TYING YOURSELF TO THE MAST: PAINFUL DEBT AS A COMMITMENT DEVICE IN SELF-FULFILLING DEBT CRISSES

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This paper studies why countries end up at a disadvantageous point of high debt and high economic cost of default. I propose that in normal times, governments may figuratively "tie themselves to the mast" by shifting to a debt portfolio that is more painful to restructure. This commits governments to repay and momentarily allows them to sustain higher debt levels, but makes the sovereign more vulnerable to large economic shocks. Empirically, I find support for this commitment channel by exploiting variation in high-frequency financial data around a series of sovereign debt litigation outcomes: a one ppt increase in creditors' expected recovery rate reduces markets' beliefs about the likelihood of a default event by 0.46 ppt. I further develop these results in a debt crisis framework in which a government can issue two bonds that differ in how painful they are to restructure. I find that governments indeed optimally shift towards more painful debt portfolios to momentarily rule out liquidity crises and increase debt levels.


1. INTRODUCTION

"Raising the cost of defaulting does indeed increase the level of debt that can be sustained if a crisis does not occur [...] ." — Cole & Kehoe (2000)

Conventional theories on belief-driven debt crises predict that governments which face the threat of a liquidity crisis optimally reduce debt levels and tighten fiscal expenditure (see e.g. Cole & Kehoe (2000), Calvo (1988)). Such theories contrast with recent evidence from low- and middle income countries (LMICs) where debt levels have both increased and become increasingly painful to restructure over the past decade (Figure 1). Many of these governments are now facing acute and severe debt crises (IMF 2023).

This paper investigates whether such debt trajectories can be reconciled with a notion of optimal policy. I propose a mechanism in which governments figuratively 'tie themselves to the mast' by choosing sovereign debt portfolios that increase the economic cost of default. Intuitively, this portfolio choice commits the government to repay its debts and allows it to sustain higher debt levels without the immediate risk of a liquidity crisis all else equal. Albeit momentarily ruling out a liquidity crisis, this mechanism can, however, lead to a dangerous spiral of increasingly high- and increasingly painful to restructure debt portfolios over time, making governments vulnerable to large, unanticipated fiscal shocks. While this is likely not the sole driver of LMICs’ past debt trajectories, I argue that it has amplified and contributed to it.

Rationalizing a government’s decision to raise debt levels is of particular interest, as it goes contrary to conventional optimal policy in standard debt crisis models where governments are expected to deleverage if they could face a liquidity crisis and to keep debt levels constant.
otherwise. A growing literature is trying to rationalize a government’s decision to increase debt levels despite the risk of a belief-driven crisis. Most closely related to my work is Conesa & Kehoe (2017) who propose that rising debt levels throughout the Euro-crisis were driven by ‘gambling for redemption’, where governments in deep recession with hopes for better future economic conditions optimally ran deficits and increased debt. Lorenzoni & Werning (2019) propose a slow-moving debt crisis framework, where the increase in borrowing cost - driven by self-fulfilling default expectations - leads to a country accumulating debt slowly over time. Similar to them, the ‘tying yourself to the mast’ dynamic which I propose, falls into this strand of the literature but puts forth a different mechanism specific to the recent situation of LMICs.

I find both empirical and theoretical support for this mechanism. Empirically, I find novel corroborative evidence that a rising economic cost of default functions as a commitment device. In line with the theoretical literature (see e.g. Cole & Kehoe (2000)), I can show that an exogenous increase in the recovery rate - the share of the bond principal creditors expect to recoup in a default event - reduces market-perceived default expectations. To do so, I jointly estimate daily recovery rates and default probabilities from a cross-section of dollar-denominated bond series for a number of Latin American economies with deep external debt markets and a history of sovereign default. I then exploit high-frequency variation in my estimates over a tight event window around a series of third-country sovereign debt litigation outcomes in 2016. Such court rulings set precedence under the respective foreign jurisdiction for shifts in bargaining power between creditors and the sovereign, which translates into exogenous shocks to creditors’ expected recovery rate. I then use this variation to capture the effect on countries’ default probabilities. I find that an increase in the expected recovery rate by one percentage point reduces the expected likelihood of a default event over the next year by 0.46 percentage points. This confirms that as the cost of sovereign default rises, markets deem the likelihood of such a default event less likely. This finding indicates that governments could use a shift towards more painful debt portfolios to reduce the risk of a liquidity crisis, or even to increase debt levels without spooking markets.

On the theory side, I develop these results in a stochastic general equilibrium model framework with an endogenous debt portfolio choice. I extend the framework by Cole & Kehoe (2000). In my model, the government can make an endogenous choice about its default penalty by issuing two types of bonds: painful-, and safe bonds. These bonds are notionally identical but differ in how painful they are to default on for the government.1 In this model setting, higher reliance on painful debt is associated with a higher economic default penalty. For example, let us imagine two governments having the same debt-to-GDP ratio. Still, one could heavily rely on ‘painful’ debt and would thereby incur a large economic penalty in case of a default, while the other government could rely heavily on ‘safe’ debt and consequently would incur a relatively smaller default penalty.

Despite exhibiting objectively worse properties, I find that if given the choice, governments in my model environment will indeed find it optimal to increase the share of painful debt under certain conditions, as it allows them to commit themselves to repay their debt. If a government’s debt-to-GDP ratio is high enough to face the risk of a liquidity crisis, I find that governments optimally issue higher shares of painful debt to commit themselves to repay and sustain their elevated debt level. For reasonable model parameters, I find that a shift towards a more painful

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1Here, the issuance of painful- or safe bonds shall reflect the choice governments have in deciding who to borrow from and what type of debt they issue. Governments could e.g. plausibly make default more economically costly for themselves, by increasingly borrowing from their domestic corporate banking sector, increasing creditor fragmentation, omitting features such as collective action clauses, or issuing tradable bonds instead of relying on concessional debt from multilateral lenders.
TYING YOURSELF TO THE MAST: PAINFUL DEBT AS A COMMITMENT DEVICE

A debt portfolio can produce a shift in sustainable debt levels, that closely resembles LMICs’ aggregate debt trajectories over the past decade: an increase from 8.7 to 10.1 percentage points compared to the actual increase from 7.7 to 11.1 percent of GDP (Figure 1a). Furthermore, I find that if an adverse MIT shock hits the GDP of a government, in normal times it is optimal to increase debt levels as well as the painful debt share of GDP. If governments are hit by a series of unanticipated adverse shocks, or are sufficiently short-sighted in their policy making, I find that they converge to a point of high debt- and high economic cost of default, not unlike the aggregate debt trajectories of LMICs over the decade pre-COVID.

This exercise is further motivated and contextualized by recent aggregate public- and publicly guaranteed (PPG) debt trends in LMICs. Between 2010 and 2020, aggregate sovereign debt to GDP ratios have risen by about 44 percent (Figure 1a). At the same time, this debt has become more economically costly to restructure across a number of dimensions. For one, LMICs’ sovereign debt portfolios have become more reliant on private creditors (Figure 1a). High reliance on private creditors can expose LMICs to substantial retaliatory risks in case of a default (Stiglitz & Rashid 2020, 2013). Further, debt obligations have become more fragmented across different geopolitical spheres of influence and groups of uncoordinated private creditors (Figure 1b). Between 2010 and 2020, the share of LMICs’ debt owed to China rose from 3.9 to 10.1 percent. The share held by lenders under Western thought leadership such as the Paris Club and the World Bank fell from 38.0 to 24.5 percent. Higher fragmentation can make it increasingly difficult to strike a restructuring deal that all creditors agree to. Lastly, an increasing share of government debt is held by domestic banks, intensifying the so-called sovereign-bank nexus. Here, LMICs’ bank-holdings of domestic sovereign bonds have surged over the past decade as visualized in figure 1c, accounting for more than 16 percent of the local banking

Figure 1.: Debt developments in low- and middle-income countries

(a) Aggregate sovereign debt-to-GDP ratios by creditor type  
(b) Share of aggregate LMICs’ debt by creditor type  
(c) PPG debt share of total domestic banking sector assets


Note: Aggregates for 123 LMICs are directly provided by the World Bank. A country is classified based on its 2021 GNI per capita. A country is low-income for a GNI below $1,045, lower-middle-income for a GNI below $4,095, and upper-middle-income for a GNI below $12.695. Advanced and Emerging Economies comprise economies classified as advanced and emerging in the IMF World Economic Outlook database respectively.
sector’s assets in 2020, reaching a 15-year high (IMF 2022). This can pose severe risks around negative feedback loops between sovereign default and financial crises (Sosa-Padilla 2018).

The rest of this paper is structured as follows. Section 2 provides empirical evidence on the causal effect between exogenous changes in the cost of economic default and market’s beliefs about default probabilities. I find evidence that - as the proposed channel suggests - an exogenous increase in the economic cost of default reduces market fears of default. Section 3 then outlines my theoretical model that motivates the ’tying yourself to the mast’ dynamic under an endogenous risk choice and provides an analytical solution that - under certain conditions - rationalizes a country’s choice to use the issuance of painful debt to rule out a liquidity crisis instead of deleveraging. Section 5 provides numerical findings for a simple calibrated model. I also show that adverse MIT shocks to output rationalize both increasing debt levels and a rising economic cost of default. I can further show that the proposed mechanism yields reasonable effects for a realistic parameter choice. Section 6 concludes.

1.1. Related Literature

The literature most closely related to my theoretical work is that on belief-driven default crises, featuring equilibria multiplicity. My model is an extension of the bond auction mechanism specified in Cole & Kehoe (2000), which explained fast and self-fulfilling rollover crises through a lack of government commitment to honoring its outstanding debt obligations before issuing new debt. A large strand of the literature has extended this idea: Among others, Da-Rocha et al. (2013) merged the Cole-Kehoe model with the Obstfeld (1996) model on self-fulfilling devaluations to analyze optimal policy. Araujo et al. (2013) extended the Cole-Kehoe model by domestic debt, allowing the government to deflate away domestic debt to avoid external debt default in a crisis. Bocola & Dovis (2019) enriched the model with maturity choices and risk-averse lenders to quantify the importance of rollover risks in debt crises empirically. Furthermore, Conesa & Kehoe (2017), Chatterjee & Eyigungor (2012), and Roch & Uhlig (2018) have introduced the feature of a fast self-fulfilling rollover crisis in models with income shocks.

Besides the classic fast rollover problem first emphasized by Cole & Kehoe (2000), there is a second strand of literature that relates to slow-moving debt crises, chiefly building on Calvo (1988), who introduced feedback between interest rates and debt levels in a two-period model. More recently, Lorenzoni & Werning (2019) and Ayres et al. (2018) built on Calvo (1988) to derive a mechanism for slow-moving self-fulfilling crises in which a shift in investors’ beliefs can lead to increased borrowing costs, causing a debt accumulation path of higher default risk, which ultimately validates the initial shift in beliefs.

Aside from these expectation-driven default models with multiple equilibria, there exists a further strand of literature on standard quantitative models of sovereign default with a single low-interest rate equilibrium. Eaton & Gersovitz (1981) developed a standard framework, which was more recently extended by Aguiar & Gopinath (2006), and Arellano (2008). In addition to the select few contributions listed above, Aguiar & Amador (2013) provided a recent survey of the sovereign debt literature.

Most closely related to my analysis is the paper by Conesa & Kehoe (2017). Similar to me, they aim to rationalize a country’s decision to increase debt levels in the presence of a crisis zone. They did so to explain the sovereign debt trajectories throughout the euro crisis, where troubled European economies such as Portugal, Ireland, Italy, Greece, and Spain, generated very large government budget deficits and increased sovereign debt levels instead of reducing them. To explain these rising debt-to-GDP ratios, Conesa & Kehoe (2017) extended the model framework of Cole & Kehoe (2000) by introducing a deep recession of stochastic duration.
In their model, a country’s decision to accumulate more debt in the crisis zone is rationalized by ‘gambling for redemption’, as the end of the recession shifts out the crisis zone, making higher debt levels sustainable again. While the objective of their analysis is similar to mine, the assumptions underpinning both models differ, as their mechanism hinges on the country being in a deep recession with high hopes for a better future, which has not been the case for LMICs over the past decade.

2. EMPIRICAL EVIDENCE

This section provides empirical evidence for the proposed commitment channel: As the cost of default increases, the probability of a liquidity crisis occurring is reduced so that the issuance of more painfully to restructure debt can allow a government to sustain higher debt-levels in normal times. Namely, I analyze the effect of a change in a country’s recovery rate on market beliefs about its default probability. Here, the recovery rate relates to the share of the bond principal that creditors expect to extract from the sovereign in case of a default or a debt restructuring. In line with the theoretical literature (e.g. Cole & Kehoe (2000)), I find that an exogenous increase in the recovery rate functions as a commitment device and decreases the market-perceived default probability.

My empirical identification strategy exploits variation in a cross section of bond prices around a series of court rulings on sovereign debt litigation cases compiled by Schumacher et al. (2021). Looking at changes around such court outcomes is informative, as they provide precedence for future debt litigation outcomes under the same jurisdiction and thereby affect how much of the bond principal creditors expect to recover in the future. I effectively use the series of court rulings as an instrument to analyze the impact of an exogenous change in bonds’ recovery rate on their default probability. I use a cross-section of USD-denominated bonds for a sample of Latin American economies with default history and deep external debt markets to jointly estimate daily recovery rates and default probabilities following Vrugt (2011). All of these bonds are raised under US jurisdiction. I then analyze how these estimates change over a two-day window around each court ruling. I find that an exogenous increase in the recovery rate by one percentage point reduces the expected default probability of the affected country by 0.46 percentage points.

2.1. Institutional Background

Over past decades, creditor lawsuits have become increasingly common and have contributed to a significant change in the cost of sovereign default and the recovery rate. Schumacher et al. (2021) provide a comprehensive summary on the history of sovereign debt litigation which I briefly summarized in this section.

