The Network Effects of Deforestation in Brazil

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1 Model

The model builds on Long & Plosser (1983) and Acemoglu et al. (2012). Consider a static economy which is populated by a representative consumer, a representative exporting firm and I competitive firms, indexed by $i \in \mathcal{I} = \{1, ..., I\}$. There is a subset of firms that are agricultural producers $\mathcal{A} \subseteq \mathcal{I}$. The output of each firm can either be consumed, exported or used as an intermediate input to the production of other goods. Each agricultural producer is endowed with a rural property of size s_i that is available to either agriculture l_i or to forest f_i and owned by a land owner.

Precipitation. Changes in the land use of rural properties result in changes in the amount of precipitation \tilde{p} over each farm. Specifically, precipitation depends on the total stock of forest as follows

$$\tilde{p} = \left(\sum_{i \in \mathcal{A}} f_i\right)^{\lambda} \tag{1}$$

where λ denotes the sensitivity of rainfall to the Amazonian land use change.

The agricultural sector. Agricultural goods are produced according to

$$y_i = \tilde{p}^{\rho_i} n_i^{\theta_i} l_i^{\gamma_i} \prod_{j \in \mathcal{I}} x_{ij}^{\alpha_i w_{ij}} \quad \forall i \in \mathcal{A}$$

$$\tag{2}$$

where n_i is labor input, l_i denotes arable land, x_{ij} is the amount of good j used in the production of good i, $\sum_{j \in \mathcal{I}} w_{ij} = 1$ and $\rho_i + \theta_i + \gamma_i + \alpha_i = 1 \forall i \in \mathcal{A}$. I assume that rainfall \tilde{p} over farm $i \in \mathcal{A}$ is exogenous to the farm. The profit of agricultural firms is given by

$$\pi_i = p_i y_i - \sum_{j \in \mathcal{I}} p_j x_{ij} - \omega n_i - r_i l_i \quad \forall \ i \in \mathcal{A}$$
(3)

where agricultural producers pay land rents r_i . Each farm has access to an *exogenously* given rural property of size s_i that is owned by a land owner.

Land owners. Land owners transform rural properties into forest f_i and arable land l_i such that

$$s_i = f_i + l_i \tag{4}$$

and supply land to the agricultural producers at a rental rate r_i . Land owners solve the following maximization problem

$$\max_{l_i} \pi_i^l = r_i l_i - \phi \ g\left(\kappa - \frac{f_i}{s_i}\right) \tag{5}$$

where r_i is the price of land for producer $i \in \mathcal{A}$ and κ is the minimum requirement of native vegetation on rural properties under the Brazilian Forest Code. ϕ is an exogenous law enforcement parameter and the function $g(\cdot)$ translates deviations from the forest code into costs that occur due to compliance with the environmental law. Land owners spends their income from renting out land outside the economy. The non-agricultural sector. Non-agricultural producers have no use for land or precipitation but are otherwise identical to agricultural producers. Non-agricultural profits are given by

$$\pi_i = p_i y_i - \sum_{j \in \mathcal{I}} p_j x_{ij} - \omega n_i \quad \forall i \notin \mathcal{A}$$
(6)

where
$$y_i = n_i^{\theta_i} \prod_{j \in \mathcal{I}} x_{ij}^{\alpha_i w_{ij}} \quad \forall i \notin \mathcal{A}$$
 (7)

and $\theta_i + \alpha_i = 1 \ \forall i \notin \mathcal{A}.$

Preferences. The representative household has the following preferences and supplies a unit amount of labour inelastically.

$$u(c_1,\ldots,c_n) = \prod_{i \in \mathcal{I}} c_i^{\eta_i} \tag{8}$$

subject to the budget constraint $\sum_{i \in \mathcal{I}} p_i c_i \leq \omega n$.

