{(\lambda + \nu)^2} \), which is ambiguous since \( \omega'(\lambda) < 0 \) by Proposition 2.2.

Proof of Proposition 2.5

Part (i). \( \frac{dp_d}{d\nu} = \frac{\kappa p \lambda (\omega'(\nu) - \nu \omega(\nu))}{(\kappa(\lambda + \nu) + \omega(\nu))^2} \Rightarrow \text{sign}\left( \frac{dp_d}{d\nu} \right) = \text{sign}\left( \omega - (\lambda + \nu)\omega'(\nu) \right) \), which can equivalently be cast as a statement about the semi-elasticity of \( \omega \) with respect to \( \nu \); that is, if \( \frac{\omega'(\nu)}{\kappa} > \frac{1}{\lambda + \nu} \), then \( \frac{dp_d}{d\nu} > 0 \). To see that \( \frac{dp_d}{d\nu} < 0 \) is possible, note that from Corollary 2.4 and Proposition 2.2, there are parameter values such that \( \omega = 0 \) but \( \omega'(\nu) > 0 \); specifically, when \( p > \bar{p} \) and \( \nu = \bar{\nu} \), increasing \( \nu > \bar{\nu} \) implies that \( \omega > 0 \), but \( \omega = 0 \) for \( \nu = \bar{\nu} \). Therefore, in this case \( \omega - (\lambda + \nu)\omega'(\nu) = -(\lambda + \nu)\omega'(\nu) < 0 \), so \( \frac{dp_d}{d\nu} < 0 \). To see that \( \frac{dp_d}{d\nu} > 0 \) is possible, note that \( \lim_{\nu \to \infty} p_d = p \) and \( p_d \leq p \), so \( p_d \) must approach \( p \) from below as \( \nu \to \infty \). This means there must be some \( \nu \) for which \( \frac{dp_d}{d\nu} > 0 \). Finally, to see that \( \nu_d \geq 0 \) such that if \( \nu \geq \nu_d \), \( \frac{dp_d}{d\nu} > 0 \) and otherwise \( \frac{dp_d}{d\nu} < 0 \), note that \( \frac{d}{d\nu}(\omega - (\lambda + \nu)\omega'(\nu)) = -(\lambda + \nu)\omega''(\nu) > 0 \) where the inequality follows since \( \omega''(\nu) < 0 \), so there at most one root for \( \omega - (\lambda + \nu)\omega'(\nu) = 0 \). Furthermore, \( \frac{d}{d\nu}(\omega - (\lambda + \nu)\omega'(\nu)) > 0 \) implies that if \( \frac{dp_d}{d\nu} \geq 0 \) for some \( \nu = \bar{\nu} \), then \( \frac{dp_d}{d\nu} > 0 \forall \nu > \bar{\nu} \). Therefore, we can define \( \nu_d > 0 \) s.t. \( \omega - (\lambda + \nu)\omega'(\nu) = 0 \) or \( \nu_d = 0 \) if there is no root and the statement holds.

Part (ii). \( \frac{dp_d}{d\nu} = \frac{\kappa p \lambda (\omega'(\nu) - \nu \omega(\nu))}{(\kappa(\lambda + \nu) + \omega(\nu))^2} \Rightarrow \text{sign}\left( \frac{dp_d}{d\nu} \right) = \text{sign}\left( \kappa + \omega - \nu \omega'(\nu) \right) \). To see that \( \frac{dp_d}{d\nu} < 0 \) is possible, note that \( \frac{d}{d\nu}(\kappa + \omega - \nu \omega'(\nu)) = -\nu \omega''(\nu) > 0 \). Therefore, if \( \frac{dp_d}{d\nu} < 0 \) is to occur, it will occur for low values of \( \nu \). Then, take the limit as \( \nu \to 0 \)
\[
\lim_{\nu \to 0}(\kappa + \omega - \nu \omega'(\nu)) = \frac{(\lambda + \rho)(k(\kappa p - \rho) + \kappa p\pi(\rho - \kappa)) - \sqrt{k^2k(1 - p)p(\lambda + \rho)^2(\kappa \pi - k)}}{\sqrt{k^2k(1 - \rho)p(\lambda + \rho)(\kappa \pi - k)}}
\]

