Incentives for Information Production in Markets where Prices Affect Real Investment\footnote{A previous version of this paper was circulated under the title: "Commitment to Overinvest and Stock Price Informativeness." We thank Sudipto Bhattacharya, Philip Bond, Jonathan Carmel, Martin Dierker, Alex Edmans, Thierry Foucault, Paolo Fulghieri, Nicolae Garleanu, Armando Gomes, Joao Gomes, Denis Gromb, Harrison Hong, Roman Inderst, Wei Jiang, Pete Kyle, Chris Malloy, Scott Schaefer, David Scharfstein, Gustav Sigurdsson, Gunter Strobl, Avanidhar Subrahmanyam, Oren Sussman, and seminar and conference participants at the European Central Bank, the Federal Reserve Bank of Philadelphia, London Business School, the University of Amsterdam, the University of Maryland, the University of Michigan, the University of Toulouse, the University of Utah, Washington University, Wharton, the 2006 EFA Meeting, the 2006 FIRS Meeting, the 2007 WFA Meeting, the NBER Market Microstructure Meeting, and the NBER Summer Institute on Capital Markets and the Economy for helpful comments. Itay Goldstein gratefully acknowledges financial support from the Rodney White Center.}

James Dow\textsuperscript{2} \quad Itay Goldstein\textsuperscript{3} \quad Alexander Guembel\textsuperscript{4}

\textit{London Business School} \quad \textit{University of Pennsylvania} \quad \textit{University of Oxford}

October 12, 2007

\textsuperscript{2}London Business School, Sussex Place, Regent’s Park, London, NW1 4SA, UK. Phone: +44 207 000 7000, email: jdow@london.edu.

\textsuperscript{3}Department of Finance, Wharton School, University of Pennsylvania, Steinberg Hall - Dietrich Hall, Suite 2300, Philadelphia, PA 19104-6367, USA. Phone: +1 215 746 0499, e-mail: itayg@wharton.upenn.edu.

\textsuperscript{4}Saïd Business School, University of Oxford, Park End Street, Oxford OX1 1HP, UK. Phone: +44 1865 288 914, e-mail: alexander.guembel@sbs.ox.ac.uk.
Incentives for Information Production in Markets where Prices Affect Real Investment

Abstract

A fundamental role of financial markets is to gather information on firms’ investment opportunities, and so help guide investment decisions. In this paper we study the determinants of information production when prices perform this allocational role. If firms cancel planned investments following poor stock market response, the value of their shares will become insensitive to information on investment opportunities, so that speculators will be deterred from producing information ex ante. We show that the amount of information in equilibrium increases in the expected profitability of the firm’s investments, and that this creates an amplification mechanism from changes in fundamentals to real value. Uncertainty about future performance has a non-trivial effect on information production. We show that information production on investment opportunities is less privately profitable than that on assets in place, and argue that some overinvestment increases firm value.
1 Introduction

Informational efficiency is a central tenet of financial economics. As Fama and Miller (1972) put it, an efficient market “has a very desirable feature. In particular, at any point in time market prices of securities provide accurate signals for resource allocation; that is, firms can make production-investment decisions ...” (p 335). Yet, although market efficiency has been extensively researched, traditional analysis of secondary financial markets limits attention to assets whose cash flows are unknown, but exogenous. In other words, traditional analysis does not study cases where informational efficiency is valuable in guiding the allocation of investment resources.1

We study a model where speculators in a secondary financial market produce information about firms. The information gets reflected in market prices and guides real investment decisions. Such feedback from market prices to investment decisions could occur either through managerial learning or through firms’ access to capital. Empirical evidence in support of such a feedback effect is provided by Baker, Stein, and Wurgler (2003), Luo (2005), Chen, Goldstein, and Jiang (2007), and Bakke and Whited (2007). We analyze the incentives of speculators to produce information in such a model. As noted by Grossman and Stiglitz (1980), for markets to be informationally efficient, speculators must have sufficient incentives to produce information. Thus, our model studies the underpinnings of market efficiency in cases where informational efficiency matters for resource allocation. We also analyze the implications that informational efficiency has for firm value and investment policy.