Exports. There is a representative exporting firm that can use the output of each firm i to produce a single good y_E designated for the export abroad at the world price p^w . The export good is produced according to the following constant returns to scale technology

$$y_E = \prod_{i \in \mathcal{I}} e_i^{\xi_i} \tag{9}$$

where e_i denotes the fraction of output of firm *i* used in the production of the export good and $\sum_{i \in \mathcal{I}} \xi_i = 1$. Profits in the exporting sector are given by $\pi^E = p^w y_E - \sum_{i \in \mathcal{I}} p_j e_j$.

Market clearing. Markets clear such that

$$y_i = c_i + e_i + \sum_{j \in \mathcal{I}} x_{ji} \quad \forall i \in \mathcal{I}$$

$$\tag{10}$$

and

$$\sum_{i\in\mathcal{I}}n_i = n = 1.$$
(11)

The equilibrium is defined in the usual way.

Proposition 1

Real GDP = $\frac{\omega + p^w y_E}{p^*}$ where $p^* = \prod_{i=1}^n (p_i^*)^{\eta_i}$ is the ideal price index, in equilibrium

$$\frac{\partial \log(\text{GDP})}{\partial \phi} = \sum_{i \in \mathcal{I}} \eta_i \left[\sum_{j \in \mathcal{A}} v_{ji} \left(\gamma_j \frac{\partial \log(l_j)}{\partial \phi} - \rho_j \frac{\lambda}{F} \sum_{k \in \mathcal{A}} \frac{\partial \log(l_k)}{\partial \phi} \right) \right] \bigg|_{l_j = l_j^*, l_k = l_k^*}$$
(12)

where $\mathbf{V} = [v_{ij}]$ is the economy's Leontief inverse and $F = \sum_{i \in \mathcal{A}} f_i$. With an increase of law enforcement, farms occur higher rental rates for their properties which reduces agricultural output. However, an increase in ϕ also results in a larger fraction of forests on agricultural properties and consequently increases the amount of rainfall and boost agricultural production. Furthermore, the shock to the agricultural sector travels through the production network of the economy and affects real GDP through the final demand of goods (see Appendix for Proof).

2 Equilibrium Conditions

The competitive equilibrium is characterised by

$$p_i^* = \exp\left(\sum_{j \in \mathcal{I}} v_{ji} \theta_j \ln(\omega) + \sum_{j \in \mathcal{A}} v_{ji} \gamma_j \ln(r_j^*) - \sum_{j \in \mathcal{A}} v_{ji} \rho_j \ln(\tilde{p}) - \sum_{j \in \mathcal{I}} v_{ji} \alpha_j \sum_{h \in \mathcal{I}} w_{jh} \ln(w_{jh}) - \chi_i\right)$$
(13)

$$l_i^* = \frac{\gamma_i m_i}{r_i^*} \tag{14}$$

$$r_i^* = \phi \frac{\partial g\left(\kappa - \frac{J_i}{s_i}\right)}{\partial l_i} \bigg|_{l_i - l^*}$$
(15)

$$m_i^* = \sum_{j \in \mathcal{I}} v_{ij} \left(\eta_i \omega + \xi_i p^w y_E \right) \tag{16}$$

$$y_{i}^{*} = \frac{m_{i}^{*}}{p_{i}^{*}}$$
(17)

$$x_{ij}^* = \frac{m_i^* \alpha_i w_{ij}}{p_i^*} \tag{18}$$

$$n_i^* = \frac{\theta_i m_i^*}{\omega} \tag{19}$$
$$e_i^* = \frac{\eta_i \omega}{\omega} \tag{20}$$