Historically, private creditors have lacked a direct enforcement device against defaulting sovereigns on grounds of sovereign immunity. This changed in 1976, when the United States passed the Foreign Sovereign Immunities Act (FSIA) under which states and state-owned firms could be held legally accountable for a breach of commercial contracts. The United Kingdom ratified the State Immunity Act shortly after in 1978 with similar effect. Notably, many LMICs issue a substantial share of their sovereign bonds under US-, or UK law. As a result, sovereigns could now be held accountable for these debt contracts they issued under foreign jurisdiction, so changing legal precedence and changes to the US and UK legal codes could affect their exposure to litigating creditors in case of a breach of contract.2 Schumacher et al. (2021) de-
scribed the history of debt litigation since the FSIA as a ‘gradual erosion of government immunity’. According to Schumacher et al. (2021), an increasing number of successful court cases for litigating creditors has set more and more precedence for enforcing more advantageous restructuring terms for creditors which would translate to an increasing recovery rate.

The first large lawsuit based on the FSIA was filed by Allied Bank in 1982 due to their disagreement with a debt restructuring proposal by Costa-Rica. While the court ruling was in favor of Allied Bank, the US government pressured Allied Bank to agree to the same terms as other creditors part of the debt restructuring. Yet, the court ruling set an important precedent that sovereigns were subject to lawsuits and that sovereign assets in the US were attachable. Throughout the rest of the 1980s, more cases against sovereigns were filed, the most prominent of which was Weltover vs. Argentina. Here the supreme court ruled the issuance of sovereign debt on international markets a commercial activity, falling under the FSIA. This effectively granted US courts jurisdiction over sovereign bond loans issued under US law. In the early 1990s, these successes of litigating creditors led to the formation of an entirely new type of plaintiff: the vulture funds; hedge funds specialized in targeting debt-distressed countries. CIBC vs. Banco Central de Brazil was the first example of a fund turning a substantial profit by buying up distressed bonds on the secondary market and successfully recouping most of the principal plus interest. This case played an important role for case law development, as it weakened the so-called Champerty defense, according to which it was prohibited to purchase bonds with the primary goal of filing a lawsuit. At this point sovereigns were very much subject to lawsuits in the United States, however, plaintiffs were still restricted to the seizure of assets that are located in the United States and that are used for commercial activity. A novel strategy to enforce lawsuits emerged in the late 1990s in Elliott vs. Republic of Peru. Elliott argued that the pari passu clause prohibits Peru from paying creditors that agreed to the debt restructuring without also paying holdouts. In doing so, Elliott successfully blocked payments to restructured creditors via a European bank and was subsequently quickly paid out in full. One of the most prominent pari passu cases of the more recent past are NML Capital vs. Argentina. Like Elliott, NML Capital interrupted payments to restructured creditors via a US Bank, which acted as the trustee for the restructured creditors. As a consequence, Argentine first defaulted on its debt and then later settled with the holdouts under a new administration in 2016. Lastly, there have also been some attempts to seize sovereign assets abroad. While most such attempts have remained legally unsuccessful, such ongoing lawsuits can still impose economic damage onto the sovereign sometimes far exceeding the litigation claims. A small group of holdouts for example successfully blocked all oil exports in the Republic of Congo for multiple years.

In summary, over the past 40 years court rulings and changes to the US-, and UK legal code have changed sovereigns’ exposure to US court rulings, the attachment of sovereign assets in the US, as well as the exertion of economic damages, such as the blocking of debt restructurings under the pari passu, blocking of exports, attachment of sovereign assets, or the exclusion from financial services. Overall, it can be concluded that court rulings and the legal precedence they set have likely had an impact on sovereigns’ and plaintiffs’ bargaining power, and by extension on the recovery rate that creditors expect to recoup after a default event.

2.2. Evidence from Brazilian CDS Spreads

Given that the changing precedence around these court rulings has plausibly affected sovereigns’ expected economic cost of default, my empirical exercise is further motivated by the following observation: The CDS spread of emerging market sovereign bonds - a commonly used proxy for how expensive it is to insure against sovereign default - predominantly drops around the series of sovereign debt litigation outcomes compiled by Schumacher et al. (2021).
I analyze changes in Brazilian CDS rates of varying maturities around the series of court outcomes. I report findings for Brazil, since Brazil exhibits the deepest external debt markets of all countries with a history of sovereign default, indicating sufficient liquidity in CDS markets of varying maturities. These findings, however, also hold for other LMICs in my sample, and I will extend the analysis to a broader cross-section of countries in section 2.3.

Figure (2) depicts by how many basis points (bps) the 5-year Brazilian CDS rate has changed one trading day after the court ruling compared to one day before the court ruling for the series of court rulings since 2007. I find that insurance against sovereign default has predominantly become cheaper around such court rulings. Under the assumption that the series of court rulings in Schumacher et al. (2021) predominantly set precedence for a strengthening of the plaintiffs’ bargaining power in sovereign debt restructurings, a drop in Brazilian CDS rates around such rulings would indicate that a rising cost of sovereign default indeed works as a commitment device, causing a reduction in the cost of insurance for such sovereign default.

Although there is some heterogeneity among the CDS rate’s reaction to court rulings, the data accords well with narrative evidence on key historical court rulings. The first big revision in Brazil’s CDS rate around a court ruling occurred on the 16th of April 2009. This revision reflects the court ruling on the suit by Montrose Capital LLC vs. Liberia. This suit stands out due to the size of the amount successfully claimed by Montrose Capital LLC, which amounted to $129.5 million: 14.7% of Liberia’s nominal GDP at the time (Schumacher et al. 2021). The litigation amount of the nine court cases filed against Liberia - all ruled on at around the same time - amounted to an unprecedented 41.6% of Liberian GDP. Around the court case, the Brazilian CDS spread dropped by 17.5 bps. Further, there exists another cluster of large and negative CDS responses around various court rulings on Argentinian litigation cases in

Note: The figure shows the five-year CDS shock series for Brazilian data around US court rulings on non-Brazilian debt litigation cases. The grey dashed lines reflect the 90 percent confidence bands. Confidence bands are based on variation in the day-to-day changes of the CDS rate, which exhibits a standard deviation of 9.03 bps.

Source: CDS rates are chosen from investing.com.
TABLE I: Brazilian CDS rate changes in bps around Court Rulings for different maturities

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Mean</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
<th>#Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 Month</td>
<td>-1.87</td>
<td>-2.18</td>
<td>-13.47</td>
<td>11.52</td>
<td>21</td>
</tr>
<tr>
<td>1 Year</td>
<td>-2.76**</td>
<td>-1.50</td>
<td>-1.544</td>
<td>-7.47</td>
<td>21</td>
</tr>
<tr>
<td>2 Years</td>
<td>-3.65**</td>
<td>-2.49</td>
<td>-17.35</td>
<td>6.77</td>
<td>21</td>
</tr>
<tr>
<td>3 Years</td>
<td>-4.18***</td>
<td>-1.00</td>
<td>-18.71</td>
<td>6.01</td>
<td>21</td>
</tr>
<tr>
<td>4 Years</td>
<td>-4.27**</td>
<td>-0.96</td>
<td>-19.86</td>
<td>5.54</td>
<td>21</td>
</tr>
<tr>
<td>5 Years</td>
<td>-4.03**</td>
<td>-0.95</td>
<td>-20.02</td>
<td>5.85</td>
<td>21</td>
</tr>
<tr>
<td>10 Years</td>
<td>-3.96**</td>
<td>-1.08</td>
<td>-19.41</td>
<td>5.72</td>
<td>21</td>
</tr>
<tr>
<td>20 Years</td>
<td>-3.86**</td>
<td>-0.29</td>
<td>-19.41</td>
<td>6.69</td>
<td>21</td>
</tr>
<tr>
<td>30 Years</td>
<td>-3.82**</td>
<td>-0.69</td>
<td>-21.41</td>
<td>-6.69</td>
<td>21</td>
</tr>
</tbody>
</table>

Note: *, **, *** refer to a p-value of 0.1, 0.5, 0.01 for the one-sample t-test that the sample is drawn from a standard normal distribution centered around zero. Data on CDS rates is drawn from Bloomberg, while data on court ruling dates are taken from Schumacher et al. (2021). Source: CDS rates are taken from investing.com.

2016, including one of the most prominent litigation cases of the recent past: NML Capital vs. Argentina. The vast majority of these shocks are negative, and two more shocks in 2016 are significant at the 10% level.5

Looking beyond just the 5-year CDS spreads, Table I summarizes key facts about the distribution of Brazilian CDS spreads of differing maturity. CDS spreads across all maturity degrees were on average negatively affected by court rulings. The effect size appears to be larger for longer CDS maturities of 3 or more years.6 Using a one-sample t-test, I can reject the hypothesis, that these effects are drawn from a standard normal distribution centered around zero. I find a statistically significant negative effect for all CDS spreads of at least one year.

However, just looking at changes in the CDS spread around court rulings is insufficient to determine a causal link between an increase in the cost of default and market beliefs about default probabilities. For one, although anecdotally, court outcomes from sovereign litigation cases seem to have provided mounting evidence for an increasing erosion of sovereign immunity and rising recovery rates for creditors in debt restructurings, individual court outcomes could set precedence one way or the other. They could sway bargaining power more in favor of either the creditor or the sovereign compared to the status-quo ex-ante. More careful analysis is needed to determine whether a court ruling increased or decreased the cost of future default. Secondly, even if a court ruling sets precedence for higher future recovery rates, a simultaneous drop in the CDS rate does not necessarily reflect a drop in the default probability. It could also reflect the rise in the recovery rate itself that the insurance then does not have to cover in case of default. To address both of these concerns, in the next section, I jointly estimate the expected recovery rate and the default probability from a cross-section of bonds. I do so to analyze how the recovery rate and the default probability have changed around a tight event window for each court ruling in my sample.

2.3. Methodology

To estimate daily default probabilities and recovery rates, I follow Vrugt (2011) and Merrick (2001) by estimating both parameters simultaneously from a daily cross-section of USD-denominated government bonds. I focus on USD-denominated bonds since all countries with

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5I classify a shock as significant at the 10 percent level, if it exceeds 1.69 standard deviations of daily CDS changes.

6In part, this can be explained, by CDS rates for longer maturities exhibit higher rates ex-ante, given that they insure over a longer period in which government default could occur.
deep external debt markets in my sample raise their debt primarily under US legislation. The underlying idea is to use the risk-neutral valuation framework of Jarrow et al. (1997) to express the price of an outstanding bond as its discounted, probability-weighted cash-flows. Given that the payment schedules of the USD-denominated bonds in my cross-section are known, and that the discount factor can be inferred from the known US yield curve at the time, one can express the price of an outstanding bond as a function of two variables: the recovery rate, and the default probability. If we have a cross-section of outstanding bonds from the same government, at each date we can determine the default probability and the recovery rate by minimizing the difference between the actual observable prices, and the estimated values implied by this valuation framework. We then repeat this estimation approach for all countries of interest on all dates in 2016.

**Estimating the default probability and the recovery rate**

Concretely, I follow Jarrow et al. (1997) in assuming that the bond price of government $i$ can be represented as the default-probability-weighted sum of promised cash-flows and the recovery value (in the following, I am omitting the country subscript $i$ and the date subscript $d$ for legibility):

$$P_0 = \sum_{n=1}^{N} df_n [(CF_n \times S_n) + (RV \times S_{n-1} \times \pi_n)]$$  \hspace{1cm} (1)

Here, $N$ is the total number of outstanding cashflow payments, the $n$-th of which is denominated by $CF_n$. $RV$ is the recovery value creditors expect to retrieve in case of a debt restructuring, $df_n$ is the risk-free discount factor for cash flow payment $n$ that can be inferred via the US yield curve. $\pi_n$ is the probability that the government defaults on coupon payment $n$ conditional on its survival up to period $n - 1$, and $S_n$ is the cumulative probability of survival related to the history of $\pi$:

$$S_n = \prod_{i=1}^{n} (1 - \pi_i) \quad \wedge \quad S_0 = 1$$  \hspace{1cm} (2)

The intuition behind this pricing equation is that if the government has not defaulted on cash flow payments $n - 1$ and does not default at payment $n$, the creditor receives a future cashflow $CF_n$ discounted by $df_n$. If the government defaults at payment $n$, the creditor receives the recovery value discounted by $df_n$. The price of a bond now reflects the probability-weighted summation over all future cash flow dates.