We show that the way in which a firm’s investment decisions respond to stock price movements has important implications for information production incentives. Our main results stem from the fact that if a firm is more likely to cancel investment projects following negative price responses, the incentives for speculators to produce information will decrease. This is because, once the

---

1See Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980), and the extensive market microstructure literature following Kyle (1985) and Glosten and Milgrom (1985). Note that general equilibrium REE models such as Radner (1981) do allow for resource allocation to be endogenous, but do not explicitly model the interdependence between a firm’s security prices and its investment policy. Also related is q-theory, the literature on investments and asset prices initiated by Tobin (1969). Despite the link between asset prices and investments in this literature, q-theory does not analyze a causal relation from the financial markets to real investment.
investment is cancelled, the security value is less sensitive to information on the project, and thus the information loses its speculative value. This exposes a fundamental limitation of the allocational role of prices: the fact that investment decisions are conditioned on information in the price may reduce speculators’ incentives to produce this information in the first place.

Based on this insight, we derive results on the effect of expected profitability and of uncertainty on the amount of information production in the financial market. We study the difference between information production on investment opportunities and on assets in place. We also analyze the effect of overinvestment and the incentive of firms to encourage it. We derive many new empirical implications that are summarized in Section 6.2.

Our main results are as follows. First, in our model, speculators have a stronger incentive to produce information when investments are ex-ante more profitable. This is because high ex-ante profitability implies that the firm is more likely to invest, and this increases expected speculative profits. This result implies that endogeneity in the level of information production amplifies small changes in fundamentals into large changes in firm values: the increased amount of information associated with improved fundamentals enables more efficient investment decisions, which increase value further. Conversely, when fundamentals deteriorate, less information is produced and resources are allocated less efficiently. The deterioration in fundamentals can lead to a discontinuous ‘collapse’ in information production, with associated large drops in investment activity and firm valuations. This provides a new explanation for the amplification of changes in fundamentals to changes in real activity over the business cycle. It is also consistent with evidence on IPO waves; initial public offerings are highly cyclical and positively correlated with stock market returns (see Ibbotson and Ritter, 1995).

Second, in standard models of financial markets, holding the cost of information fixed, the incentive to produce information will increase in the degree of uncertainty about the underlying fundamentals. This is because higher uncertainty implies greater profit opportunities. In our model, where the investment decision is affected by the information in the market, this is not always true. Indeed, we find that when the ex-ante NPV of the investment is positive, an increase in uncertainty makes the investment less likely to be undertaken. As a result, the incentive to
produce information (and hence the equilibrium amount of information) is sometimes decreasing in the level of uncertainty.

Third, our analysis has implications for incentives to collect information about assets in place versus new projects. Because new projects may be cancelled following adverse market signals, the incentive to produce information about them is lower than for assets in place (holding other things constant). In Hirshleifer’s (1971) terminology, information on new projects is discovery, while information on assets in place is foreknowledge. He emphasizes that foreknowledge has no social value, while discovery is valuable, and argues that economic forces do not guarantee optimal information production. Hirshleifer’s argument is compelling, although it is not derived as a result of a detailed economic model. We derive the result that foreknowledge is more privately profitable than discovery using an explicit economic model of a financial market.

Fourth, our analysis implies that firms with a tendency to overinvest will attract more speculative information production. Deviations from ex-post optimal investment decisions increase ex-ante firm value in our model. Since more investment increases the production of information, and since information increases firm value, it is in the best interest of shareholders to have managers who tend to overinvest. The finance literature has argued that managers are ‘empire builders’ who use free cash flow to overinvest (Jensen (1986)). Our analysis shows that there is a positive side to this phenomenon. Another implication of this is that shareholders could design the firm’s financial structure to control the level of free cash flow in a way that achieves the ex-ante optimal level of overinvestment. Shareholders could also design managerial contracts to generate the optimal level of overinvestment.²

Our paper emphasizes the role of information produced by financial markets in firms’ investment decisions. The justification for the usefulness of information in financial markets for firms’ investment decisions is that markets gather information from many different participants, who are too numerous to communicate with the firm outside the trading process (see Subrahmanyam and Titman (1999)). This idea goes back to Hayek (1945), who argues that markets provide an efficient mechanism for information production and aggregation. The ability of financial markets to produce