$$e_i^* = \frac{\xi_i}{r_i^*}$$

$$(20)$$

$$(21)$$

$$f_i^* = s_i - l_i^* \tag{22}$$

$$\tilde{p}^* = \left(\sum_{i \in \mathcal{A}} f_i^*\right)^{\lambda} \tag{23}$$

where all values are anchored to the nominal wage ω and $m_i = p_i y_i$ are sales of firm *i*. The value of exports $p^w y_E$ is exogenous. χ_i is a constant $\chi_i = \sum_{j \in \mathcal{I}} v_{ji} (\theta_j \ln(\theta_j) + \alpha_j \ln(\alpha_j)) + \sum_{j \in \mathcal{A}} v_{ji} \gamma_j \ln(\gamma_j)$. $\mathbf{V} = (\mathbf{I} - (\mathbf{AW})')^{-1} = [v_{ij}]$ is the economy's Leontief inverse where $\mathbf{W} = [w_{ij}]$ is the economy's input–output matrix and \mathbf{A} is a $n \times n$ matrix with the *i*th diagonal entry equal to α_i and every other entry equal to zero (see Appendix for Derivation).

3 Calibration

To discipline my quantitative exercises, I either calculate parameter values directly from the data or set the parameters such that the model is consistent with some moments of the economy of Brazil in 2018.

External calibration. To construct W, I use the input-output table from Guilhoto et al. (2010) who estimate the total value of transactions between 67 sectors in Brazil during the year 2018. The data includes two agricultural sectors: livestock and crop production. The estimation also includes sector level production values, wage bills, exports and final demand. I use production values and wage bills to estimate sector-level labor shares ϕ_i via expenditure shares. Further, I back out η_i and ξ_i for each sector by calculating the share used for final demand and exports, respectively. The minimum requirement of native vegetation on rural properties under the Brazilian Forest Code in the legal Amazon is 80%, hence $\kappa = 0.8$. I normalize $\sum_{i \in \mathcal{A}} s_i = 1$ and calculate s_i with data from the FAO as the share of total area used for cropland and pastures for the crop producing and the livestock sector in 2018, respectively. For the land shares γ_i , I use municipality level data to calculate the share of land leasing expenditures for the livestock and the crop-production sector from the 2006 Agricultural Census. Lastly, I assume that for the livestock sector $\rho_i = 0$. This means that rainfall is irrelevant for the production of cattle. For ρ_i in the crop production sector, I exploit geo-spatial monthly precipitation anomalies from McNally (2018) and municipality-year-crop level production values from the Pesquisa Agrícola Municipal (PAM) dataset. Therefore, I aggregate precipitation anomalies to the municipality-year level between 1985 and 2018 and back out precipitation shortages in deviations from their long-term average. I estimate ρ_i with the following regression:

$$\log y_{mt} = \alpha_m + \gamma_t + \rho \log \tilde{p}_{mt} + \varepsilon_{it} \tag{24}$$

where y_{mt} denotes the total value of agricultural production across all crops in municipality m at time t. \tilde{p}_{mt} denotes precipitation shortages. α_m and γ_t denote municipality level fixed and time effects, respectively, and ε_{it} is the error term. The result of this regression is depicted in the Table below for the effect on the total production value and the three biggest crops by land use separately.

(1)(2)(3)(4) $(\log) y$ total $(\log) y$ soybean $(\log) y$ maize $(\log) y$ sugarcane -0.041*** -0.030*** -0.085*** $\log \tilde{p}$ -0.005(0.003)(0.006)(0.004)(0.004)8.047*** 7.093*** 4.785*** 5.495*** Constant (0.032)(0.069)(0.046)(0.053)**Fixed Effects** yes yes yes yes Time Effects yes yes yes yes Obs. 92,712 86,791 59,613 28,867

FIXED EFFECT REGRESSION OF RAINFALL SHORTAGES ON AGRICULTURAL PRODUCTION

Standard errors in parentheses.

* p < 0.10, ** p < 0.05, *** p < 0.01

For the calibrated value of ρ , I use the estimate on the total production value 0.041.