where \( \omega > 0 \) has been assumed. This expression is negative if \( p \in \left(\frac{k \rho p}{\pi \kappa p - k k(1 - 2p)}, 1\right) \). However, the \( \omega > 0 \) solution may be invalid. From Corollary 2.1, a sufficient condition to take \( \nu \to 0 \) while maintaining \( \omega > 0 \) is that \( p < \bar{p} = \frac{k(\rho + \kappa)}{(\pi \rho + k(k + 2p))} \). Both of these conditions on \( p \) can be met simultaneously if \( \kappa > 2\sqrt{\bar{p}} \), which is admissible. Therefore, \( \lim_{\nu \to 0}(\kappa + \omega - \nu \omega'(\nu)) < 0 \) in some part of the parameter space and so \( \frac{dp}{d\nu} \) is feasible. To see that \( \frac{dp}{d\nu} > 0 \) is possible, note that \( \lim_{\nu \to \infty} p_s = p \) and \( p_s \leq p \), so \( p_s \) must approach \( p \) from below as \( \nu \to \infty \). The remainder of the proof is analogous to that for part (i).

**Part (iii) and (iv).** \( \frac{dp_d}{dp} = \frac{\kappa(\lambda + \nu)(\kappa(\lambda + \nu) - \lambda \omega'(p) + \lambda \omega)}{(\kappa(\lambda + \nu) + \lambda \omega)^2} \) and \( \frac{dp_s}{dp} = \frac{\nu}{\lambda + \nu} \frac{dp_d}{dp} \). By Proposition (2.2), \( \omega'(p) < 0 \), so \( \frac{dp_d}{dp} > 0 \) and \( \frac{dp_s}{dp} > 0 \). Furthermore, \( \frac{d}{dp} \frac{\kappa(\lambda + \nu)}{\omega(\lambda + \nu)} = \frac{-\kappa (\lambda + \nu) \omega'(p)}{(\kappa(\lambda + \nu) + \lambda \omega)^2} > 0 \) since \( \omega'(p) < 0 \). The same holds for \( p \), holding \( p \times \pi \) constant. ■

**Proof of Proposition 2.7**

This follows directly from an application of the implicit function theorem applied to the steady state conditions, with entry held constant. For clarity, the equations are given by

\[
\mu_d^p : \quad \mu_s^p \mu_c^\beta = (\kappa + \omega_1) \mu_d^p \\
\mu_d^\nu : \quad \kappa(1 - p)\mu_d^\nu = \omega_2 \mu_d^\nu \\
\mu_s : \quad \omega_1 \mu_d^p + \Lambda = (\lambda + \mu_s^{-1} \mu_c^\beta) \mu_s
\]

where I have given subscripts to \( \omega \) in order to highlight separately the rebalancing and reallocation effects. Isolating the rebalancing effect implies holding \( \omega_2 \) constant, whereas isolating the reallocation effects implies holding \( \omega_1 \) constant. Then, treating \( (\mu_s, \mu_d^p, \mu_d^\nu) \) as an implicit function of \( \omega_1 \) and \( \omega_2 \), it is simple to show that the total effect of changes in \( \omega \) can be written as the sum of the effects of \( \omega_1 \) and \( \omega_2 \). The result in the main body of the paper then obtains.

**Proof of Proposition 2.8**

Consider again the steady state conditions, but now write \( \omega \) as a function of \( \nu \), which in turn depends on \( M \). Specifically,

\[
\mu_d^p : \quad \mu_s \nu = (\kappa + \omega(\nu)) \mu_d^p \\
\mu_d^\nu : \quad \kappa(1 - p)\mu_d^\nu = \omega(\nu) \mu_d^\nu \\
\mu_s : \quad \omega(\nu) \mu_d^p + \Lambda = (\lambda + \nu) \mu_s
\]

where \( \nu = \mu_s^{-1} (M - \mu_d^p - \mu_d^\nu) \). Treating \( (\mu_s, \mu_d^p, \mu_d^\nu) \) as an implicit function of \( M \), we can obtain an expression for \( \frac{dm}{dM} \), where \( \frac{m(\mu_s, M - \mu_d^p - \mu_d^\nu)}{dM} \) is the flow of matches. It is then simple to show that \( \frac{dm}{dM} \) is increasing in \( \omega'(\nu) \). By Proposition 2.2, \( \omega'(\nu) > 0 \). Then, compare two economies. In the first,
permit contracts to respond to movements in capital supply, so that \(\omega'(\nu) > 0\). In the second, keep contracts at their initial level, so that \(\omega'(\nu) = 0\). It is clear that \(\frac{dm}{dM}\) is larger in the former economy, since it is increasing in \(\omega'(\nu)\).