²In parallel work, Strobl (2006) makes a related point. We discuss this further in Section 5.
information that accurately predicts future events has also been demonstrated empirically. For example, the literature on prediction markets shows that markets provide better forecasts than polls and other devices (see Wolfers and Zitzewitz (2004)). Roll (1984) shows that private information of citrus futures traders regarding weather conditions gets impounded into citrus futures' prices, so that prices improve even public predictions of the weather. By focusing only on information produced in financial markets, we do not deny the importance of alternative information producers such as banks or large shareholders, which have been extensively analyzed in the corporate finance literature (see Allen and Gale (2000), for a review). Rather, we try to extend the analysis to a class of information providers which has been somewhat neglected in the corporate finance literature.

A small number of theory papers study market equilibrium in the presence of a feedback effect from asset prices to the real economy. They include Fishman and Hagerty (1992), Leland (1992), Khanna, Slezak, and Bradley (1994), Boot and Thakor (1997), Dow and Gorton (1997), Subrahmanyan and Titman (1999), Fulghieri and Lukin (2001), Dow and Rahi (2003), Foucault and Gehrig (2007), and Goldstein and Guembel (2007). Among these papers, only Boot and Thakor (1997), Dow and Gorton (1997), and Fulghieri and Lukin (2001) have endogenous production of information. In these papers, however, the feedback effect serves only to compare between bank financing and market financing or between debt and equity. Our paper is the first one to show the implications that the feedback effect has for the nature of information production. In particular, we are the first to show how firm and project characteristics affect the equilibrium amount of information and the resulting investment decision and firm value in the presence of a feedback effect.

The remainder of the paper is organized as follows. In Section 2, we describe the basic model of feedback. Section 3 derives the equilibrium outcomes. In Section 4, we analyze the effect of expected profitability and uncertainty on information production, and discuss the implications for firm value. Section 5 shows that firms benefit from deviating from ex-post optimal investment decisions. In Section 6, we consider the differences between investment opportunities and assets in

---

3The IPO literature has also used the assumption that stock-market participants have information about some aspects of the firm, which is not available to the firm’s managers. See, for example, Rock (1986), Benveniste and Spindt (1989), Benveniste and Wilhelm (1990), and, Biais, Bossaerts, and Rochet (2002).
place and summarize the main empirical implications of the paper. Section 7 concludes. All proofs are relegated to the appendix.

2 A Model of Feedback

2.1 Modelling assumptions

There is a firm with an investment opportunity. The investment requires a fixed amount $I$. The investment decision is taken by a manager who acts in the interest of shareholders. Since we assume that there is no agency problem, we can use the terms ‘firm’ and ‘manager’ interchangeably. The final payoff of the investment $R$ is binary and takes realizations $R_h$ and $R_l$ with equal probability, depending on the underlying state of the world $\omega \in \{l,h\}$. Assume that $R_h > I > R_l$, i.e., the investment is worthwhile undertaking when the state of the world is $h$ but not when it is $l$.

The shares of the firm are traded in the financial market, where three types of agents are present: noise traders, speculators, and a market maker. Speculators are atomistic, risk neutral, and indexed by $i \in [0, \infty)$. Each speculator can choose to become informed about $\omega$ at cost $c > 0$, in which case he receives a fully-revealing signal. After deciding whether to acquire information or not, each speculator can trade $x_i$, where $x_i \in [-1,1]$.\footnote{That is, there are frictions (such as limited wealth) that constrain trade size to a maximum of 1.} Denote by $\alpha$ the measure of speculators that become informed about $\omega$. Noise trading $\tilde{n}$ is normally distributed with 0 mean and variance $\sigma^2$. We use $X$ to denote the total order flow. It is given by:\footnote{Clearly, uninformed speculators optimally choose not to trade.}

$$X = \tilde{n} + \int_0^\alpha x_i \, di.$$ \hfill (1)

The total order flow is submitted to a risk neutral market maker, who observes $X$, but not its components. He then sets the price equal to the expected value of the firm conditional on the information contained in the order flow. As in Kyle (1985), this can be justified as a result of a perfectly-competitive market-making industry. Unlike Kyle (1985), here the value of the firm is not exogenous, but rather depends on the information revealed in the trading process – i.e., there is a
feedback effect from the financial market to the value of the firm. This is because the decision on whether to take the investment or not is conditioned on this information.