Proposed calibration at the farm-level. If I assume the cost function for the land owner problem to be $g(x) = (x)^{\psi}$ such that

$$\max_{l_i} \pi_l = r_i l_i - \phi \left(\kappa - \frac{f_i}{s_i}\right)^{\psi} \tag{25}$$

Then I can solve for the equilibrium land allocation $l_i^* \forall i \in \mathcal{A}$ by solving the following implicit functions computationally

$$m_i^* \gamma_i = \phi \psi \left(\kappa - 1 + \frac{l_i^*}{s_i}\right)^{\psi - 1} \frac{l_i^*}{s_i}$$
(26)

Since s_i , f_i , l_i , κ and sales m_i (approx. all transaction to firm i)¹ are known, I can estimate ψ with the following fixed effects regression for producer i at time t

$$\log(\text{sales})_{it} = \alpha_i + \gamma_t + \beta \log\left(\kappa - \frac{f}{s}\right)_{it} + \delta \log\left(\frac{l}{s}\right)_{it} + \epsilon_{it}$$
(27)

Then $\hat{\beta} + 1 = \hat{\psi}$.

¹This follows from the fact that in equilibrium $\sum_{j \in \mathcal{I}} p_i x_{ji} = m_i - \eta_i \omega - \xi_i p^w y_E$ and I assume that changes in $\log(\eta_i \omega + \xi_i p^w y_E)$ are absorbed by time and fixed effects.

Internal calibration. First, I calibrate the total value of exports $p^w y_E$ to match the exports as the % of GDP in 2018 with data from the WDI. Second, I calibrate the enforcement parameter such that the total stock of forest F equals to the percentage of intact Amazon rainforest in 2018 compared to 1970. Third, choose a quadratic loss function $g(x) = x^2$ for the land owners problem. Lastly, for the calibration of λ , I exploit data from the meta-analysis by Spracklen & Garcia-Carreras (2015) who synthesize results from 96 regional and global climate model simulations of the impact of Amazonian deforestation on Amazon basin rainfall. The match between the calibrated model and a quadratic fit of the estimates is shown in the Figure below.



Note. The figure shows model simulation results from different journal publications synthesized by the meta-analysis of Spracklen & Garcia-Carreras (2015). The black line shows a quadratic fit to these estimates and the dotted line shows the effect deforestation on rainfall changes in the model economy. The vertical line shows the Amazonian forest loss in 2018 since 1970.

4 Growth Accounting

As a growth accounting exercise, I compute a series of equilibria by linearly decreasing the enforcement parameter ϕ until the total stock of forest reaches its 2022 level while keeping all remaining parameters and exogeneous variables fixed. This is the network effect of deforestation in Brazil during the presidency of Jair Bolsonaro.



TOP 20 WINNERS



All Sectors



5 Afforestation Counterfactual

As a counterfactual exercise, instead of decreasing law enforcement relative to 2018, I increase ϕ to reflect a 1% gain in afforestation. Moreover, I assume CES production technologies. This follows the argument in the literature that farms intensify their production if they can not expand land. We would observe this as a reallocation from the factor land to either labor or intermediate inputs (e.g., machinery, fertilizers etc.). Note that under Cobb-Douglas, income and substitution effect cancel out exactly.

$$y_{i} = \left(\rho_{i}^{\frac{1}{\sigma}}\tilde{p}^{\frac{\sigma-1}{\sigma}} + \theta_{i}^{\frac{1}{\sigma}}n_{i}^{\frac{\sigma-1}{\sigma}} + \gamma_{i}^{\frac{1}{\sigma}}l_{i}^{\frac{\sigma-1}{\sigma}} + \alpha_{i}^{\frac{1}{\sigma}}\sum_{j\in\mathcal{I}}w_{ij}^{\frac{1}{\sigma}}x_{ij}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \quad \forall i \in \mathcal{A}$$

$$(28)$$

with the elasticity of substitution parameter σ . I assume that, again, non-agricultural producers have identical CES production functions but no use for land or precipitation. Everything else is kept constant and I use the same external calibration as in the benchmark economy. Further, for every level of σ , I perform the same internal calibration strategy as described above to match the model to moments in the data from 2018.