Proof of Proposition 2.9

In steady state, the values of the co-states associated with (16) are given by

\[
\begin{align*}
\gamma_d^p &= \frac{(\rho + \omega) \left[ \kappa \pi - \left( 1 + \frac{\kappa(1-p)}{\rho+\omega} \right) k \right] \left[ \rho + \lambda + \frac{\partial m}{\partial \mu_s} \right]}{(\rho + \omega)(\rho + \lambda) \left( 1 + \frac{\kappa(1-p)}{\rho+\omega} \right) \frac{\partial m}{\partial \mu_t} + (\rho + \omega) \left[ (\rho + \kappa) \frac{\partial m}{\partial \mu_v} + (\rho + \lambda) (\rho + \kappa + \omega) \right]} \\
\gamma_d^u &= - \frac{(\rho + \kappa) \frac{\partial m}{\partial \mu_s} k + (\rho + \lambda) \left[ \kappa \pi \frac{\partial m}{\partial \mu_v} + (\rho + \kappa + \omega) k \right]}{(\rho + \omega)(\rho + \lambda) \left( 1 + \frac{\kappa(1-p)}{\rho+\omega} \right) \frac{\partial m}{\partial \mu_v} + (\rho + \omega) \left[ (\rho + \kappa) \frac{\partial m}{\partial \mu_s} + (\rho + \lambda) (\rho + \kappa + \omega) \right]} \\
\gamma_s &= \frac{(\rho + \omega)(\rho + \lambda) \left( 1 + \frac{\kappa(1-p)}{\rho+\omega} \right) \frac{\partial m}{\partial \mu_v} + (\rho + \omega) \left[ (\rho + \kappa) \frac{\partial m}{\partial \mu_s} + (\rho + \lambda) (\rho + \kappa + \omega) \right]}{(\rho + \omega)(\rho + \lambda) \left( 1 + \frac{\kappa(1-p)}{\rho+\omega} \right) \frac{\partial m}{\partial \mu_v} + (\rho + \omega) \left[ (\rho + \kappa) \frac{\partial m}{\partial \mu_s} + (\rho + \lambda) (\rho + \kappa + \omega) \right]}
\end{align*}
\]

where \(m\) refers to the flow of matches. Noting that \(\frac{\partial m}{\partial \mu_v} = \beta \nu \theta\) and substituting these expressions into equation (17) delivers the key result, following some algebra. As an aside, a similar consider for

\[\Lambda = \frac{1}{\sigma} \frac{\alpha \nu \left[ \kappa \pi - \left( 1 + \frac{\kappa(1-p)}{\rho+\omega} \right) k \right]}{(\beta \theta (\rho + \lambda) \left( 1 + \frac{\kappa(1-p)}{\rho+\omega} \right) - (1 - \alpha) (\rho + \kappa) + [(\rho + \kappa) (\rho + \lambda + \nu) + (\rho + \lambda) \omega]}\]

In the decentralised equilibrium, the equivalent expression is

\[\Lambda = \frac{1}{\sigma} \frac{\nu \left[ \kappa \pi - \left( 1 + \frac{\kappa(1-p)}{\rho+\omega} \right) \cdot k \right]}{(\rho + \kappa) \cdot (\rho + \lambda + \nu) + (\rho + \lambda) \cdot \omega}.
\]

so that entry is generally inefficient.

B Data and estimation

B.1 Definitions

Thomson Reuters define funding rounds in the following way.