The firm’s objective is to maximize expected value. The firm’s manager does not observe $\omega$ and the investment decision is taken after observing the order flow and price. Clearly, if $\alpha > 0$, these will contain information about the profitability of the project. The firm will use this information and undertake the investment if and only if the updated NPV is at least 0. (In Section 5, we consider the case where the investment decision is not necessarily optimal based on the available information.)

The modelling assumptions that noise trade is normally distributed, that there is a continuum of risk neutral speculators, and that each one of them trades up to one unit are similar to Fulghieri and Lukin (2001). The informational structure is kept deliberately simple. First, we assume that speculators who choose to acquire information observe the true state of the world. Second, we assume that the firm’s manager does not receive any signal on the state of the world. Our goal is to study the implications of the feedback effect in a very parsimonious model without unnecessary complications. We believe that endowing the speculators with noisy heterogenous signals and allowing the manager to acquire a noisy signal himself will not change the qualitative results we obtain here.

Finally, it is important to note that while we write this paper to describe a firm that learns from its own stock price, there is an alternative interpretation of our model that does not require this assumption. According to this alternative interpretation, the firm does not have capital to pursue the investment, and relies on external providers of capital, who learn the information from the market. They will provide capital to finance the investment if and only if, based on the available information, they at least break even. If there is perfect competition among external providers of capital (i.e., when they exactly break even), the analysis of our model will remain exactly the same. In particular, the investment will be undertaken if and only if its NPV is at least zero, and the value of the firm will reflect the full profit from the investment if it is undertaken.
2.2 Trading decisions and investment policy

From risk neutrality we know that if speculators acquire information, they will trade the maximum size possible. So, when the true state is \( \omega = h \) all informed speculators optimally choose to submit buy orders of size 1 and total order flow is then \( X = \tilde{n} + \alpha \). Conversely, when \( \omega = l \) informed traders submit sell orders and total order flow is \( X = \tilde{n} - \alpha \).

Suppose that the belief of the market maker and the firm is that \( \alpha^m \) speculators acquire information in equilibrium. We define \( \theta(X, \alpha^m) \equiv \Pr(\omega = h | X) \) – the probability that the state of the world is \( h \) given order flow \( X \) and measure of informed speculators \( \alpha^m \). We can write

\[
\theta(X, \alpha^m) = \frac{\varphi(X - \alpha^m)}{\varphi(X - \alpha^m) + \varphi(X + \alpha^m)}, \tag{2}
\]

where \( \varphi(n) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}(\frac{n}{\sigma})^2} \) is the density function of the normal distribution.

An investment policy is a mapping from the firm’s belief about the high state \( \omega = h \) onto its decision to invest. If the firm maximizes value conditional on \( \theta \), then it will invest if and only if it believes that doing so generates a NPV of at least zero. Thus, the firm will invest if and only if

\[
\theta(X, \alpha^m) \geq \delta \equiv \frac{I - R_l}{R_h - R_l}. \tag{3}
\]

We will use \( \delta \) as a measure of the profitability of the investment. A high \( \delta \) indicates low ex-ante NPV. In particular, when \( \delta > \frac{1}{2} \), the ex-ante NPV is negative, whereas when \( \delta < \frac{1}{2} \), the ex-ante NPV is positive.