TOP 10 LOOSERS





Real output change for a 1% increase in forest cover



All Sectors

6 Appendix

Derivation of Equilibrium Conditions

The first order conditions of agricultural firms $i \in \mathcal{A}$ are

$$\theta_i m_i = \omega n_i \tag{29}$$

$$\gamma_i m_i = r_i l_i \tag{30}$$

$$\alpha_i w_{ij} m_i = p_j x_{ij} \tag{31}$$

where $m_i = p_i y_i$ are sales. Note that the first order conditions of non-agricultural firms are equivalent without the FOC for land l_i . Now take the logarithm of the agricultural production functions

$$\ln(y_i) = \rho_i \ln(\tilde{p}) + \theta_i \left(\ln(\theta_i m_i) - \ln(\omega)\right) + \gamma_i \left(\ln(\gamma_i m_i) - \ln(r_i)\right) + \alpha_i \sum_{j \in \mathcal{I}} w_{ij} \ln\left(\frac{m_i \alpha_i w_{ij}}{p_j}\right) \quad \forall i \in \mathcal{A}$$
(32)

Now subtract $\ln(m_i)$ from both sides such that $\ln(y_i) - \ln(m_i) = -\ln(p_i)$ and use the fact that $\sum_{j \in \mathcal{I}} w_{ij} = 1$. Then

$$\ln(p_i) = -\chi_i^A - \rho_i \ln(\tilde{p}) + \theta_i \ln(\omega) + \gamma_i \ln(r_i) - \alpha_i \sum_{j \in \mathcal{I}} w_{ij} \ln(w_{ij}) + \alpha_i \sum_{j \in \mathcal{I}} w_{ij} \ln(p_j^*) \quad \forall i \in \mathcal{A}$$
(33)

where $\chi_i^A = \theta_i \ln(\theta_i) + \gamma_i \ln(\gamma_i) + \alpha_i \ln(\alpha_i)$ Following the same steps as above, for non agricultural firms we arrive at

$$\ln(p_i) = -\chi_i^N + \theta_i \ln(\omega) - \alpha_i \sum_{j \in \mathcal{I}} w_{ij} \ln(w_{ij}) + \alpha_i \sum_{j \in \mathcal{I}} w_{ij} \ln(p_j^*) \quad \forall \, i \notin \mathcal{A}$$
(34)

where $\chi_i^N = \theta_i \ln(\theta_i) + \alpha_i \ln(\alpha_i)$

Now stack all equations $\forall i \in \mathcal{I}$:

$$\ln(\boldsymbol{p}) = -\boldsymbol{\chi} - \boldsymbol{\rho} \ln(\boldsymbol{\tilde{p}}) + \boldsymbol{\theta} \ln(\omega) + \boldsymbol{\gamma} \ln(\boldsymbol{r}) - \boldsymbol{A}(\boldsymbol{W} \circ \ln(\boldsymbol{W}))\mathbf{1} + \boldsymbol{A}\boldsymbol{W'}\ln(\boldsymbol{p})$$
(35)

where $\boldsymbol{\chi}$ is the stacked vector of all χ_i^N and χ_i^A and similar for $\boldsymbol{\gamma}$, $\tilde{\boldsymbol{p}}$, and \boldsymbol{r} such that the ith elements equal to zero if $i \notin \mathcal{A}$. \circ denotes the Hadamard product and 1 is the unit vector. $\boldsymbol{W} = [w_{ij}]$ is the economy's input–output matrix and $\boldsymbol{A} = \boldsymbol{A'}$ is a $n \times n$ matrix with the *i*th diagonal entry equal to α_i and every other entry equal to zero. Then,

$$\ln(\boldsymbol{p}) = -\boldsymbol{V}'\left(\boldsymbol{\chi} + \boldsymbol{A}(\boldsymbol{W} \circ \ln(\boldsymbol{W}))\boldsymbol{1} + \boldsymbol{\rho}\ln(\boldsymbol{\tilde{p}}) - \boldsymbol{\theta}\ln(\omega) + \boldsymbol{\gamma}\ln(\boldsymbol{r})\right)$$
(36)