In line with Vrugt (2011) and Merrick (2001), I assume a time-variant default rate. I follow Vrugt (2011) in employing a flexible parametrization for the probability of default that takes the form:

$$\pi_i = \alpha + \beta \times (1 - \exp(-t_i)) / t_i$$  \hspace{1cm} (3)

Here $\pi_i$ is the probability that the government will default until the next cash-flow payment at time $i$, with $t_i$ being the number of years until the next cash-flow from time $i$. As argued by Vrugt (2011), the 2-parameter specification of the default penalty allows for various slopes in the term structure of default rates.
Assuming that the expected recovery value at a given date is constant over time and across each outstanding bond, combining equations (1) through (3) results in the bond pricing equation

\[
P_0 = \sum_{n=1}^{N} df_n \left[ (CF_n \times \prod_{i=1}^{n} (1 - \{\alpha + \beta \times (1 - e^{-t_i}) / t_i\})) + (RV \times \prod_{i=1}^{n} (1 - \{\alpha + \beta \times (1 - e^{-t_i}) / t_i\}) \times \{\alpha + \beta \times (1 - e^{-t_n}) / t_n\}) \right]
\]

This is an equation in three unknowns: \(\alpha\), \(\beta\) and \(RV\). Note that the cash flow schedule for each bond and the associated discount value are known at every date. If we have a cross-section of \(J\) bonds issued by the same government with associated known prices, this yields a system of \(J\) non-linear equations in three unknowns. To solve for the three unknown parameters, I minimize the sum of squares pricing error:

\[
\min_{\alpha, \beta, RV} \sum_{j=1}^{J} \left( P_j - \hat{P}_j \right)^2
\]

Here at a given date, \(P_j\) is the observed market price of bond \(j\), while \(\hat{P}_j\) is the fitted price of bond \(j\) based on equation (4). I minimize this error by using the Broyden–Fletcher–Goldfarb–Shanno algorithm in the ’optim’ optimization suit of R. On the first day of my sample, I set initial guesses for all three parameters. On every consecutive day, the initial guess for the parameter is the estimate from the previous day. Since such non-linear optimization algorithms are not guaranteed to converge to the global minimum but may get stuck on a local one, I try out different starting values in my optimization to which my findings are robust.\(^7\)

Changes Around Court Rulings

Based on these daily estimates of country \(i\)'s recovery rate as well as the parameters alpha and beta, we can capture the implied recovery rate, as well as the implied term structure that the government will default over the next \(t\) years:

\[
\hat{RV}_d^i
\]

\[
\hat{p}_d^i(t) \equiv \hat{\alpha}_d^i + \hat{\beta}_d^i(1 - \exp(-t))/t
\]

Here, \(d\) indicates the day in 2016, while \(i\) indicates the country. I now proceed to capture the change in the estimated default probability, as well as the estimated recovery rate around each court ruling in my sample. I take the difference in the two estimates one trading day before each court ruling and one trading day after each court ruling as made explicit in equations (8) and (9). In other words, I opt for a two-day event window around each court ruling. Longer windows mean that the risk of capturing background noise increases. Shorter windows risk not capturing the entire response to the court ruling if effects are not transmitted instantaneously in financial markets. I choose not to use a 30-minute or 1-day window as is common in the monetary policy literature. Firstly, there are practical limitations. Times of court rulings are not available at an hourly granularity, which rules out event windows of less than a day. Secondly, debt litigation

\(^7\)In Figure A.1 in the appendix, I provide a graphical representation of how well the price estimates fit the actual bond prices on each date for the example of Brazil. While there are some discrepancies, this is to be expected to a certain extent. While the estimates do not exactly coincide with the observed bond prices, deviations are relatively minor.
cases do not communicate as clearly as a central bank and markets usually require some time to process what the ruling or the out-of-court settlement means. Furthermore, unlike central bank announcements where no information is shared before the official announcement, for court cases there are instances where the outcome of the case can be inferred before the official ruling. Thus, I take the day before the ruling as my baseline and compare it to the day after the ruling:

\[
\Delta \hat{RV}^{i}_{CR} = \hat{RV}^{i}_{CR+1} - \hat{RV}^{i}_{CR-1} \tag{8}
\]

\[
\Delta \hat{p}^{i}_{CR}(t) = \hat{p}^{i}_{CR+1}(t) - \hat{p}^{i}_{CR-1}(t) \tag{9}
\]

here, \( CR \) indicates the day of the court ruling, while \( CR - 1 \) and \( CR + 1 \) represent the first trading day before- and after the court ruling. \( \Delta \hat{RV}^{i}_{CR} \) reflects the change in the recovery rate around the court ruling on day \( d \) in country \( i \), and \( \Delta \hat{p}^{i}_{CR}(t) \) reflects the associated change in the default probability as laid out in equation (7).

Given these estimates, I then run a simple OLS regression of the change in the estimated default probability on the change in the estimated recovery value:

\[
\Delta \hat{p}^{i}_{CR}(t) = \gamma + \zeta \times \Delta \hat{RV}^{i}_{CR} + \epsilon^{i}_{CR} \tag{10}
\]

In this regression, I interpret the coefficient \( \zeta \) as the causal effect of a one percentage point increase in the recovery rate on the default probability. I argue causality in this setting, as the high-frequency change in the recovery rate around a court ruling plausibly represents exogenous variation in my explanatory variable. For this to hold, my instrument - the series of court rulings - must satisfy both the exclusion restriction and be relevant. Since I restrict my sample to court rulings under US jurisdiction, such court outcomes plausibly affect the recovery rate of USD-denominated Brazilian bonds as argued in section 2.1. The court rulings also plausibly satisfy the exclusion restriction. To best account for this, I only consider changes around court outcomes in lawsuits against other third countries. Only considering court rulings on third countries makes it unlikely, that a court ruling can affect the default probability through any channel other than through its effect on the recovery rate in US sovereign debt litigation cases. I further check that no other major domestic macroeconomic events overlap with the sovereign debt litigation outcome series. To do so, I check that there are no meaningful central bank press releases on such days.

2.4. Data

In my analysis, I focus on a sub-sample of the court rulings compiled by Schumacher et al. (2021). Namely, I analyze the court rulings from the year 2016. I chose this year as the scope of my analysis for two reasons. Firstly, 2016 exhibits a large number of court rulings, many of which were associated with significant changes in the CDS rate (Figure 2). This indicates a larger variation in the recovery rates and default probabilities which my approach can exploit. Secondly, high-frequency and granular data availability on a wide cross-section of historic bond prices becomes increasingly scarce the further back one goes. This negatively affects the accuracy of estimates for the default probability and the recovery rate for the next closer court rulings in the early 2010s.\(^8\)

\(^8\)I summarize the sovereign debt litigation cases that ended in 2016 and thus form the basis of my analysis in table A.II in the appendix.
Regarding the regional scope of my empirical exercise, I consider the effect of changes in the recovery rate on the default probability in Latin American Economies. Since all court rulings in 2016 were filed against Argentina, I expect to see the largest effects in Latin American bond markets, due to plausible structural similarities in the way in which debt is raised. Namely, I consider Latin American countries that exhibit a history of sovereign default as well as sufficiently deep external debt markets. Since granular data on outstanding amounts of external USD-denominated sovereign debt in 2016 are not available to me, I proxied deep debt markets by considering large economies with a gross domestic product of at least $50 billion and at least three eligible outstanding bond series in 2016 listed on Refinitiv Eikon.\(^9\) I will elaborate on the eligibility of bond series in the next paragraph. I further exclude Argentina from my analysis, since the court rulings and restructuring agreements in 2016 were predominantly filed against Argentina. Thus, to avoid possible endogeneity around court rulings, as well as volatility in estimates due to unreliable bond price data in this tumultuous year for Argentinian bonds, I omit Argentina. This leaves me with five countries as outlined in Table II.

Finally, I consider a bond series as ‘eligible’ if it is denominated in US dollars, and if it was trading in 2016. I further follow Vrugt (2011) in excluding zero-coupon bonds, inflation-linked bonds, bonds without daily prices, and bonds with a maturity of less than one month. I further select only a singular bond in case of dual-listings on different exchanges which exhibit the same bond price. I obtain all of my bond data from Reuter’s Eikon Refinitive platform. Table A.I in the appendix lists all bonds used in my analysis for which price data was available on all trading days of 2016. I limit my analysis to bonds denominated in USD, as court rulings under US jurisdiction likely have little to no impact on the recovery value of domestically issued bonds in local currency, or bonds issued in currencies other than USD.

Maturity dates for bonds in my sample range from 2017 to 2050, and the current coupon rate ranges from 3.0 percent of the par value to 12.75 percent. All bonds pay half of this coupon rate biannually in the form of fixed coupon payments. While there is no data available on the daily trading volume of these government bonds, all bond series comprise of substantial original issue amounts of around or mostly between $500 million and $1.5 billion. This indicates that the secondary markets in which these are traded exhibit some degree of liquidity so that the price likely reflects the underlying price. Notably, none of the local central banks engaged in quantitative easing or large-scale bond purchasing programs before 2020, which could distort prices. Indeed I do not observe significant discrepancies between the bid-, and ask price of these

### TABLE II: Regional Scope of the Analysis

<table>
<thead>
<tr>
<th>Country</th>
<th>Number of ‘eligible’ bonds</th>
<th>2016 GDP [billion USD]</th>
<th>#debt litigation cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>13</td>
<td>$1,795.7</td>
<td>1</td>
</tr>
<tr>
<td>Panama</td>
<td>13</td>
<td>$57.9</td>
<td>2</td>
</tr>
<tr>
<td>Peru</td>
<td>6</td>
<td>$191.9</td>
<td>8</td>
</tr>
<tr>
<td>Costa Rica</td>
<td>5</td>
<td>$58.9</td>
<td>2</td>
</tr>
<tr>
<td>Guatemala</td>
<td>3</td>
<td>$66.15</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: ‘Number of ‘eligible’ bond refers to external bond series traded in 2016, which have par-, and coupon payments denominated in USD with a plain vanilla fixed coupon payment, a maturation date of more than a month, for which data is available on Reuters. 2016 GDP refers to the countries’ 2016 GDP in billion current USD. # debt litigation cases refers to the number of litigation claims filed against the country according to Schumacher et al. (2021).

\(^9\)I require at least three bond prices to pin down the three parameters as laid out in section 2.3. For less than three bond series, the optimization problem is under-identified.
bonds at any date in my sample of bonds. I further find no major central bank announcements on fiscal or monetary policy that overlap with my date series of court rulings.

I obtain the coupon payment schedule from the bond description on Eikon. All bonds pay coupon rates biannually between the date of the first coupon payment and their maturity date. To proxy the risk-free discount factor, I use daily data on the par yield curve rates from the U.S. Department of Treasury.

### 2.5. Results

Table III shows regression coefficients and r-squareds for equation (10). I report the effect of a one percentage point increase in the recovery rate on the default probability of a sovereign over one-, five-, and ten years (on the left side). I do so for both a simple OLS regression, as well as for a country fixed effects regression. Each row in the table reflects the effect on a different time horizon, while the two columns reflect the two different regression approaches employed. The first column runs a simple OLS regression with an intercept, while the second column runs a regression with country-fixed effects.

Indeed I find strong empirical support that an exogenous increase in the recovery value due to a change in legal precedence causes a decrease in the market-perceived default probability across all specifications. Concretely, my sample of court rulings indicates, that a one percentage point increase in the recovery rate decreases the probability that a sovereign will default over the next year by 0.46 percentage points. The effect is statistically significant at the one percent level. Effect sizes grow as the time horizon over which we study default grows. A one percentage point increase in the recovery rate likewise decreases the probability that a sovereign will default over the next five (ten) years by 1.61 (1.89) percentage points. Effect sizes are similar when including country fixed effects. Effects are more statistically significant for shorter time windows. Default probabilities over the next five or ten years are statistically significant, but much less so than the effect on the probability of default over the next year. This is likely linked to more noise over longer time windows. Furthermore, changes in the recovery rate around court rulings can explain about one-third of the variation in changes in the default probability over the next year.

To conclude, the empirical evidence provided in this section underlines that as the economic cost of default exogenously increases for the sovereign, such an increase functions as a commitment device and reduces markets’ beliefs about the likelihood of a default event. This could help explain the two stylized facts on LMICs’ sovereign debt trajectories discussed in the introduction: rising debt-to-GDP ratios and a shift towards debt portfolios that are more painful

<table>
<thead>
<tr>
<th></th>
<th>-</th>
<th>( \Delta p ) Effect</th>
<th>( \Delta p ) ( R^2 )</th>
<th>Fixed Effects</th>
<th>( \Delta p ) Effect</th>
<th>( \Delta p ) ( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta p ) (1)</td>
<td>-0.46***</td>
<td>0.34</td>
<td>-0.44***</td>
<td>0.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td></td>
<td>(0.09)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta p ) (5)</td>
<td>-1.61*</td>
<td>0.05</td>
<td>-1.47</td>
<td>0.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.94)</td>
<td></td>
<td>(0.99)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta p ) (10)</td>
<td>-1.89*</td>
<td>0.05</td>
<td>-1.73</td>
<td>0.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.09)</td>
<td></td>
<td>(1.14)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: *\( p<0.1 \); **\( p<0.05 \); ***\( p<0.01 \). \( N = 55 \) in all regressions corresponding to the eleven court rulings in 2016 in five countries. I use HAC standard errors clustered at the entity level to calculate the standard errors for my fixed effects regression.
to restructure. In section 3, I now proceed to analyze what happens in a model setting where governments can endogenously choose their debt portfolio and by extension the economic cost associated with defaulting on this portfolio.

3. MODEL ENVIRONMENT

Moving to a setting where the government can make an endogenous choice about how painful it wants to make default for itself, consider a model that features a self-fulfilling debt crisis framework à la Cole & Kehoe (2000). I follow Conesa & Kehoe (2017) in omitting the representative households’ consumption-investment choice; Introducing capital into the model would be computationally straightforward, but I omit it to keep the model partly tractable and focus on the endogenous default penalty choice.

The major innovation of my proposed model environment is that I incorporate heterogeneity on the international lender’s side by including two types of creditors and bonds: painful debt and safe debt purchased by a unit mass of ‘painful’- and ‘safe’ creditors respectively. Here, the issuance of painful- or safe bonds shall reflect the real portfolio choice governments have in deciding who to borrow from and what types of debt they issue. This debt portfolio choice about painful- and safe debt shares then determines the default penalty which is modeled as a function of the painful debt-to-GDP ratio (ϕ): a higher (lower) reliance on painful debt is associated with a higher (lower) default penalty.10

On the production side, labor is supplied in-elastically and TFP is for now assumed to be constant over time, so aggregate output is based on ϕ, and whether a default has occurred in the past (omitting time subscripts for legibility):

\[ y(z_{-1}, \phi) = Z(\phi)^{1-z_{-1}} \bar{y} \] (11)

Here, \( z_{-1} \) is an indicator variable that equals one if no default has occurred in past or present periods, and zero if default has occurred. \( 1 - Z(\phi) \) reflects the default penalty \((0 < Z(\phi) < 1)\). The penalty for default is bigger, the smaller \( Z(\phi) \) is. Since I assume high reliance on painful debt to be more hazardous in sovereign debt crises, I assume \( Z(\phi) \) to be a strictly decreasing function in \( \phi \).