Since the normal distribution function satisfies the monotone likelihood ratio property, we know that \( \theta(X, \alpha^m) \) is strictly increasing in \( X \). Thus, we can define a cut-off value \( \overline{X} \) for total order flow, such that the investment is undertaken if \( X \) is above \( \overline{X} \) and rejected if \( X \) is below \( \overline{X} \). Threshold \( \overline{X} \) is determined as a function of \( \alpha^m \) by \( \theta(\overline{X}, \alpha^m) = \delta \). Hence, using the properties of the normal distribution function, we get:

\[
\overline{X}(\alpha^m) = \frac{\sigma^2}{2\alpha^m}\ln \frac{\delta}{1 - \delta}. \tag{4}
\]

The price that is set by the market maker is then given as a function of \( X \) and \( \alpha^m \) as follows:

\[
P(X, \alpha^m) = \begin{cases} 
R_h\theta(X, \alpha^m) + R_l(1 - \theta(X, \alpha^m)) - I & \text{if } X \geq \overline{X}(\alpha^m) \\
0 & \text{if } X < \overline{X}(\alpha^m)
\end{cases}. \tag{5}
\]
That is, when the order flow is below $\bar{X}$, the firm does not invest so its value is 0. On the other hand, when the order flow is above $\bar{X}$, the firm invests, and its expected value is the NPV of the project conditional on the information contained in the order flow.

2.3 Trading profits

We define a function $\pi(\alpha, \alpha^m)$, which gives the expected trading profits of speculators who choose to become informed as a function of the actual mass of informed speculators $\alpha$ and the mass $\alpha^m$ that the market maker and the firm believe is present. We will require that in equilibrium $\alpha = \alpha^m$. To confirm the equilibrium, however, it will be important to make a distinction between $\alpha$ and $\alpha^m$. This is because we will need to consider potential deviations by speculators, and this requires calculating the derivative of $\pi(\alpha, \alpha^m)$ with respect to $\alpha$.

To derive the function $\pi(\alpha, \alpha^m)$ note that when $\omega = h$, $X = \tilde{n} + \alpha$, and thus the investment will be undertaken if and only if $n \geq \bar{X}(\alpha^m) - \alpha$. Similarly, when $\omega = l$, $X = \tilde{n} - \alpha$, and thus the investment will be undertaken if and only if $n \geq \bar{X}(\alpha^m) + \alpha$. Then, the expected trading profits can be written as:

$$\pi(\alpha, \alpha^m) = \frac{1}{2} \int_{\bar{X}(\alpha^m) - \alpha}^{\infty} \varphi(n) (R_h - I - P((n + \alpha), \alpha^m)) \, dn + \frac{1}{2} \int_{\bar{X}(\alpha^m) + \alpha}^{\infty} \varphi(n) (P((n - \alpha), \alpha^m) - R_l + I) \, dn.$$  \hspace{1cm} (6)

Here, if an informed speculator gets good news and buys (with probability $\frac{1}{2}$), he will make a profit if the noise traders’ order is high enough ($n \geq \bar{X}(\alpha^m) - \alpha$) for the firm to invest. In this case, for each realization of $n$, his profit is the difference between the true value of the firm $R_h - I$ and the price $P((n + \alpha), \alpha^m)$. On the other hand, if the noise traders’ order is low ($n < \bar{X}(\alpha^m) - \alpha$), then the investment will not be made, and the speculator ends up buying an asset with liquidation value 0 at price 0, so he makes no profit. A parallel explanation applies for a speculator who sells on bad news.
Using the price function (5), we can write:

\[
\int_{\mathcal{X}(\alpha^m) - \alpha}^{\infty} \varphi(n) (R_h - I - P(n + \alpha, \alpha^m)) \, dn = (R_h - R_l) \int_{\mathcal{X}(\alpha^m) - \alpha}^{\infty} \frac{\varphi(n + \alpha + \alpha^m)}{\varphi(n + \alpha - \alpha^m) + \varphi(n + \alpha + \alpha^m)} \, dn
\]

\[
= (R_h - R_l) \int_{\mathcal{X}(\alpha^m)}^{\infty} \frac{\varphi(x - \alpha) \varphi(x + \alpha^m)}{\varphi(x - \alpha^m) + \varphi(x + \alpha^m)} \, dx,
\]

where the second equality follows from the change of variable \( x = n + \alpha \). Similarly (and using a change of variable \( x = n - \alpha \)) we can write:

\[
\int_{\mathcal{X}(\alpha^m) + \alpha}^{\infty} \varphi(n) (P((n - \alpha), \alpha^m) - R_l + I) \, dn = (R_h - R_l) \int_{\mathcal{X}(\alpha^m) + \alpha}^{\infty} \frac{\varphi(x - \alpha) \varphi(x + \alpha^m)}{\varphi(x - \alpha^m) + \varphi(x + \alpha^m)} \, dx.
\]