Seed stage. This stage is a relatively small amount of capital provided to an inventor or entrepreneur to prove a concept. This involves product development and market research as well as building a management team and developing a business plan, if the initial steps are successful. This is a pre-marketing stage.
**Early stage.** This stage provides financing to companies completing development where products are mostly in testing or pilot production. In some cases, product may have just been made commercially available. Companies may be in the process of organizing or they may already be in business for three years or less. Usually such firms will have made market studies, assembled the key management, developed a business plan, and are ready or have already started conducting business.

**Expansion stage.** This stage involves working capital for the initial expansion of a company that is producing and shipping and has growing accounts receivables and inventories. It may or may not be showing a profit. Some of the uses of capital may include further plant expansion, marketing, working capital, or development of an improved product. More institutional investors are more likely to be included along with initial investors from previous rounds. The venture capitalist’s role in this stage evolves from a supportive role to a more strategic role.

**Later stage.** Capital in this stage is provided for companies that have reached a fairly stable growth rate; that is, not growing as fast as the rates attained in the expansion stages. Again, these companies may or may not be profitable, but are more likely to be than in previous stages of development. Other financial characteristics of these companies include positive cash flow. This also includes companies considering IPO.

**B.2 Data processing**

**Missing raised amounts.** For the US, the amount of capital raised is not available for 2,741 of the 34,360 rounds (9%). For the UK, the issue is slightly more severe, with 455 of the 2,419 funding rounds missing data on amounts raised (19%). Data on amounts raised is used to compute the burn rate and exit multiples, both of which are used in model calibration. In order to overcome this issue, I follow the literature and impute amounts raised in these cases. The methodology I use follows Jagannathan et al. (2022) closely. Specifically, I estimate the following regression

$$\log(\text{Amount raised}_{i,r}) = \beta_0 + \beta_1 \log(\text{Amount raised}_{i,r-1}) + \beta_2 X_{i,r} + u_{i,r}$$  \hspace{1cm} (29)$$

where Amount raised$_{i,r}$ is the amount raised by firm $i$ in round $r$ (in 2015 Mn USD) and $X_i$ is a list of firm-specific controls and fixed effects. I include the investment stage, industry, year-quarter fixed effects, and fixed effect for the number of investors, which I cap at ten investors (i.e. all rounds with 10 or more investors are lumped together). I run this regression separately for the US and UK. In addition, for some rounds I do not observe the previous funding amount and so run the specification with only the controls, $X_{i,r}$.

The results are displayed in Table 6. I run this on all deals between 2005 and 2022, which is why the observation counts exceed the sample used in the calibration. Using the results of these regression I impute missing amounts raised using fitted values. I run this on all the data, regardless of whether previous funding information is available.
Table 6: Imputation model

<table>
<thead>
<tr>
<th>US</th>
<th>Log(Amount&lt;sub&gt;r&lt;/sub&gt;)</th>
<th>UK</th>
<th>Log(Amount&lt;sub&gt;r&lt;/sub&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log(Amount&lt;sub&gt;r-1&lt;/sub&gt;)</td>
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<td>-</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.43</td>
<td>0.49</td>
<td>0.54</td>
</tr>
<tr>
<td>Industry FE</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Stage FE</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td># Investors FE</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>YQ FE</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
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<td>0.50</td>
<td>0.26</td>
</tr>
<tr>
<td>Observations</td>
<td>74,882</td>
<td>43,459</td>
<td>7,133</td>
</tr>
</tbody>
</table>

Notes. The table reports regression results from estimation of equation (29). ***, p<0.01.

B.3 Moment selection

In this section, I provide details of how I estimate the values of $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$, which are then included as targeted moments in the model estimation. At a high level, the idea is to leverage Proposition 2.4 which characterises the model-implied distribution of the duration between funding rounds. Generalising the results from the baseline model, the distribution between any two early-stage funding rounds is $T_{br,e} \sim Hypo(\kappa + \omega_e, \lambda + \nu_e)$ and between any two late-stage funding rounds is $T_{br,l} \sim Hypo(\kappa + \omega_l + \hat{\phi}_d, \lambda + \nu_l + \hat{\phi}_s)$. By estimating this distribution, I can recover estimates of $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$ in the data.