The state of the economy in every period is \( s_t = (B_t, z_{-1}, \zeta_t, \phi_t) \), where further \( B_t \) is the total level of government debt, and \( \zeta_t \) is the value of a sunspot variable à la Cole & Kehoe (2000). Note that the total debt stock \( B_t \), past default decisions \( z_{-1} \) and the painful debt GDP-share \( \phi_t \) uniquely pin down total safe debt \( B_t^S \) and total painful debt \( B_t^P \). Thus the two states \( s_t = (B_t, z_{-1}, \zeta_t, \phi_t) \) and \( s_t = (B_t^S, B_t^P, z_{-1}, \zeta_t) \) carry the same information, so I use them interchangeably throughout the paper to simplify notation.

There are three types of agents in this model: Households, the government, as well as creditors which can be of two observable types: 'safe'- and 'painful'. Households derive utility from individual consumption and government spending. In line with the literature, I assume a constant and exogenous tax rate \( \tau \). As a result, for the household, the consumption choice is static, and uniquely pinned down by output:

\[ c(z_t, \phi_t) = (1 - \tau) y(z_t, \phi_t) \] (12)

---

10 I define \( \phi_t = \frac{B_t^P}{B_t} \), where \( B_t^P \) is the amount of painful debt and \( y_t \) is GDP.
For the government, revenue equals $\tau y(z_t, \phi_t)$. I assume a benevolent government whose only objective is to maximize the lifetime household utility. Hence, the government’s optimization problem amounts to:

$$\max_{\{B^P_{t+1}, B^S_{t+1}, z_t, g_t\}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c(z_t, \phi_t), g_t) \right]$$

subject to the government’s budget constraint each period

$$g_t + z_t(B^P_t + B^S_t) = \tau y(z_t, \phi_t) + q^P_t B^P_{t+1} + q^S_t B^S_{t+1} \quad \forall t$$

Here, the government chooses how much safe- and painful debt to raise next period at prices $q^S_t$ and $q^P_t$, whether to default or not ($z_t$), and it chooses its expenditure ($g_t$). Note that it can only choose to default on all of its government debt, so a selective default on only safe-, or only painful bonds is not possible.

Both ‘safe’ and ‘painful’ creditors face the same optimization problem in the model. There is a continuum of measure one of both creditor types, who are both risk neutral and maximize their respective discounted future consumption $x^i_t$ for $i \in \{R, S\}$ subject to a budget constraint:

$$\max_{b^i_{t+1}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t x^i_t \right] \quad \text{s.t.} \quad x^i_t + q^i_t B^i_{t+1} = w^i_t + z_t b^i_t \quad \forall t$$

Here $i$ indexes the type of lender, and $w^i_t$ refers to the constant income of both creditor types. In line with the literature, I assume $w^i$ to be sufficiently large, to rule out corner solutions. I further impose $b^i_t \geq -A^i$ to rule out a Ponzi scheme, but set $A^i$ high enough to not bind in any other case. Setting up the lagrangian and differentiating with respect to $b^i_{t+1}$ yields

$$q^i_t = \beta E_t[z_{t+1} | s_t]$$

So by the static first-order condition of the creditors, prices for new bonds equal the expected discounted payoff determined by the expected default probability next period. Notably, prices for both bonds equal, so in the following I do omit the index to simplify notation. Prices equal, because I assume the same discount factor for both types of creditors and because both types of creditors have the same information set; that is, their beliefs about a government’s ability to repay next period are correlated.

Alongside the three agents and their maximization problems, I follow Cole & Kehoe (2000) in including a sunspot variable to deal with equilibria multiplicity. $\zeta_t$ is drawn from the uniform distribution on $[0, 1]$ in each period. Namely, if $\zeta_t > 1 - \pi$, both painful-, and safe creditors expect a crisis to occur and do not lend to the government if such a crisis would be self-fulfilling, i.e. if it would actually be optimal for the government to default. Here $\pi$ is exogenous. As described before, the sunspot for both types of investors are correlated, so I assume that they form their beliefs based on the same (exogenous) information set. The timing within each period is like that in Cole & Kehoe (2000):

1. The shock $\zeta_t$ is realized. The aggregate state is $s_t = (B_t, z_{-1}, \zeta_t, \phi_t)$, and the government chooses how much debt to sell $B^P_{t+1}, B^S_{t+1}$ which of course determines next period’s debt composition $\phi_{t+1}$.
2. Painful-, and safe creditors choose how much debt to buy respectively $b^P_{t+1}, b^S_{t+1}$. In equilibrium markets clear, so $b^P_t = B^P_t$ and $b^S_t = B^S_t$.
3. The government chooses whether or not to default on the outstanding debt it has to repay that period $B_t$. This decision determines $y_t, c_t$, and $g_t$. 

TYING YOURSELF TO THE MAST: PAINFUL DEBT AS A COMMITMENT DEVICE

15
3.1. Recursive Competitive Equilibrium

Recall that we defined our state to be \( s = (B, z_{-1}, \zeta, \phi) \). An equilibrium is a value function \( V_{g}(s) \), and policy functions \( B'(s), \phi'(s), z(B', \phi', s, q), g(B', \phi', s, q) \) for the government, a value function \( V_{i}^{*}(b, B', \phi', s) \), as well as policy functions \( b^{i*}(b, B', \phi', s) \) for painful and safe creditors, and a bond price function \( q(B', \phi', s) \) such that

1. Given the policy functions \( z(B', \phi', s, q), g(B', \phi', s, q) \) and the price function \( q(B', \phi', s) \), the government’s value and policy functions \( V_{g}(s), B'(s), \phi'(s) \) solve the government problem at the beginning of their period:

\[
V(B, \phi, z_{-1}, \zeta) = \max_{B', \phi'} \left[ \max_{c} u(c, g(B', \phi', s, q(B', \phi', s))) + \beta E[V(B', \phi', z, \zeta')] \right]
\]

\[
\text{s.t. } c = (1 - \tau)y(z(B', \phi', s, q), \phi) \tag{17}
\]
\[
g(B', \phi', s, q(B', \phi', s)) + z(B', \phi', s, q(B', \phi', s))B = \tau y(z(B', \phi', s, q), \phi) + q(B', \phi', s)B'
\]

2. \( b^{i*}(b, B', \phi', s) \) solves the creditors’ problems and \( q(B', \phi', s) \) is consistent with market clearing and rational expectations:

\[
B'(s) \in b^{i*}(b, B', \phi', s) \tag{18}
\]
\[
q(B', \phi', s) = \beta E[z(B', \phi', s, q(B', \phi', s))] \tag{19}
\]

3. Given the value function \( V_{g}(s) \), policy functions \( z(B', \phi', s, q), g(B', \phi', s, q) \) solve the government’s problem at the end of the period:

\[
\max_{z, g} u(c, g) + \beta E[V(B', \phi', z, \zeta')] \]

\[
\text{s.t. } c = (1 - \tau)y(z, \phi)
\]
\[
g + zB = \tau y(z, \phi) + qB' \tag{20}
\]
\[
z = 0 \text{ or } z = 1 \quad \text{but} \quad z = 0 \text{ if } z_{-1} = 0
\]

3.2. The Crisis Zone

Just like in Cole & Kehoe (2000), there are two cutoff levels for debt that define the crisis zone. In my model, these cutoff levels depend on \( \phi \), as \( \phi \) determines the default penalty. The cutoff levels are \( \bar{b}(\phi), \bar{B}(\phi) \):

1. If \( B \leq \bar{b}(\phi) \), the government never defaults, no matter what lenders do. Namely, even if lenders do not extend new credit \( (q = 0) \), it remains optimal for the government to repay the current debt level \( B \) and not default.

2. If \( \bar{b}(\phi) \leq B \leq \bar{B}(\phi) \) then the government does not default if and only if lenders extend new credit. So if lenders expect a default to happen, and do not lend \( (q = 0) \), then the value of defaulting actually exceeds the value of repaying debt. If lenders extend credit, the value of repaying exceeds the value of default. This is called the crisis zone for \( B \), as lenders’ default expectations become self-fulfilling. In the crisis zone, lenders’ beliefs about default are driven by the sunspot variable \( \zeta \): during a period, where \( \bar{b}(\phi) \leq B \leq \bar{B}(\phi) \) creditors expect a crisis and stop lending when \( \zeta > 1 - \pi \).

3. If \( B \geq \bar{B}(\phi) \) then the government always finds it optimal to default, no matter whether lenders extend new credit or not.
Since prices depend on the probability of default next period, and since the probability of default is constant in each of the three previously defined zones, we can write prices as a function of \(B'\) and \(\phi'\): In the no default zone, default never occurs, in the crisis zone, default occurs only if the sunspot variable is smaller than \(\pi\), which happens with probability \(\pi\), and in the always default zone, default always occurs.

\[
q(B', \phi', s) = \begin{cases} 
\beta & \text{if } B' \leq \bar{b}(\phi') \\
\beta(1 - \pi) & \text{if } \bar{b}(\phi') \leq B' \leq \bar{B}(\phi') \\
0 & \text{if } \bar{B}(\phi') \leq B' 
\end{cases} \tag{21}
\]

4. ANALYTICAL RESULTS

Before turning to the model calibration to derive global estimates for the government’s policy functions, I first analytically derive two key findings for governments’ optimal debt policy that hold in general. Namely, I find that by proposition 1, the lowest debt level at which a liquidity crisis can occur increases in the painful debt-to-GDP ratio. Secondly, proposition 2 shows that if a government finds itself in the crisis zone, a shift towards a more painful debt portfolio while keeping debt levels constant at the same time can be optimal for a government. Proofs of my propositions can be found in the appendix.

Turning to proposition 1 first, the lowest debt level at which a liquidity crisis can occur is defined as the debt level where a government is indifferent between defaulting and repaying if international bankers do not extend new credit. This is because in the safe-zone even if international bankers do not extend new credit, governments will prefer repaying all maturing debt obligations that period over defaulting. Likewise, in the crisis zone, a liquidity crisis can occur because the government prefers defaulting to repaying if it can’t roll over debt. Thus at the lower bound of the crisis zone - the lowest debt level at which a liquidity crisis can take place - a government must be indifferent between repaying and defaulting if international bankers do not buy bonds \((q = 0)\).

Under \((q = 0)\), I can find closed-form expressions for both the utility of defaulting \((V_g^d)\), as well as the utility of not defaulting \((V_g^n)\). For the latter, if creditors do not extend new credit, but the government does not default, it needs to repay all outstanding debt that period and is debt-free thereafter. For the former, if it defaults, the default penalty takes effect immediately. The associated utilities are:\(^{11}\)

\[
V_g^n(B, \phi, q = 0) = u((1 - \tau)\bar{y}, \tau\bar{y} - B) + \frac{\beta u((1 - \tau)\bar{y}, \tau\bar{y})}{1 - \beta} \tag{22}
\]

\[
V_g^d(B, \phi, q = 0) = \frac{u((1 - \tau)Z(\phi)\bar{y}, \tau Z(\phi)\bar{y})}{1 - \beta} \tag{23}
\]

Now, for a given \(\phi\), I denote the level of debt that makes a country indifferent between defaulting and repaying \((if q = 0)\) as \(\bar{b}(\phi)\). While it is not possible to find a closed-form expression for \(\bar{b}(\phi)\) in this general setting, I can capture the impact of \(\phi\) on \(\bar{b}(\phi)\) as laid out in proposition 1.

\(^{11}\)Note that I define the value function as a function of the debt level and the price that period. While the value function is a function of the state, the state ultimately pins down \(B', \phi',\) and through those also \(q\). So while the value function is a function of the state, the relevant components that pin down utility are the debt level and the price.
PROPOSITION 1: The lower bound of the crisis zone $\bar{b}(\phi)$ is increasing in the painful-debt-to-GDP ratio $\phi$.

PROOF: Appendix section A.1.

The intuition behind the proof of proposition 1 is as follows. First note that the utility of not defaulting ($V^n_g$) is decreasing in the total debt level $B$ and it is independent of the painful debt share $\phi$, as long as a change in $\phi$ does not affect the debt level $B$.\footnote{This can be achieved by the government as long as the safe debt-to-GDP ratio falls one-to-one with a rise in the painful-debt-to-GDP ratio.} This is because if the country repays all its outstanding debt in one period, the higher the debt, the lower government spending will be in the repayment period. However, since the government does not default, the default penalty does not apply. The utility of default ($V^d_g$) on the other hand is decreasing in the painful debt share ($\phi$), but it is independent of the total debt level $B$. This is because if the government defaults, a higher $\phi$ raises the default penalty which the government incurs for all remaining periods, but the default penalty is independent of the amount of debt defaulted on. Thus, if $\phi$ rises, to remain indifferent between defaulting and repaying under $q = 0$, $B$ must rise. As a result, $\bar{b}(\phi)$ is rising in $\phi$.

Proposition 1 is a key finding. It tells us that for constant debt levels $B$, the government can shift out the crisis zone by increasing $\phi$. Note that even though increasing the painful debt-to-GDP share $\phi$ would increase the total debt-to-GDP ratio $B$ under constant issuance of safe debt, a government can issue less safe debt and increase $\phi$ without changing $B$. Thus, by affecting the bond portfolio, the government can indeed affect the crisis zone without changing $B$.