Then, we can rewrite \( \pi(\alpha, \alpha^m) \) as follows:

\[
\pi(\alpha, \alpha^m) = \frac{1}{2}(R_h - R_l) \int_{\mathcal{X}(\alpha^m)}^{\infty} \frac{\varphi(x - \alpha) \varphi(x + \alpha^m)}{\varphi(x - \alpha^m) + \varphi(x + \alpha^m)} \, dx
\]

\[
+ \frac{1}{2}(R_h - R_l) \int_{\mathcal{X}(\alpha^m)}^{\infty} \frac{\varphi(x + \alpha) \varphi(x - \alpha^m)}{\varphi(x - \alpha^m) + \varphi(x + \alpha^m)} \, dx.
\]

### 2.4 Equilibrium

In equilibrium, \( \alpha = \alpha^m \). That is, the mass of informed speculators that the market maker and the firm believe is present – on the basis of which the real-investment threshold and the price are set (see (4) and (5)) – is equal to the actual mass. Moreover, at the equilibrium level of \( \alpha \), no further speculator has an incentive to acquire information. Let us define a function \( \pi(\alpha) \equiv \pi(\alpha, \alpha) \), which gives the expected trading profits of each trader if the equilibrium mass of speculators who have become informed is \( \alpha \). Following (9), we get:

\[
\pi(\alpha) = (R_h - R_l) \int_{\mathcal{X}(\alpha)}^{\infty} \frac{\varphi(x - \alpha) \varphi(x + \alpha)}{\varphi(x - \alpha) + \varphi(x + \alpha)} \, dx.
\]

Using \( \hat{\alpha} \) to denote the equilibrium level of \( \alpha \), there are two sets of conditions that may determine \( \hat{\alpha} \). First, an equilibrium with no production of information (\( \hat{\alpha} = 0 \)) is obtained when the cost to
produce information is strictly greater than the expected trading profit given that no speculator produces information, i.e.,
\[ \pi(0) < c. \] (11)

Second, an equilibrium with positive production of information (\( \hat{\alpha} > 0 \)) is obtained when, given that \( \hat{\alpha} \) speculators choose to produce information, each speculator who acquires information breaks even, and no further speculator has an incentive to acquire information, i.e.,
\[ \pi(\hat{\alpha}) = c \]
\[ \text{and} \]
\[ \frac{d\pi(\alpha, \hat{\alpha})}{d\alpha} \bigg|_{\alpha=\hat{\alpha}} < 0. \] (12)

### 2.5 Firm value

Our paper studies the interaction between the information in financial markets and real investment decisions. Ultimately, we are interested in the effect of stock price informativeness on the value of the firm. We can compute the value of the firm as a function of \( \hat{\alpha} \) as follows:
\[ \frac{1}{2} \int_{X(\hat{\alpha})}^{\infty} (R_h - I) \varphi(x - \hat{\alpha}) + (R_l - I) \varphi(x + \hat{\alpha}) \, dx. \] (13)

Intuitively, when the state of the world is high (\( \omega = h \)), the firm’s investment generates a net payoff of \( (R_h - I) > 0 \), while investment takes place if and only if the realization of noise trade \( n \) is above \( X(\hat{\alpha}) - \hat{\alpha} \) (so that total order flow is above \( X(\hat{\alpha}) \)). Similarly, when \( \omega = l \), the firm’s investment generates a net payoff of \( (R_l - I) < 0 \), while investment takes place if and only if the realization of noise trade \( n \) is above \( X(\hat{\alpha}) + \hat{\alpha} \).
3 Equilibrium Outcomes

3.1 Positive NPV Investment

In analyzing our model we find that the characterization of equilibrium outcomes is distinctly different for the case where the NPV of the investment is ex-ante positive than for the case where it is ex-ante negative. We first analyze the case where the investment project has a positive NPV ex ante. In this case, if no information arrives, the firm chooses to invest. This means that \( I \leq \frac{1}{2}(R_h + R_l) \), or in other words \( \delta \leq \frac{1}{2} \). Proposition 1 characterizes the equilibrium outcomes for this case. It says that in this case there is a unique equilibrium: if the cost of information production is high, no information is produced, whereas if it is not high, a positive measure of speculators choose to become informed.