To see how this is implemented in practice, I need to introduce some notation. I denote by $t_{i,n}^s$ the time of the $n$th funding round for firm $i$ in state $s \in \{e, l\}$ and the duration between round $n$ and round $n+1$ for firm $i$ in stage $s$ by $\tau_{i,n}^s = t_{i,n+1}^s - t_{i,n}^s$. Firm $i$ completes $N_i^e \in \{1, 2, 3, \ldots\}$ early-stage rounds and $N_i^l \in \{0, 1, 2, \ldots\}$ late-stage rounds, but $N_i^e$ and $N_i^l$ are unobserved because the data is truncated.

The objective is to estimate $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_s$ based on the data, $\{\tau_{i,n}^s\}$. However, there are potential concerns of sample selection. The duration between rounds in stage $s$, $\tau_{i,n}^s$, is only observed for firm $i$ if: (i) firm $i$ has at least $n+1$ funding rounds in stage $s$, $n+1 \leq N_i^s$; and (ii) both round $n$ and round $n+1$ occur within seven years of the firm’s first funding round. Therefore, the duration data that we observe over-samples: (a) durations of firms that have more funding rounds; (b) durations that correspond to funding rounds relatively early in a firm’s maturation process; and, relatedly, (c) durations corresponding to low round-number funding rounds (in the sense of $n$).

To be more concrete, the objective is to estimate $Pr(\tau_{i}^s \leq \tau)$ for $s \in \{e, l\}$, where $j$ indexes funding rounds, but the data that we observe is drawn from the following distribution

$$Pr(\tau_{i,n}^s \leq \tau | n+1 \leq N_i^s \text{ and } t_{i,n+1}^s = t_{i,n}^s + \tau_{i,n}^s \leq T)$$

67It is also possible to characterise the distribution of the time between an early and late-stage funding round as $T_{br,e-l} \sim Hypo(\kappa + \omega_e, \lambda + \nu_l + \hat{\phi}_s)$, but I will not use this result in the estimation.

68$N_i^e$ starts at 1, not 0, because the sample is firms that have at least one funding round, which is necessarily an early-stage round.
where $T$ is the truncation date, set at 7 years following the firm’s first funding round. To the extent that these two distributions differ in a way we cannot control for, estimates of $\lambda + \nu_e$ and $\lambda + \nu_l + \hat{\phi}_a$ based on the data, $\{\tau_{i,n}^e\}$, will be biased.

The structure of the model provides a route forwards. To this end, note that we can write

$$
Pr(\tau_{i,n}^e \leq \tau | n + 1 \leq N_i^e \text{ and } t_{i,n+1}^e = t_{i,n}^e + \tau_{i,n}^e \leq T) = Pr(\tau_{i,n}^e \leq \tau | \tau_{i,n}^e \leq T - t_{i,n}^e) = Pr(\tau_{j}^e \leq \tau | \tau_{j}^e \leq T - t_{j}^e) = Pr(\tau_{j}^e \leq \tau | \tau_{j}^e \leq T_j^e)
$$

The first line uses the fact that, in the model, the probability of a subsequent funding round within the current stage is independent of the current round number, $n$. Specifically, the number of funding rounds within a given stage has a geometric distribution, which is memoryless. Therefore, conditional on observing the $n$th funding round, the probability that the firm has an additional funding round in the same stage is independent of $n$ and, therefore, there is no need to condition on $n + 1 \leq N_i^e$. The second line makes use of the observation that we do not need to treat the unit of observation as a firm-round tuple, but rather can treat funding rounds themselves, indexed by $j$, as the unit of observation. This follows because, conditional on the stage, the time to the next funding round is independent of the round number. This follows from the Markov property of the model and also implies that we need not be concerned with oversampling firms with more funding rounds in any given stage, under the assumptions of the model. A final concern is that the time $t_j^e$ of the current funding round might affect the time to the next round. However, conditional on the funding stage, this is not a concern because of the Markov property. Therefore, in the third line I define the truncation time $T_j^e = T - t_j^e$ for a given funding round.