Now turning to proposition 2, I can also show that shifting out the crisis zone is optimal policy for the government under certain conditions. To capture the government value function in the crisis zone, we need to account for the different options governments have for leaving the crisis zone. Similar to Cole & Kehoe (2000), I will assume that a government in the crisis zone will want to leave the crisis zone over $T$ periods depending on the current state $s_t$. In addition to the model by Cole & Kehoe (2000), a government now, however, can not only choose $T$ and the trajectory of $B$, but it can also choose the trajectory of $\phi$. In my model specification, a repayment plan consists of a choice over how many periods $T$ the government plans to leave the crisis zone, alongside a choice about the dynamics for debt levels and painful-credit shares over this window. The defining condition to leave the crisis zone after $T$ periods is

$$B_{t+T} = \bar{b}(\phi_{t+T})$$

(24)

The government chooses a repayment plan from the following set of all possible repayment plans

$$\mathcal{P} \equiv \{T, B_{t+1}, ..., B_{t+T}, \phi_{t+1}, ..., \phi_{t+T} \mid B_{t+T} = \bar{b}(\phi_{t+T})\}$$

PROPOSITION 2: If a country is in the crisis zone, that is if $\bar{b}(\phi) < B \leq \bar{B}(\phi), \zeta \leq 1 - \pi$, then if there exists a $\phi_{t+1}$ such that (i) $\bar{b}(\phi_{t+1}) \geq B_t$, (ii) $\phi_{t+1} \times y_t \equiv B_{t+1}^{R} \leq B_t$, then it is optimal for the government to choose the repayment plan $T = 1$ and $\phi_{t+1}$ s.t. $\bar{b}(\phi_{t+1}) \geq B_t$.

PROOF: Appendix section A.1.

The intuition for this finding is as follows. Note first that only a small subset of these possible repayment plans is optimal; If the government acts optimally, the choice of a repayment plan is
equivalent to simply choosing over how many periods $T$ to run down debt, and what $\phi_{t+T}$ to run down debt to. This is because by equation (24), $B_{t+T}$ is uniquely pinned down by the choice of $\phi_{t+T}$. Furthermore, as part of the proof for proposition 2, I find that it remains optimal policy to keep government spending constant along the repayment path to smooth household consumption, which uniquely pins down the dynamics for the debt levels given $B_t$, and $B_{t+T}$. I can further show that the utility from any repayment plan is strictly decreasing in $\{\phi_t, ..., \phi_{t+T-1}\}$, since these do not affect the level of debt to which the government deleverages, but just increases the probability weighted disutility from default throughout the repayment plan. As a result, these are optimally set as low as possible. So given $B_t$, and $\phi_t$, an optimal repayment plan is uniquely pinned down by the choice $T$ and $\phi_{t+T}$.

Finally, I can show, that across all these possible optimal repayment plans $\{T, \phi_{t+T} \mid T \in \mathbb{N}, \phi_{t+T} \in \mathbb{R}\}$, the government’s utility in the crisis zone can never be greater than the utility it derives from the repayment plan as part of which the government keeps debt levels constant and shifts out the crisis zone beyond its current debt level in one period. That is to say, if it is possible for a government to issue enough painful debt while keeping $B$ constant, it is optimal policy to do so if this rules out a liquidity crisis.

This is a key finding, which is visualized in figure (3). We can see that if a country can increase its default penalty enough to shift out the lower bound of the crisis zone to or beyond its current debt level, it finds it optimal to do so within one period. Intuitively, it finds it optimal to do so, as this attains the optimal utility it receives outside the crisis zone without any default risk and without the need to deleverage. Note that when increasing the painful debt-to-GDP share, the government will decrease the safe debt-to-GDP share one-to-one to sustain a constant debt level.

In figure (3), the black trajectory represents optimal policy in a setting, where the government starts out in the crisis zone, but can not affect its default penalty endogenously. Here, as in the standard Cole & Kehoe (2000) setting, it is optimal policy to run down debt over a number of periods until the debt level reaches the lower bound of the crisis zone and the threat of a liquidity crisis is eliminated. The blue trajectory on the other hand resembles optimal policy for the same initial debt level and painful debt-to-GDP share, but here the government can shift

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**Figure 3:** Optimal debt policy with for $\phi < \phi'$

![Figure 3](image-url)
out the crisis zone sufficiently far by increasing the share of painful debt \( \phi' \) they will hold. Here, by proposition 2, it is optimal policy to shift out the crisis zone beyond the current debt level. This way, the government can sustain the elevated debt level and avoids deleveraging as it would have under the scenario of an exogenous default penalty. Note however, that if the effect of \( \phi \) on \( Z \) is bounded, or the effect of \( Z \) on \( \bar{b}(\phi) \) is bounded, a country might not be able to shift out the crisis zone all the way to its current debt level. In this case, it must deleverage as it then has to choose an optimal repayment plan \( T, \phi_{t+T} \) where \( B_t > \bar{b}(\phi_{t+T}) \).

5. QUANTITATIVE RESULTS

In this section, I globally solve for the government’s policy functions, and further numerically quantify and motivate some of the key dynamics from section 3. To that end, I solve the full model computationally using global methods to evaluate when governments find it optimal to rely on painful credit and how much of a debt increase this might explain. I externally calibrate the model as described in section 5.1 and then use a value function iteration algorithm laid out in appendix B to derive value-, and policy functions of the recursive competitive equilibrium. The key contribution of this section is not to derive exact estimations of effects. Rather, I aim to show that the proposed mechanism yields reasonable effects for realistic parameter choices, and thus has plausibly contributed to the rising debt levels in LMICs over the past decade. I further show how a government will react if it is hit by an MIT shock to output at the beginning of the period.

5.1. Model Parametrization

To solve the model, I need to specify the functional form of the households’ utility function. Similar to Conesa & Kehoe (2017), I choose an isoelastic utility function for households, where the agents value consumption and government spending subject to a minimum government expenditure constraint. The minimum government expenditure constraint can be interpreted as the subsistence level of government spending that the society needs to function. Unlike Conesa & Kehoe (2017), however, I allow for various degrees of households’ risk aversion with regards to consumption and government spending as proxied by parameters \( \rho \) and \( \sigma \) in a constant relative risk aversion (CRRA) setting:

\[
u(c, g) = \frac{c^{1-\rho}}{1-\rho} + \gamma \frac{(g - g)^{1-\sigma}}{1-\sigma} - 1 \quad \text{where} \quad c \geq 0, g \geq g\]

Notably, the minimum government expenditure constraint implies a key property of the ‘tying yourself to the mast’ dynamic: a country can not shift out the crisis zone indefinitely as formalized in proposition 3.

PROPOSITION 3: In general, the lower bound of the crisis zone can never exceed \( \tau \bar{y} - g \).

\[\forall \phi \in \mathbb{R}^+ \quad \bar{b}(\phi) \leq \tau \bar{y} - g\]

PROOF: Appendix section A.1.

Intuitively, this result holds because in each period the government can never spend more than \( \tau \bar{y} - g \) on bond repayments without violating the minimum spending constraint. So if the debt level exceeds this value, the government will always prefer defaulting over repaying all outstanding debt if this can’t be rolled over.
With regard to the parameters of my model, I calibrate them to a period length of one year. The parameters chosen are laid out in table IV. Although these parameters are not derived from careful model calibration, they are illustrating that the proposed mechanism produces plausible effects for reasonable parameter choices in the context of LMICs.

As it turns out, one of the most central parameters in my calibration is the Arrow-Pratt measure of relative risk aversion $\sigma$. The size of the shift in the crisis zone is very closely linked to $\sigma$ as shown in figure 4: The higher the degree of relative risk aversion, the larger is the shift of the lower bound of the crisis zone $\bar{b}(\phi)$ if the default penalty increases. This means, that a government’s ability to shift out the crisis zone via an increased reliance on painful debt, depends on the government’s relative degree of risk aversion. Intuitively, only for high enough $\sigma$ does a change in the default penalty trigger a shift in the crisis zone. Intuitively, this is because, for small degrees of relative risk aversion, a government does not dislike a one-period slump in government spending strongly enough to be indifferent between the slump, and incurring the default penalty indefinitely. The higher the governments dislike for the temporary slump, the larger the shift in the lower bound of the crisis zone if the default penalty increases.

Although the literature on measuring relative risk aversion at the individual level is vast, there is not one commonly accepted estimate. In a standard CRRA utility function setting, estimates for the Arrow-Pratt coefficient of relative risk aversion generally lie between 1 and 3 but can be as high as 10 in some empirical analyses. Gandelman & Porzecanski (2013) for example find estimates between 0 and 3 for a group of 52 developing economies, whose distribution is skewed more heavily towards higher coefficients relative to the estimates for developed economies. Szpiro (1986) analyze a set of 20 developed economies and 11 developing economies and find estimates that lie between 1 and 5 with a mean of 2.89. In my main calibration, I follow the empirical work by Szpiro (1986) and choose degrees of relative risk aversion of 2.89 for both consumption and government spending.

This value is on the higher side of estimates for the Arrow-Pratt coefficient, but I also report how the crisis zone shifts out for lower values of $\sigma \in [0, 10]$ in figure 4 where all other parameters remain as described in table IV. Here I plot the lowest debt-to-GDP level at which a liquidity crisis could occur for different default penalties ($Z$), and different degrees of government risk aversion ($\sigma$). As shown in proposition 1, we see that the lower bound of the crisis zone is increasing in the default penalty: for any given $\sigma$ a higher default penalty is associated with a higher $\bar{b}(\phi)$. However we can further see, that for relatively small values of $\sigma$, this difference becomes negligible: for small values of $\sigma$, even for relatively low default penalties, $\bar{b}(\phi)$ has already nearly attained the upper bound specified in proposition 3. Hence for low values of $\sigma$, a higher default penalty cannot shift out $\bar{b}(\phi)$ substantially. Intuitively this happens, because households do not mind suffering low temporary government spending for a low degree of risk aversion as much, as for a higher degree of risk aversion. As a result, the higher the risk aversion, the lower the debt level at which a government is indifferent between repaying and defaulting if it can not issue new debt during that period. In my calibrated setting, I only observe sizable shifts in the lower bound of the crisis zone for $\sigma > 1.5$ (Figure 4).

As figure 4 indicates, the shift in the crisis zone is also very closely linked to the default penalty that the government faces for different portfolio choices. Again, there is a vast literature on economic penalties incurred by countries as a result of defaulting. Cole & Kehoe (1996)

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14 I assume equal degrees of relative risk aversion for both consumption and government spending, as these are thought of as highly substitutable in my model setting.
for example assume a relatively conservative default penalty of 5 percent in their numerical calibration for Mexico’s 1994-1995 debt crisis. More recent work by Mendoza & Yue (2012), or Sosa-Padilla (2018) find default penalties in the range of 6 – 12 percent, where the economic cost of default is endogenized: a sovereign default forces the private sector to use less efficient resource (Mendoza & Yue 2012) or disrupts domestic lending via domestic banks’ exposure to sovereign bonds (Sosa-Padilla 2018). Empirical estimates on the economic cost of government default also vary widely due to the endogenous relationship between sovereign debt crises and output contractions: default usually occurs as a result of a strong economic contraction. Empirical results in the literature are thus to be interpreted with caution and vary in magnitude.15

The empirical estimate that I will rely on in my model specification is based on Furceri & Zdzenicka (2012) who attempt to address the endogeneity concerns by employing a two-step GMM-system estimator, as well as a two-step GMM only for those debt crises that occurred in relatively good economic conditions. By analyzing data from 154 default episodes between 1970 and 2008, they find that debt crises produce significant and long-lasting output losses of about 10 percent. Their point estimate has a variance of 2.65. Thus, in my model setting, I will consider a low default penalty, that lies two standard deviations below the point estimate, as well as a high default penalty that lies 2 standard deviations above the point estimate. This yields $Z_{low} = 4.7$ and $Z_{high} = 15.3$.

For the exogenous probability of a self-fulfilling crisis in the crisis zone $\pi$, I rely on emerging markets’ credit default swap (CDS) spreads. The CDS spread is related to financial markets’ perceived probability that a country will default over the bond maturity. As is common in the literature, I calibrate the probability of a self-fulfilling crisis to Mexican data; concretely I look at the CDS spread on 5-year Mexican bonds on March 11, 2020, to proxy for financial markets beliefs about the default probability after a severe drying up of international credit after Figure 4.: The lower Bound of the crisis zone for different default penalties and degrees of risk aversion

Note: The model parameters other than $Z$ and $\sigma$, $\rho$ are as described in table IV. For a given $\sigma$ this figure shows by how much the lower bound of the crisis zone shifts if the default penalty increases from $Z_{low}$ to $Z_{high}$. The dashed line indicators $\sigma = 2.89$ as in my main model calibration.

15Sturzenegger (2002) suggest an accumulated 4 percent drop in output over the 4 years following a sovereign default. Borensztein & Panizza (2009) find only a small short-term impact of between 0.6 and 2.5 percent.
the world health organization declared COVID-19 a global pandemic. The CDS spread of 192.9 basis points corresponds to a perceived 3.2 percent default probability by market participants assuming a 40 percent recovery rate. This is in line with the exogenous probability of a self-fulfilling crisis of 4 percent reported in Conesa & Kehoe (2017) for distressed southern-European economies during the height of the euro crisis. My results are also robust to other plausible values of \( \pi \).

For the international bankers’ discount factor \( \beta \), I follow Conesa & Kehoe (2017), who assume a yield on safe bonds of 2 percent, which corresponds to \( \beta = 0.98 \) given a yearly frequency. Likewise, I follow them in assuming a government revenue share of 36 percent of output and a minimum government spending of 25 percent of GDP. Hence, in periods where no default penalty is in place, government revenue is 36 so a government will never choose to spend more than 11 percent of GDP on debt repayments in order to not violate the spending constraint.