Proposition 1 When \( \delta \leq \frac{1}{2} \), there exists a unique equilibrium. For \( c < \pi(0) \), a positive measure of speculators become informed, i.e., \( \hat{\alpha} > 0 \), and for \( c \geq \pi(0) \), no information is produced, i.e., \( \hat{\alpha} = 0 \).

3.2 Negative NPV Investment

Consider now the case where \( \delta > \frac{1}{2} \). Proposition 2 characterizes the equilibrium outcomes for this case. It says that in this case there may be multiple equilibria. There is always an equilibrium with no production of information. If the cost of information production is high, this is the only equilibrium, whereas if it is not high, there are additional equilibria (more than one) with positive measures of speculators that choose to become informed.

Proposition 2 When \( \delta > \frac{1}{2} \):

(i) For any \( c > 0 \), there exists an equilibrium with \( \hat{\alpha} = 0 \).

(ii) For \( c \leq \max_{\alpha \in \mathbb{R}^+} \pi(\alpha) \), there also exist equilibria with \( \hat{\alpha} > 0 \).

(iii) For \( c > \max_{\alpha \in \mathbb{R}^+} \pi(\alpha) \), the equilibrium with \( \hat{\alpha} = 0 \) is unique.
3.3 Discussion

Figure 1 depicts the expected trading profits as a function of the measure of speculators who choose to acquire information. The solid curve is for the case where the investment has a negative NPV ex ante (here, $\delta = 0.55$), while the dotted curve is for the case where the investment has a positive NPV ex ante (here, $\delta = 0.45$). We can see that when the ex-ante NPV is negative, the profit function is hump-shaped, whereas when the ex-ante NPV is positive, the profit function is monotonically decreasing. This illustrates why the negative NPV case may have multiple equilibria, while the positive NPV case has a unique equilibrium.

To understand the differences between the two cases, it is useful to isolate the different underlying economic effects that a change in the number of informed traders has on each traders’ profits. First, there is the standard effect in models of informed trading with exogenous investment (e.g., Grossman and Stiglitz (1980)). As more speculators become informed, the equilibrium price becomes closer to the value of the stock, and profits are reduced. This causes a downward slope in the profit function. We call this the competitive effect. In our model, it generates strategic substitutabilities in agents’ decisions to produce information.

Second, there is the effect caused by the endogeneity of the firm’s investment decision. The direction of this effect depends on whether the project has a positive or a negative NPV ex ante. In case of a positive ex-ante NPV, without any information, the firm makes the investment, and more information decreases the probability of the investment being undertaken. This reinforces the competitive effect because as more speculators produce information, the investment is undertaken less often and the value of the stock becomes less sensitive to the information, so the trading profit decreases. This is why in the positive NPV case, the profit function is downward sloping and the equilibrium is unique.

In case of a negative ex-ante NPV, this effect is reversed to produce strategic complementarity in information production. In this case, without any information, the firm does not make the investment. As more speculators produce information, the investment is undertaken more often and the value of the stock becomes more sensitive to the information. There is an informational leverage
Figure 1: The figure shows trading profits as a function of $\alpha$ for the case of $\delta = 0.45$ (dotted line) and for the case of $\delta = 0.55$ (solid line). The other parameters are set at $\sigma = 1$ and $R_h - R_l = 1$. The figure also shows two different costs of information production $c'$ and $c''$. $\alpha_1$ and $\alpha_2$ are two equilibrium values of the measure of informed traders when $\delta > \frac{1}{2}$ and the cost of information production is $c'$. 
effect,\(^6\) where information becomes more valuable as more agents produce it. The interaction between the standard competitive effect and the informational leverage effect causes the profit function to be non monotone.\(^7\)