Given the model implied distribution for the duration, $\tau_j^e \sim Hypo(h_{1}^e, h_{2}^e)$, the parameters $h_{1}^e$ and $h_{2}^e$ can be estimated via maximum likelihood. The likelihood for the data is given by

$$
L^e = \prod_{j=1}^{N} \frac{f(\tau_j^e; h_{1}^e, h_{2}^e)}{F(T_j^e; h_{1}^e, h_{2}^e)}
$$

where $f(\cdot)$ and $F(\cdot)$ are, respectively, the PDF and CDF of the hypoexponential distribution. The estimates are displayed in Table 7.

<table>
<thead>
<tr>
<th></th>
<th>UK</th>
<th>95% CI</th>
<th>N</th>
<th></th>
<th>USA</th>
<th>95% CI</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_{1}^e$</td>
<td>1.49</td>
<td>(1.16, 1.82)</td>
<td>517</td>
<td>1.54</td>
<td>(1.48, 1.61)</td>
<td>11,094</td>
<td></td>
</tr>
<tr>
<td>$h_{2}^e$</td>
<td>2.70</td>
<td>(1.68, 3.71)</td>
<td>517</td>
<td>3.28</td>
<td>(3.00, 3.55)</td>
<td>11,094</td>
<td></td>
</tr>
<tr>
<td>$h_{1}^l$</td>
<td>0.85</td>
<td>(0.66, 1.04)</td>
<td>384</td>
<td>0.90</td>
<td>(0.86, 0.94)</td>
<td>7,097</td>
<td></td>
</tr>
<tr>
<td>$h_{2}^l$</td>
<td>3.58</td>
<td>(1.76, 5.40)</td>
<td>384</td>
<td>5.96</td>
<td>(5.12, 6.80)</td>
<td>7,097</td>
<td></td>
</tr>
</tbody>
</table>

Table 7: Estimates of hypoexponential distribution

A problem remains, because the parameters of the hypoexponential distribution are interchangeable,
so that there is no strict guidance on whether to pick $h_1^s$ or $h_2^s$ (the parameters are ranked based on size in Table 7). In the moment matching exercise, I target $h_2^s$, the larger of the two parameter estimates, for three reasons. Firstly, suppose instead I were to target $h_1^s$. Then, the meeting rates $\nu_s$ would typically be lower, implying that it would be more difficult for a firm to access follow-on funding and by Proposition 2.2, this would imply a longer contract; a low value for $\omega_s$. But if the model is to provide a good fit to the data, then the (untargeted) value of $\kappa + \omega_e$ and $\kappa + \omega_l + \hat{\phi}_d$ should be close to $h_2^s$ and $h_1^l$, respectively, which is relatively high. Therefore, such a procedure is unlikely to provide a good fit to the data because the value of $\omega_s$ would be inconsistent with $h_1^s$. Secondly, consider that targeting $h_1^l = \lambda + \nu_l + \hat{\phi}_s$ in the late-stage would imply that the time in late-stage search, $T_{s,l} \sim \text{Exp}(h_1^l)$, which means that, for the US, the average time spent in search is 1.11 years ($=1/0.90$), or roughly 13 months, whereas the average amount of time spent in development would be 0.17 years ($=1/5.96$), or just over two months. A similar calculation can be made for the early stage, as well as for the UK. Beyond noting that this would be a highly dysfunctional market, it also contradicts conventional wisdom that venture capital firms provide start-ups with enough capital for one to two years of development. Finally and relatedly, the choice to target $h_2^s$ to identify $\nu_s$ is a conservative approach: it implies that start-ups spend only a relatively short period of time in search and therefore limits the quantitative impact of the search friction.

As validation of this approach, Table 8 displays output from the model for the relevant counterparts to the estimates in Table 7, ordered for ease of comparison. As also shown in Table 8, the targetted values are close to the empirical estimates. Furthermore, although not perfect, the model-implied values for $h_1^e$ and $h_1^l$ fall in the correct region, especially in terms of their magnitude relative to the estimates for $h_2^e$ and $h_2^l$. In addition, Figure 4 shows that the model is able to closely match the distribution of the duration between funding rounds in the data, which provides further confidence in approach adopted.