5.2. Tying Yourself to the Mast

Given this model parametrization, I will now solve for the agents’ policy functions in the recursive competitive equilibrium laid out in section 3 where the government can endogenously issue two types of bonds: painful-, and safe bonds. To model the link between the bond portfolio choice and the default penalty, I further assume that each period the government has a binary choice for its painful debt share. It can either pick some \( \phi_{\text{low}} \) or some \( \phi_{\text{high}} \). These two debt shares are respectively associated with the two default penalties \( Z_{\text{low}} \) and \( Z_{\text{high}} \). If the share of painful debt is high, governments incur a large default penalty of \( 1 - Z = 15.3 \) percent. If the reliance on painful debt is low, they incur the low default penalty of \( 1 - Z = 4.7 \) percent. The default penalties are in line with the estimates from Furceri & Zdzienicka (2012).

While the reduction of the government’s painful debt-to-GDP ratio choice to a binary problem may seem restrictive at first, I do so for two reasons. First, if one allows for a continuous choice of \( \phi \), this also means that one must take a stance on the functional form of the relationship between \( \phi \) and the default penalty at every point of \( \phi \). Given that the painful-debt-to-GDP ratio is a simplifying abstraction from the government’s overall debt portfolio choice,

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16While the COVID-19 shock reflects both a drying up in international credit, as well as a fundamental shock to countries’ fiscal space, I assume that in the very early days of the pandemic, before the major asset purchasing programs of the Federal Reserve Board and the European Central Bank were announced - and while the economic costs of COVID-19 were largely speculative, the CDS spread more closely reflects the risk of a liquidity crisis. However, as it is likely an overestimate, I choose the CDS spread of a rather stable LMIC such as Mexico.
this seems over-assumptious and is impossible to link to actual data. The binary choice better reflects that the government can abstractly choose between different debt portfolios that affect its economic cost of default for two reasonable estimates of the default penalty based on Furceri & Zdzienicka (2012). Secondly, in my one-period bond setup, a binary choice of $\phi$ will yield identical results to a continuous choice of $\phi \in [\phi_{\text{Low}}, \phi_{\text{High}}]$. This is because a government would only ever optimally pick the border case. To see this, note that governments will always prefer to choose the lowest $\phi'$ possible as long as $\phi_{\text{High}}$ does not shift out the crisis zone beyond its current debt level exposing the government to the risk of a liquidity crisis. Likewise, the government will be indifferent between all $\phi'$ high enough (including $\phi_{\text{High}}$) s.t. the crisis zone is shifted beyond their current debt level because once the government leaves the crisis zone, default is off the equilibrium path. I arbitrarily set $\phi_{\text{low}} = 0$ and $\phi_{\text{high}} = 5.5$ however the results carry through for any choice of $\phi_{\text{low}} < \phi_{\text{high}}$ as long as $\phi_{\text{high}}$ is lower $\bar{b}(\phi_{\text{low}})$.

For this calibrated model, I can now use the value function iteration algorithm laid out in the appendix to derive value-, and policy functions for the agents in my economy. The algorithm used is an extended version of value function iteration algorithm used in Conesa & Kehoe (2017). In figure 5, I plot the policy function of next-period’s debt and next period’s painful debt share if the government that hasn’t defaulted before is currently at debt level $B$, at $\phi = \phi_{\text{Low}}$, and at $\zeta$ such that no liquidity crisis takes place this period. Here, we can see the core model dynamic at play. For a government that starts out in the lower range of the crisis zone at debt-to-GDP ratios of between 8.7 and 10.1 percent, we see a lack of deleveraging as governments keep their debt levels constant. They can so so, because they simultaneously increase the share of painful debt they rely on. Indeed, I find that governments with debt-to-GDP ratios between 8.7 and 14.1 percent (the shaded red region), choose $\phi_{t+1} = \phi_{\text{high}}$ to shift out the lower bound of the crisis zone in order to avoid deleveraging (for debt levels between 8.7 and 10.1 percent), or in order to deleverage less than they would have to otherwise (for debt levels between 10.1 and 14.1 percent). This is the first core finding, where governments ‘tie themselves to the mast’: they use the issuance of painful debt to sustain higher debt-to-GDP ratios and avoid deleveraging.

Figure 5.: Policy function for binary Choice of $\phi'$

Note: The red vertical lines represent the lower- and upper bound of the crisis zone. The solid red lines represent the crisis zone associated with the painful-debt-to-GDP ratio $\phi_{\text{Low}}$, while the dashed red vertical lines (upper bound is out of bound) represent the crisis zone for the elevated painful-debt-to-GDP ratio $\phi_{\text{High}}$. The shaded area represents the window of current debt, for which a country finds it optimal to choose $\phi' = \phi_{\text{High}}$. 
If their debt level is too high to shift out the lower bound of the crisis zone to their current debt level, governments use the issuance of painful debt to deleverage by less than they otherwise would have.

In my one-period bond setup, the amount by which the crisis zone shifts is still relatively small at about 1.5 percent of GDP. This is because, in the one-period bond setup, the lower bound of the crisis zone is still bounded above by a debt-to-GDP ratio of 11 percent according to my model calibration. Yet, these figures are not unlike what one can see for the aggregate debt data of LMICs of the past decade, where debt-to-GDP ratios have risen from about 7.7 to just under 11.1 percent between 2010 and 2020 (Figure 1a).

While this model solution can help explain why governments that face the risk of a liquidity crisis do not deleverage but rather rely on painful debt as a commitment device, the model does not yet explain why governments may increase their sovereign debt levels, or converge to such a point of high debt and high economic cost of sovereign default. To that end, I will now incorporate an unexpected MIT shock to output that hits the government at its steady state.

5.3. Unexpected Shocks to Output

So far the model has taken a somewhat static approach to modeling output, which has been assumed to be constant subject to a default penalty. I will now examine what happens when the government faces the same endogenous bond choice - choosing both how much debt to raise, and how painful this debt will be to restructure - but on top, the economy is then being hit by an unexpected one-off shock to output. The timing here is identical to the one laid out in section 3, but right before the sunspot variable \( \zeta \) is realized, output is hit by an unexpected shock \( \epsilon \). The timing is now as follows:

1. The economy is hit by a shock \( \epsilon \) so that output becomes \( y(z_{-1}, \phi) = Z(\phi)^{1-z_{-1}} \bar{y} - \epsilon \).
2. The shock \( \zeta_t \) is realized. The aggregate state is \( s_t = (B_t, z_{-1}, \zeta_t, \phi_t) \), and the government chooses how much debt to sell \( B_{t+1}^P, B_{t+1}^S \) which of course determines next period’s debt composition \( \phi_{t+1} \).
3. Painful-, and safe creditors choose how much debt to buy respectively \( b_{t+1}^P, b_{t+1}^S \). In equilibrium markets clear, so \( b_{t+1}^P = B_{t+1}^P \) and \( b_{t+1}^S = B_{t+1}^S \).
4. The government chooses whether or not to default on the outstanding debt it has to repay that period \( B_t \). This decision determines \( y_t, c_t, \) and \( g_t \).

Now, in the recursive competitive equilibrium, given policy functions for \( g(\cdot) \) pinned down by the government budget constraint and \( z(\cdot) \) pinned down by the sunspot variable and the crisis zone for each \( \phi \) as laid out in section 3, the government problem becomes

\[
V(B, \phi, z_{-1}, \zeta, \epsilon) = \max_{B', \phi'} \left( (1 - \tau) \left[ y(z(\cdot), \phi) - \epsilon, g(\cdot) \right] + \beta E \left[ V(B', \phi', z, \zeta') \right] \right) \\
\text{s.t.} \quad g(\cdot) + z(\cdot)B = \tau[y(z(\cdot), \phi) - \epsilon] + q(B', \phi', s)B'
\]

Note that \( \epsilon \) is an unexpected one-off shock to output, and \( V(B', \phi', z, \zeta') \) is known from the previous value function iteration for the static case without an MIT shock. Thus assuming that the government can again choose between \( \phi_{low} = 0 \) and \( \phi_{high} = 5.5 \) with associated default penalties of \( Z_{low} = 4.7 \) or \( Z_{high} = 15.3 \), one can solve the government problem with an MIT shock by maximizing over a grid of \( B' \) and \( \phi' \) to find the optimal choice of \( B' \) and \( \phi' \) given that one knows \( E[V(B', \phi', z, \zeta')] \) for each combination of \( B' \) and \( \phi' \).

Doing so yields the following policy functions for government debt and painful-debt-to-GDP shares summarized in Figures (6a) and (6b). A notable difference emerges. If governments are
hit by an unexpected shock to GDP at a debt level that places them in the safe-zone, they will optimally smooth out this perceived one-off shock by increasing debt - and if necessary, increasing their painful debt-to-GDP ratio. Given that the policy function for next period’s debt is consistently above the 45 degree line in the safe zone for both moderate and severe MIT shocks, if hit by a series of unanticipated MIT shocks, governments will converge to a high debt level at $B = \bar{b}(\phi_{\text{High}})$ while simultaneously shifting their sovereign debt portfolio from $\phi_{\text{low}}$ to $\phi_{\text{high}}$. This is the case both for a series of moderate and severe shocks, although the convergence to this point occurs faster if the economy is hit by larger consecutive and unanticipated shocks.

Thus, I can show that an endogenous portfolio choice and a shift towards more painfully to restructure debt portfolios not only allow governments to sustain alleviated debt levels that would otherwise make them prone to liquidity crises. If hit by an unexpected adverse shock to GDP, the issuance of debt that is more painful to restructure even allows governments to smooth consumption by increasing debt to an extent without facing the threat of a liquidity crisis by committing themselves to repay. If hit by a series of unexpected adverse shocks, in my model, governments will converge to a point of high debt and high economic cost of default. These trajectories qualitatively closely resemble the aggregate sovereign debt trajectories of LMIC’s throughout the decade preceding COVID-19.

6. CONCLUSION

In this paper, I provide a theory on self-fulfilling debt crises that accounts for the build-up of increasingly large and increasingly painful sovereign debt portfolios. I propose a mechanism in which governments figuratively 'tie themselves to the mast' by choosing sovereign debt portfolios that increase the economic cost of default. This portfolio choice commits the government to repay its debts and thus allows it to sustain higher debt levels without the immediate risk of
a liquidity crisis. Albeit momentarily ruling out a liquidity crisis, this mechanism can however lead to a dangerous spiral of increasingly high-, and increasingly painful to restructure debt portfolios over time.

The contribution I make in this paper is twofold. Empirically, section 2 shows that a rising economic cost of default does indeed function as a commitment device and reduces the expected likelihood of a default event. I can show that an increase in the recovery sovereigns expect to pay in a debt restructuring by one percentage point reduces the expected likelihood of a default event over the next year by 0.46 percentage points. Secondly, I can show that when the government can make an endogenous choice about the type of bond portfolio it issues, it can be optimal for a government to use this commitment channel, and ‘tie themselves to the mast’ to avoid liquidity crises at higher debt levels. I extend the framework by Cole & Kehoe (2000) to show that if governments can make an endogenous choice about their default penalty via their bond portfolio choice, under certain conditions they will find it optimal to increase their share of painful debt. In my model, I can show that governments optimally use the issuance of painful debt to sustain higher debt-to-GDP ratios and avoid deleveraging, or deleverage by less than they otherwise would have if their debt level is too high to shift out the lower bound of the crisis zone to their current debt level. When introducing an MIT shock to output to this static system, I can show that governments who face an unexpected GDP contraction issue more painful debt as they increase their debt-to-GDP ratio to smooth out the economic slump. This could explain aggregate sovereign debt trajectories as observed in LMICs throughout the decade preceding the COVID-19 pandemic. When calibrating my model for a one-period bond setting, the switch from a low painful debt to GDP ratio to a high painful debt to GDP ratio can explain increases in the total debt-to-GDP ratio from 8.7 to 10.1 percent of GDP. The size of this effect can account for a substantial share of the aggregate increase of LMICs’ debt-to-GDP ratios which have risen from about 7.7 to just under 11.1 percent between 2010 and 2020.

Now, while the shift towards a more painful sovereign debt portfolio does allow governments to temporarily avoid a liquidity crisis, there are severe risks associated with it. Namely, the extent to which a government can use larger default penalties to sustain higher debt levels is bounded. This means that a government that experiences a series of fiscal shocks will converge to a point of high debt, at a high default penalty beyond which it can not shift out the crisis zone any further to avoid deleveraging. If a government at this point is hit by further shocks, it is caught between a rock and a hard place. It needs to deleverage - possibly at a very inconvenient point in time - or it faces the threat of a very painful debt restructuring due to the portfolio choice it has made in the past. This scenario is not unlike the impact of the COVID-19 pandemic and other recent economic shocks for LMICs. Over the decade preceding the pandemic, many LMICs had already increased debt-, and default-penalty-levels. This debt trajectory may well have deprived LMICs from a stronger fiscal response to the pandemic, leaving very little margin for error. Given that there were severe concerns about another major debt crisis in developing economies due to the precarious debt-levels- and default-risks throughout much of 2020, governments had to keep fiscal spending low, and avoid running large deficits, to not risk a very painful self-fulfilling debt crisis (Stiglitz & Rashid 2020). Other LMICs that were less prudent, are currently facing very painful sovereign debt crises IMF (2023).