where $\boldsymbol{V} = (\boldsymbol{I} - (\boldsymbol{A}\boldsymbol{W})')^{-1} = [v_{ij}]$. Unstacking yields:

$$\ln(p_i) = \sum_{j \in \mathcal{I}} v_{ji} \theta_j \ln(\omega) + \sum_{j \in \mathcal{A}} v_{ji} \gamma_j \ln(r_j^*) - \sum_{j \in \mathcal{A}} v_{ji} \rho_j \ln(\tilde{p}) - \sum_{j \in \mathcal{I}} v_{ji} \alpha_j \sum_{h \in \mathcal{I}} w_{jh} \ln(w_{jh}) - \chi_i \quad \forall i \in \mathcal{I}$$
(37)

where $\chi_i = \sum_{j \in \mathcal{I}} v_{ji} (\theta_j \ln(\theta_j) + \alpha_j \ln(\alpha_j)) + \sum_{j \in \mathcal{A}} v_{ji} \gamma_j \ln(\gamma_j)$. Now plug m_i , the FOC of the household problem and the FOC of the exporting firm in the market clearing condition gives

$$m_i = \eta_i \omega + \sum_{j \in \mathcal{I}} p_i x_{ji} + \xi_i p^w y_E \quad \forall \ i \in \mathcal{I}$$
(38)

with the FOC of the firms gives

$$m_i = \eta_i \omega + \sum_{j \in \mathcal{I}} \alpha_j m_j w_{ji} + \xi_i p^w y_E \quad \forall \ i \in \mathcal{I}$$
(39)

In Matrix form:

$$\boldsymbol{M} = \boldsymbol{\eta}\boldsymbol{\omega} + \boldsymbol{\xi}p^{w}\boldsymbol{y}_{E} + (\boldsymbol{A}\boldsymbol{W})'\boldsymbol{M} + \boldsymbol{\xi}$$

$$\tag{40}$$

$$= \boldsymbol{V}(\boldsymbol{\eta}\omega + \boldsymbol{\xi}p^w y_E) \tag{41}$$

yields

$$m_i = \sum_{j \in \mathcal{I}} v_{ij} (\eta_i \omega + \xi_i p^w y_E) \quad \forall \ i \in \mathcal{I}$$

$$\tag{42}$$

The rest of Proposition 1 follows directly from the FOCs of firms, the household and the land owners.

Proof Proposition 1

First note that

$$\frac{\partial \ln(m_i)}{\partial \phi} = 0 \quad \text{and} \quad \frac{\partial \ln(p_i)}{\partial \phi} = \sum_{j \in \mathcal{A}} v_{ji} \gamma_j \frac{\partial \ln(r_j)}{\partial \phi} - \sum_{j \in \mathcal{A}} v_{ji} \rho_j \frac{\partial \ln(\tilde{p})}{\partial \phi} \quad \forall \ i \in \mathcal{I}$$
(43)

From firm FOCs we have $\ln(r_i) = \ln(\gamma_i) + \ln(m_i) + \ln(l_i)$. Then,

$$\frac{\partial \ln(r_i)}{\partial \phi} = -\frac{\partial \ln(l_i)}{\partial \phi} \quad \forall \ i \in \mathcal{I}$$
(44)

and from the rainfall equation we get that

$$\frac{\partial \ln(\tilde{p})}{\partial \phi} = -\frac{\lambda}{F} \sum_{j \in \mathcal{A}} \frac{\partial \ln(l_i)}{\partial \phi}$$
(45)

Then real GDP is GDP = $\frac{\omega + p^w y_E}{p^*}$ where $p^* = \prod_{i=1}^n (p_i)^{\eta_i}$. Hence,

$$\frac{\partial \ln(\text{GDP})}{\partial \phi} = -\sum_{i \in \mathcal{I}} \eta_i \frac{\partial \ln(p_i)}{\partial \phi}$$
(46)

and plugging in the above expressions gives the desired result.

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