<table>
<thead>
<tr>
<th></th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa + \omega_e$</td>
<td>0.75</td>
<td>0.85</td>
</tr>
<tr>
<td>$\lambda + \nu_l$</td>
<td>2.65</td>
<td>3.29</td>
</tr>
<tr>
<td>$\kappa + \omega_l + \hat{\phi}_d$</td>
<td>1.30</td>
<td>1.71</td>
</tr>
<tr>
<td>$\lambda + \nu_l + \hat{\phi}_s$</td>
<td>3.69</td>
<td>6.05</td>
</tr>
</tbody>
</table>

Table 8: Model values

C General model

In this section, I provide the set of equations that characterise the model of section 3 in partial equilibrium. An overview of the model is already provided in the main text, so I keep the discussion brief.

The main point of divergence from the baseline model is that the expected capital cost associated with a contract in the late-stage needs to account for the role of acquisitions. Recall that potential acquirors arrive at rate $\hat{\phi}$, which is exogenous. I think of the process of an acquisition as having a

Future versions will estimate the model in equilibrium and so provide the full equilibrium model.
number of steps. Firstly, the potential acquiror approaches the company and, at this point, the match-
specific synergy, $\epsilon \sim F(\epsilon)$ is unknown to both the start-up and potential acquiror. Next, the potential
acquiror does some due-diligence in order to determine $\epsilon$. However, this process of due-diligence, the
potential acquiror also learns whether the project has hit a major roadblock and appears unlikely to
reach fruition; that is, they learn whether the start-up is productive or unproductive. Finally, having
learned this information, the parties negotiate over an acquisition price.

I make the simplifying assumption that an unproductive entrepreneur cannot prevent the acquiror
from conducting due diligence and learning that its project has failed. The venture capital firm knows
that this due diligence process is ongoing and, therefore, also learns this information. Having learned
this information, the venture capital firm cuts off funding. In effect, this implies that the arrival of
potential acquirors acts as a monitoring device. Intuitively, if the entrepreneur were to prevent all
potential acquirors from conducting due diligence, the venture capital firm might learn that the start-
up is of low quality. Similarly, if the VC never learned of any interest from potential acquirors - perhaps
because the entrepreneur keeps this information private - they might downgrade their prospects about
the firm and have an incentive to monitor. The assumption hopes to capture these ideas in a simple
way. With this in mind, the total expected capital commitment associated with a late-stage investment
contract $(\omega_l, \varsigma_l)$ is given by

$$K_l(\omega_l) = \int_0^\infty \left( \int_0^T e^{-\rho t} k \, dt \right. $$

$$+ \left. \frac{\kappa (1 - p)}{\kappa + \omega + \hat{\phi}_d} \int_0^\infty \left( \int_T^{T+T_{\omega_l}} e^{-\rho s} k \, ds \right) (\omega + \phi) e^{-(\omega_l + \phi) T_{\omega_l}} dT_{\omega_l} \right)$$

$$\times (\kappa + \omega + \hat{\phi}_d) e^{-(\kappa + \omega + \hat{\phi}_d) T} dT$$

$$= \left( 1 + \frac{\kappa (1 - p)}{\rho + \omega + \phi} \right) \frac{k}{\rho + \kappa + \omega + \hat{\phi}_d}$$

which has the same form as equation (2). However, the wedge now includes $\phi$ in the denominator,
reflecting the monitoring role of acquisitions. Furthermore, the frictionless expected capital cost is now
affected by the (endogenous) effective arrival rate of acquisitions for a firm in productive development,
because this reduces the expected time horizon over the VC provides funding to the firm.

Given the expected capital cost associated with a late-stage contract, the conditions characterising the
model in partial equilibrium are given by

\[ \rho V_{d,e}(\omega_e) = \kappa [\hat{p}_e V_{s,e} - V_{d,e}(\omega_e)] + \omega_e \left[ V_{s,e} - V_{d,e}(\omega_e) \right] \]  
\[ (\rho + \lambda) V_{s,e} = \nu_e \left[ V_e^M - V_{s,e} \right] \]  
\[ \quad \text{for } i \in \{s, d\}, \text{ and } (\omega_e, \varsigma_e) \text{ and } (\omega_l, \varsigma_l) \text{ are chosen optimally in the sense of equations (33) and (37), respectively.} \]