One way to prevent this debt-risk spiral from reoccurring would be the implementation of a more comprehensive global framework for sovereign debt. These would need to regulate how much painful debt the sovereign can take on. This would limit countries in their ability to heavily rely on painful debt, even though they may want to in the short run. Strengthening the sovereign’s legal stance in debt restructurings under foreign jurisdiction for example, could help reduce the risk stemming from the issuance of bonds to private creditors. The broad adoption of collective action clauses (CACs) and enhanced CACs in most emerging markets’ bond
contracts is a promising step. For bond contracts with a collective-action clause, changes to the bond agreement are binding for all holders of the debt security if the proposed modification is approved by a supermajority of bondholders. In case of a default or a debt restructuring, prior inclusion of a CAC reduces the risk of individual hold-outs, who block orderly debt restructuring proposals. This generally reduces the economic cost associated with full- or partial default. With regard to risks from fragmentation, a unified framework for bringing together creditors in debt restructurings could help reduce risk. The G20 has launched the Common Framework for Debt Treatment to address this threat of creditor fragmentation. However, countries requesting debt relief under the scheme have all experienced severe delays, which still reflect difficulties coordinating Paris Club and other creditors (IMF 2021). Likewise, more strict and unified regulations for commercial banks’ holdings of domestic sovereign bonds could reduce the risks associated with a sovereign-bank nexus. This is because if domestic banks are highly exposed to sovereign debt - particularly as part of their regulatory capital - then a sovereign default poses severe risks of an adverse feedback loop where a sovereign debt crisis causes a banking crisis, which further depresses output and tax revenue for the sovereign. If however, the exposure of domestic banks to sovereign debt is limited, or if they are required to also hold other safe assets, such as hard currencies, foreign bonds, or other reserve assets, this adverse feedback loop to a country’s macro-economic stability is less likely to manifest. These propositions could help limit the extent to which a government can double down on painful debt via an increased share of debt issued to private creditors or domestic banks. In general, unified frameworks that regulate economic costs from a sovereign default could be beneficial in helping to prevent the debt-risk spiral.

An interesting direction for further research could lie in the design of commitment devices for the government that are disliked by the government, but could yield societal benefits in the long run. This could come in the form of commitments to better governance, or climate protection that governments may dislike in the short run, but are beneficial in the long run.

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APPENDIX A: PROOFS, FIGURES, AND TABLES

A.1. Proofs of Propositions

Proof of Proposition 1

PROOF: The lower bound of the crisis zone $\bar{b}(\phi)$ is defined as the debt level at which a country is indifferent between defaulting or repaying if no new credit is extended; that is if $q = 0$. Recall the utilities for defaulting and repaying in this scenario as laid out in equation (22), and (23). Then $\bar{b}(\phi)$ must solve:

$$u((1 - \tau)\bar{y}, \tau\bar{y} - \bar{b}(\phi)) + \frac{\beta u((1 - \tau)\bar{y}, \tau\bar{y})}{1 - \beta} = u((1 - \tau)Z(\phi)\bar{y}, \tau Z(\phi)\bar{y})$$

(26)

One can see that the left-hand side of equation (26) is decreasing in $\bar{b}(\phi)$ but is independent of the default penalty $Z(\phi)$ conditional on the debt level, while the right-hand side is decreasing in the default penalty, but is independent of the debt level $\bar{b}(\phi)$. Thus, increasing $\phi$ decreases $Z(\phi)$ and $V'_{dg}(B, \phi, 1 = 0)$, so for the equation to hold, the left-hand side of equation (26) must decrease too, which it does for a higher $\bar{b}(\phi)$. Q.E.D.

Proof of Proposition 2

PROOF: In the following section, I will show that it is optimal policy to shift out the crisis zone beyond the current debt level and not deleverage if it is possible to do so. To that end, I will derive closed-form expressions for government utility in the safe-zone, in the crisis zone, and for a general repayment plan $\{T, \phi_{t+1}\}$. I will then show, that the utility of all possible repayment plans is bounded above by the safe-zone utility of the same debt level, and that this utility can be attained by a repayment plan where $T = 1$ and $\bar{b}(\phi_{t+1}) \geq B_t$.

To that end, I first show that in my simple model, optimal government consumption is constant along the equilibrium path in which no default takes place.

LEMMA 1: Optimal government spending is constant along the equilibrium path in which no default takes place at:

$$\bar{g} = \tau\bar{y} - (1 - \beta)B_t$$

(27)

PROOF: The government problem when default doesn’t occur (i.e. $\pi = 0$ and $B_t \leq \bar{B}(\phi)$) has the following associated Lagrangian:

$$\mathcal{L} = \sum_{t=t_0}^{\infty} \beta^t \{u((1 - \tau)\bar{y}, g_t)\} + \lambda_t[\tau\bar{y} + \beta B_{t+1} - g_t - B_t]$$

(28)

The first order conditions with respect to $g_t$ and $B_{t+1}$ imply

$$\beta^t u_g((1 - \tau)\bar{y}, g_t) = \lambda_t$$

$$\lambda_{t+1} = \beta \lambda_t$$

$$u_g((1 - \tau)\bar{y}, g_t) = u_g((1 - \tau)\bar{y}, g_{t+1})$$

(29)
By equation (29), along the equilibrium path, \( g_t = \bar{g} \). Further, the transversality condition requires

\[
\lim_{t \to \infty} \lambda_t B_{t+1} = \lim_{t \to \infty} \beta^t u_g((1 - \tau)\bar{y}, \bar{g}) B_{t+1} = 0
\]

(30)

Iterating the government budget constraint forward, and using the TVC (30) I find:

\[
B_t = \lim_{\tau \to \infty} \left[ \sum_{t=0}^{\infty} \beta^t (\tau\bar{y} - \bar{g}) + \beta^{T+1} B_{T+1} \right] = \frac{\tau\bar{y} - \bar{g}}{1 - \beta}
\]

(31)

So if at time \( t \), we have some debt level \( B_t \), s.t. the country is not in the crisis zone, it is optimal policy to keep debt levels constant with

\[
\bar{g} = \tau\bar{y} - (1 - \beta) B_t
\]

(32)

Q.E.D.

Given that we know \( \bar{g} \) from Lemma 1, and \( \bar{c} \) is trivial, along the equilibrium path where no default occurs, we can write down a closed form expression for the government’s value function in the safe zone:

\[
V_g(B, \phi, z^{-1} = 1, \zeta) = \frac{u(((1 - \tau)\bar{y}, \tau\bar{y} - (1 - \beta)B)}{1 - \beta} \quad \text{for} \quad B < \bar{b}(\phi)
\]

(33)

Likewise, trivially, we know the utility in the always default zone:

\[
V_g(B, \phi, z^{-1} = 1, \zeta) = \frac{u((1 - \tau)Z(\phi)\bar{y}, \tau Z(\phi)\bar{y})}{1 - \beta} \quad \text{for} \quad B > \bar{B}(\phi)
\]

(34)

Thus, we can derive closed-form expressions for government utility if the government is in the safe-zone, or in the always default zone. In the following I will find a closed form expression for all possible optimal repayment paths a government can choose if it finds itself in the crisis zone.

In general, for all repayment plans In the crisis zone, I similarly find that optimal government spending along a repayment path is constant:

**Lemma 2:** If a government has debt level \( B_t \) such that it could face a liquidity crisis, if it chooses to reduce debt over \( T \) periods to \( B_{t+T} = \bar{b}(\phi_{t+T}) \), then the constant government expenditure over all such periods \( t, ..., t + T - 1 \) is constant at

\[
\bar{g}(B_t) = \tau\bar{y} - \frac{1 - \beta(1 - \pi)\bar{b}(\phi_{t+T})}{1 - \beta(1 - \pi)}
\]

\[
(B_t - \beta(1 - \pi)^{T-1}\bar{b}(\phi_{t+T}))
\]

(35)

**Proof:** Lemma 1 yields that under optimal policy, along the equilibrium path on which no default occurs, government spending must remain constant, smoothing consumption, as the government adjusts debt levels and bond composition \( \phi \):
The government budget constraints are:

$$\bar{g}(B_t) + B_t = \tau \bar{y} + \beta (1 - \pi) B_{t+1}$$
$$\bar{g}(B_t) + B_{t+1} = \tau \bar{y} + \beta (1 - \pi) B_{t+2}$$
$$\vdots$$

$$\bar{g}(B_t) + B_{t+T-2} = \tau \bar{y} + \beta (1 - \pi) B_{t+T-1}$$
$$\bar{g}(B_t) + B_{t+T-1} = \tau \bar{y} + \beta \bar{b}(\phi_{t+T})$$

(36)

Multiplying each equating by $(\beta (1 - \pi))^k$ for $k = 0, \ldots, T - 1$ and adding, I obtain

$$\sum_{t=0}^{T-1} (\beta (1 - \pi))^t \bar{g}(B_t) + B_t = \sum_{t=0}^{T-1} (\beta (1 - \pi))^t \tau \bar{y} + (\beta (1 - \pi))^{T-1} \beta \bar{b}(\phi_{t+T})$$

$$\bar{g}(B_t) = \tau \bar{y} - \frac{1 - \beta (1 - \pi)}{1 - (\beta (1 - \pi))^T} (B_t - (\beta (1 - \pi))^{T-1} \beta \bar{b}(\phi_{t+T}))$$

(37)

Q.E.D.

Based on Lemma 2, we know the entire path of both government spending and debt based on the observable level of $B_t$ and the choice $B_{t+T}$, which is uniquely determined by $\phi_{t+T}$. Knowing this, we can now also find a closed form expression for the government’s utility associated with every possible optimal repayment plan $\{T, \phi_{t+T}\}$.

**Lemma 3:** The closed-form expression for the utility of leaving the crisis zone over $T$ periods and reaching debt level $\phi^{t+T}$ is:

$$V_{T, \phi^{t+T}} (s_t) = \frac{1 - (\beta (1 - \pi))^T}{1 - \beta (1 - \pi)} u \left( (1 - \tau) \bar{y}, \bar{g}(B_t) \right) + (\beta (1 - \pi))^{T-1}$$

$$\times \frac{\beta u((1 - \tau) \bar{y}, \tau \bar{y} - (1 - \beta) \bar{b}(\phi_{t+T}))}{1 - \beta}$$

$$+ \sum_{j=0}^{T-1} (\beta (1 - \pi))^j \frac{\beta \pi}{1 - \beta} u((1 - \tau) Z(\phi_{t+j}) y, \tau Z(\phi_{t+j}) \bar{y})$$

Proof: Define the utility of repaying over $T$ periods and reaching debt level $\phi^{t+T}$ as $V_{T, \phi^{t+T}} (s_t)$. Now let $V_{k, \bar{T}, \phi^{t+T}} (B_t)$ be the value of the optimal repayment plan where a country reduces debt to $B_{t+T} = \bar{b}(\phi_{t+T})$ over $T$ periods when there are still $k$ periods to go in changing debt and potentially increasing the risky credit share. From Lemma 2: we know that government spending is constant. The respective value functions are:

$$V_{T} (B_t, \phi_t) = u \left( (1 - \tau) \bar{y}, \bar{g}(B_t) \right) + \beta (1 - \pi) V_{T-1} (B_t, \phi_t)$$

$$+ \frac{\beta \pi u((1 - \tau) Z(\phi_t) y, \tau Z(\phi_t) \bar{y})}{1 - \beta}$$

(38)

$$V_{T-1} (B_t, \phi_t) = u \left( (1 - \tau) \bar{y}, \bar{g}(B_t) \right) + \beta (1 - \pi) V_{T-2} (B_t, \phi_t)$$
of debt that needs to be shed in order to leave the low as possible in this setting of one-period bonds. Intuitively, only: they raise the default penalty without reducing after reaching . represents the expected discounted utility from repayment periods in which no default occurs, expected utility from Lemma 3 can be broken down into three components: the first term, which increases from \( \bar{y} \) to \( \tau \bar{y} - (1 - \beta) \bar{b}(\phi_{t+T}) \) in period \( T \). We can use backwards induction to find an expression for \( V^T_t(B_t) \):

\[
V^T_t(B_t, \phi_t) = u \left( (1 - \tau) \bar{y}, \bar{g}(B_t) \right) + \frac{\beta u ((1 - \tau) \bar{y}, \tau \bar{y} - (1 - \beta) \bar{b}(\phi_{t+T}))}{1 - \beta}
\]

Note that \( g \) increases from \( \bar{g}(B_t) \) to \( \tau \bar{y} - (1 - \beta) \bar{b}(1) \) in period \( T \). Based on Lemma 3, we know that the utility of being at state \( s_t \) in the crisis zone is the maximum over all possible repayment plans; that is the maximum over all possible combinations
of $T, \phi_{t+T}$, which represents all possible optimal repayment plans. Thus the general value of not defaulting in the crisis-zone is defined as

$$V_{g}^{n}(B, \phi, q = (1 - \pi)\beta) = \max[V_{1,}\star(B, \phi), ..., V_{\infty,}\star(B, \phi)]$$  \hspace{1cm} (44)$$

where $\star$ denotes the optimal choice of $\phi_{t+T}$ for each $T$.

Now in a last step, I show that this set of utilities across possible repayment plans is bounded above by the utility a government derives from shifting out the crisis zone beyond its current debt level without adjusting the debt level. This is to show, that if possible, it is indeed optimal policy to shift the crisis zone out instead of deleverage.

To that end, first note, that the value of being in the crisis zone from equation (44) must be bounded above by the value of having the same amount of debt, but being in the safe zone (equation 33). Consider if it were not: a government in the crisis zone would need to choose a certain repayment plan $T, \phi_{t+T}$. The counterfactual government at the same debt level $B$ that is not in the crisis zone, could choose to follow the exact same trajectory of $B$ and $\phi$ and get the exact same utility for each possible repayment plan. Yet we showed earlier in Lemma 1, that it prefers not to do so. Thus the utility that the government derives from sustaining constant debt in the safe zone (equation 33) must be greater or equal to the value in the crisis zone. So $V_{g}^{n}(B, \phi, q = (1 - \pi)\beta)$ must be bounded above by the safe-zone utility from equation (33) for the same debt level. This is to say that for $\bar{b}(\phi) < B \leq \bar{B}(\phi)$, $\zeta \leq 1 - \pi$ - a government in the crisis zone, that does not face a self-fulfilling rollover-crisis in the present period - we have

$$V(B, \phi, z_{-1} = 1, \zeta) \equiv \max[V_{1,}\star(B, \phi), ..., V_{\infty,}\star(B, \phi)] \leq \frac{u((1 - \tau)\bar{y}, \tau \bar{y} - (1 - \beta)\bar{b})}{1 - \beta}$$

Now I can lastly show, that if a government is in the crisis zone and if it can shift out the crisis zone sufficiently far, the utility it receives from doing so equals this upper bound. Indeed it is the case, that the following repayment plan

$$\{T, \phi_{t+1} \mid T = 1, B_{t} \leq \bar{b}(\phi_{t+1})\}$$

for which we know the associated optimal government spending by Lemma 2

$$\bar{g}(B_{t}) = \tau \bar{y} - (1 - \beta)\bar{b}(\phi_{t+1})$$

yields the following utility for the government, which equals the associated safe-zone utility:

$$V_{1,}\star(B_{t}, \phi_{t}) = u((1 - \tau)\bar{y}, \bar{g}(B_{t})) + \frac{\beta u((1 - \tau)\bar{y}, \tau \bar{y} - (1 - \beta)\bar{b}(\phi_{t+1}))}{1 - \beta} = \frac{u((1 - \tau)\bar{y}, \tau \bar{y} - (1 - \beta)\bar{b}(\phi_{t+1}))}{1 - \beta} \equiv V(B \leq \bar{b}(\phi), \phi, z_{-1} = 1, \zeta)$$  \hspace{1cm} (45)$$

Q.E.D.

**Proof of Proposition 3**

Define $B(\tau, \bar{y}, g)$ as the set of all possible lower bounds of the crisis zone for any combination of model parameters given values for $\tau$, $\bar{y}$, and $g$.  

**PROOF:** Proof by contradiction that $\tilde{b}$ is an upper bound for $B$: Assume that there exists a $B_t > \tau \bar{y} - g$ s.t. $B_t \in \mathcal{B}$. Given that $B_t$ is a lower bound of the crisis zone, there must be some model parametrization, s.t. at $B_t$ the government is indifferent between repaying and defaulting if no new credit is extended ($B_{t+1} = 0$). According to the government’s budget constrained in equation (14)

$$g = \tau y(z) - zB_t < \tau y(z) - z(\tau y(z) - g) = \begin{cases} \frac{g}{\tau Z \bar{y}} & \text{if } z = 0 \\ \frac{g}{\tau Z \bar{y}} & \text{if } z = 0 \end{cases}$$

Note however, that if the government doesn’t default, this implies that $g < g$, which violates the minimum government spending constraint. Under no model parameterization would a government be indifferent between violating this constraint, or defaulting - the government would always choose to default. Thus, for all $B_t > \tau \bar{y} - g$, $B_t \notin \mathcal{B}$, which makes it an upper bound.

$Q.E.D.$
### TABLE A.I: Eligible bond series in selected Latin American countries

<table>
<thead>
<tr>
<th>ISIN</th>
<th>First Coupon Payment</th>
<th>Maturity Date</th>
<th>Current Coupon</th>
<th>Original Issue Amount (USD m)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Brazil</strong></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>US105756BV13</td>
<td>Jan 06 2014</td>
<td>Jan 06 2025</td>
<td>4.25</td>
<td>3,250</td>
</tr>
<tr>
<td>US105756BU30</td>
<td>Jan 04 2013</td>
<td>Jan 04 2023</td>
<td>2.625</td>
<td>1,350</td>
</tr>
<tr>
<td>US105756BQ28</td>
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<td>Jan 14 2019</td>
<td>5.875</td>
<td>1,025</td>
</tr>
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<td>Jan 21 2021</td>
<td>4.875</td>
<td>788</td>
</tr>
<tr>
<td>US105756BH29</td>
<td>Jan 14 2006</td>
<td>Jan 14 2018</td>
<td>8</td>
<td>4,509</td>
</tr>
<tr>
<td>US105756BF62</td>
<td>Aug 03 2005</td>
<td>Feb 03 2025</td>
<td>8.75</td>
<td>1,250</td>
</tr>
<tr>
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<td>Apr 13 2005</td>
<td>Oct 13 2019</td>
<td>8.875</td>
<td>1,000</td>
</tr>
<tr>
<td>US105756AZ36</td>
<td>Oct 14 2003</td>
<td>Apr 14 2024</td>
<td>8.875</td>
<td>833</td>
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<td>Apr 14 2024</td>
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<td>Jan 14 2020</td>
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<tr>
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<td>Jul 20 2004</td>
<td>Jul 20 2033</td>
<td>8.25</td>
<td>1,500</td>
</tr>
<tr>
<td><strong>Panama</strong></td>
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<td>PAL63444SHA6</td>
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<td>Dec 05 2018</td>
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<td>PAL63444SPA9</td>
<td>Aug 07 2014</td>
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<td>Mar 30 2027</td>
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<td>US698299AY01</td>
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<td>Dec 24 2017</td>
<td>5.0</td>
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<td>Mar 22 2024</td>
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<td>Sep 16 2024</td>
<td>3.75</td>
<td>1,250</td>
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<tr>
<td><strong>Peru</strong></td>
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<td>US715638AP79</td>
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<td>May 21 2033</td>
<td>8.75</td>
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<td>Jan 21 2024</td>
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<td><strong>Costa Rica</strong></td>
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<td>US221597BR74</td>
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<td>Sep 12 2044</td>
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<td>700</td>
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Note: These are all bond series issued by Brazil that were traded in 2016, which have par-, and coupon payments denominated in USD with a plain vanilla fixed coupon payment for which data is available on Reuters. All of these bond series pay coupon payments semi-annually.
TABLE A.II: Sovereign Debt Litigation cases that ended in 2016

<table>
<thead>
<tr>
<th>End of litigation</th>
<th>Case citation</th>
<th>Debtor country</th>
<th>Plaintiff</th>
<th>Litigation outcome</th>
<th>Face value (USD m)</th>
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</thead>
<tbody>
<tr>
<td>Dec 20 2016</td>
<td>07 Civ. 08000</td>
<td>Argentina</td>
<td>Banca Nazionale Del Lavoro</td>
<td>OCS</td>
<td>23</td>
</tr>
<tr>
<td>Dec 04 2016</td>
<td>04 Civ. 07643 et al.</td>
<td>Argentina</td>
<td>Greylock Global Distressed Debt Master Fund Ltd. and Greylock Global Opportunity Master Fund Ltd.</td>
<td>OCS</td>
<td>205.85</td>
</tr>
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<td>Nov 28 2016</td>
<td>05 Civ. 00277</td>
<td>Argentina</td>
<td>Banca Amer S.A.</td>
<td>OCS</td>
<td>-</td>
</tr>
<tr>
<td>Nov 07 2016</td>
<td>06 Civ. 14339</td>
<td>Argentina</td>
<td>BNP Paribas</td>
<td>OCS</td>
<td>147.94</td>
</tr>
<tr>
<td>Aug 11 2016</td>
<td>06 Civ. 01839</td>
<td>Argentina</td>
<td>ARTAL Alternative Treasury Management</td>
<td>OCS</td>
<td>103.8</td>
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<td>Jun 27 2016</td>
<td>07 Civ. 05593</td>
<td>Argentina</td>
<td>Andarex Ltd.</td>
<td>OCS</td>
<td>2.63</td>
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<td>May 04 2016</td>
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<td>Capital Markets Financial Services Inc.</td>
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<td>05 Civ. 10201 et al.</td>
<td>06 Civ. 05887</td>
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<td>Los Angeles Capital</td>
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<td>OCS</td>
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<td>02 Civ. 03804</td>
<td>Argentina</td>
<td>Lightwater Corporation Ltd.</td>
<td>OCS</td>
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<td>EM Ltd.</td>
<td>OCS</td>
<td>203.35</td>
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<td>03 Civ. 08845 et al.</td>
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<td>05 Civ. 03328</td>
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<td>09 Civ. 08757 et al.</td>
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<td>A. Gandola &amp; C. S.P.A.</td>
<td>OCS</td>
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Note: These are all court rulings against sovereigns under US Jurisdiction which ended in 2016. None of these litigation cases included court cases under jurisdictions other than the United States. OCS is short for out-of-court settlement.

Figure A.1.: Fit of the bond pricing model for Brazil in 2016
APPENDIX B: THE ALGORITHM FOR COMPUTING DEBT MATURITY EQUILIBRIA

The proposed algorithm is a modified version of the algorithm proposed by Conesa & Kehoe (2017) and provides the debt thresholds defining the crisis zone, value functions, policy functions, and bond prices, for the government problem.

1. For two values of \( Z (Z_{\text{High}} \text{ and } Z_{\text{Low}}) \), conduct steps 1.a to 1.e
   
   (a) For the parametrization laid out in table IV, compute the value function of being in the default state as defined in equation 23. The value function takes this value whenever the state of the economy implies a crisis in the past, or in the present. Since we know the value function in the case of default, from this point on, I follow Conesa & Kehoe (2017) in dropping \( z_{-1} \) and \( \zeta \) from the state space to simplify notation and just focus on computing the value function in the case of no default. That is, I compute \( V(B) \) if default hasn’t happened in the past, and is not happening in the present.
   
   (b) I guess initial values for the threshold levels of the crisis zone \( \bar{b} \) and \( \bar{B} \). Since I can compute \( \bar{b} \) for any given combination of parameters, my initial guess for \( \bar{b} \) is its theoretical value as per Equation 26.
   
   (c) Now I perform Value Function Iteration on a finite grid of 100 points of debt. I guess an initial value function \( \tilde{V}(B) \), and an optimal debt policy. Prices are given by equation (21) and only depend on whether a grid point lies in the safe-, crisis-, or default zone.

   • For initial debt level \( B \leq \bar{B} \) we know that by assumption a government does not find it optimal to default this period, so the value function is

     \[
     V(B) = \max[V_1(B), V_2(B)]
     \]

     where

     \[
     V_1(B) = \max u((1 - \tau)\bar{y}, \tau \bar{y} + \beta B' - B) + \tilde{V}(B') \quad \text{s.t.} \quad B' \leq \bar{b}
     \]

     \[
     V_2(B) = \max u((1 - \tau)\bar{y}, \tau \bar{y} + \beta (1 - \pi) B' - B)
     \]

     \[
     + \beta(1 - \pi)\tilde{V}(B') + \beta \pi V^d_g \quad \text{s.t.} \quad \bar{b} < B \leq \bar{B}
     \]

     So for each node of \( B \), we loop through the grid of \( B' \) and based on the guesses for the thresholds, update \( V(B) \) and optimal debt policy based on the best choice of \( B' \) in the safe zone \( (V_1) \), or in the crisis zone \( (V_2) \).

   • For debt values in the default zone, we set the value function to the default utility calculated earlier.

   • If \(|V(B) - \tilde{V}(B)| > \epsilon\) for some tolerance level \( \epsilon \), then repeat step c. else go to d.

   (d) Update the threshold values:

   • Choose \( \bar{b} \) to be the highest point in the grid for which

     \[
     u((1 - \tau)\bar{y}, \tau \bar{y} - B) + \beta V(0) \geq V^d_g
     \]

   • Choose \( \bar{B}_{\text{New}} \) to be the highest point in the grid for which

     \[
     V(B) \geq u((1 - \tau)Z\bar{y}, \tau Z\bar{y} + \beta (1 - \pi)B') + \beta V^d_g
     \]

   (e) If \(|\bar{b}_{\text{New}} - \bar{b}| > \epsilon\), or \(|\bar{B}_{\text{New}} - \bar{B}| > \epsilon\), then \( \bar{b} = \bar{b}_{\text{New}} \) and \( \bar{B} = \bar{B}_{\text{New}} \) and go back to step c. Else exit and retrieve the value and policy functions alongside the thresholds for each point on the grid for \( Z \).
2. I take the threshold levels of the crisis zone $\bar{b}(\phi)$ and $\bar{B}(\phi)$ for given. Assuming that the government is at the state where $\phi = \phi_{\text{Low}}$ and for the parametrization laid out in table IV, I also compute the value function of being in the default state according to starting at $Z_{\text{Low}}$.

3. Now I again perform Value Function Iteration on a finite grid of 100 points of debt and two grid points. I guess an initial value function $\tilde{V}(B, \phi_{\text{Low}})$, and an optimal debt policy $B'$ and a policy for next periods painful debt-to-GDP ratio $\phi'$.

Now for both $\phi' \in \{\phi_{\text{Low}}, \Phi_{\text{High}}\}$, do the following:

- For the corresponding bounds of the crisis zone $\bar{b}(\phi')$ and $\bar{B}(\phi')$, for initial debt levels $B < \bar{B}$, we again know by assumption that the government finds it optimal to default this period, so the value function is

$$V(B, \phi) = \max[V_1(B, \phi), V_2(B, \phi)]$$

where

$$V_1(B, \phi) = \max u((1 - \tau)\bar{y}, \tau y + \beta B' - B) + \tilde{V}(B', \phi') \quad \text{s.t.} \quad B' \leq \bar{b}(\phi')$$

$$V_2(B, \phi) = \max u((1 - \tau)\bar{y}, \tau y + \beta(1 - \pi)B' - B) + \beta(1 - \pi)\tilde{V}(B', \phi') + \beta \pi V_{d}(\phi') \quad \text{s.t.} \quad \bar{b}(\phi') < B \leq \bar{B}(\phi')$$

So for each node $B$ we loop through the grid of $B'$ and $\phi'$ and based on the guesses for the thresholds, update $V(B)$ and optimal debt policy based on the best choice of $B'$ and $\phi'$ in the safe zone ($V_1$), or in the crisis zone ($V_2$).

- For debt values in the default zone, we set the value function to the default utility calculated earlier.

- If $|V(B) - \tilde{V}(B)| > \varepsilon$ for some tolerance level $\varepsilon$, then repeat step 3. else finish.