As a result of the non monotonicity, we have multiple equilibria in the case of an ex-ante negative NPV. First, there always exists an equilibrium in which no information is produced. This happens for the following reason. When nobody produces information, the firm does not invest. Then, it does not pay for an individual to become informed, since the firm’s securities never gain exposure to the information that the speculator collected. Second, when the cost of information production is not too high, there are equilibria with a positive amount of information. From Figure 1, we can see that for \(c < \max_{\alpha \in \mathbb{R}^+} \pi(\alpha)\), the profit function will intersect twice with the cost of producing information, generating two equilibria with positive amount of information. For example, if \(c = c'\), we can see in the figure that one equilibrium \((\alpha_1)\) is obtained at the upward sloping part of the profit function, representing less information than the other equilibrium \((\alpha_2)\) that is obtained at the downward sloping part of the profit function. Essentially, there is a coordination problem in information production among multiple speculators. A coordination failure may obtain if they coordinate on producing no information or if the amount of information is \(\alpha_1\) rather than \(\alpha_2\).

Models of multiple equilibria are often used to explain market volatility and other phenomena like herds and frenzies. These phenomena are attributed to jumps between equilibria and the arbitrariness with which one equilibrium is played for some firms and another equilibrium is played for other firms. It is interesting that the introduction of feedback from the financial market to real investments is sufficient to generate such results. Even more interesting is the fact that this is expected to happen only when the NPV of the investment is ex-ante negative. Thus, a model of feedback predicts that volatility, herds and frenzies will only characterize firms with less promising investment opportunities. This helps distinguishing the feedback effect from other channels proposed in the literature for the presence of multiple equilibria in financial markets.\(^8\)

\(^{6}\)We thank Rohit Rahi for suggesting this terminology.

\(^{7}\)The model of Boot and Thakor (1997) exhibits a similar non-monotonicity, although they do not explore this feature and base their analysis on selecting one of the three equilibria we consider.

\(^{8}\)See Veldkamp (2006a, b) for an explanation based on fixed costs in the production of information and Froot,
4 The Comparative Statics of Information Production

4.1 The effect of expected profitability

4.1.1 Expected profitability and information production

To analyze the effect of expected profitability on information production, we study the effect of $\delta$ on the equilibrium amount of information $\hat{\alpha}$. Recall that $\delta \equiv \frac{I - R_l}{R_h - R_l}$; a high $\delta$ indicates a high ex-post threshold for undertaking the investment and thus a low ex-ante profitability. In varying $\delta$, we wish to consider only the effect of the profitability of the investment without changing anything else in the factors that determine $\hat{\alpha}$. Inspecting the profit function in (10), which is the main determinant of equilibrium, we can see that this amounts either to changing $I$ or to changing $R_h$ and $R_l$ by the same amount. Thus, an increase in $\delta$ can be interpreted as an increase in $I$ or as a decrease in $R_h$ and $R_l$ while holding $(R_h - R_l)$ constant.

Another issue to consider is how to analyze the effect of $\delta$ on the equilibrium amount of information when there are multiple equilibria. In our analysis, in case of multiple equilibria, we will focus on the most informative equilibrium, i.e., the equilibrium that features the highest $\hat{\alpha}$. We will sometimes use the notation $\hat{\alpha}_{\text{max}}$ to denote the highest $\hat{\alpha}$. Note that the most informative equilibrium Pareto dominates the others when we consider the value of the firm, the market maker, and the speculators. This is because, as will become clear later, the value of the firm increases in the amount of information, while the speculators and the market maker always make a profit of 0.

Proposition 3 establishes the effect of $\delta$ on $\hat{\alpha}$ (or $\hat{\alpha}_{\text{max}}$). Trivially, when the equilibrium is at the corner $\hat{\alpha} = 0$, small changes in the model parameters will not affect $\hat{\alpha}$. For the comparative statics presented in the following proposition we therefore focus on the case where $\hat{\alpha} > 0$.

**Proposition 3** Suppose $\hat{\alpha} > 0$.

(i) If $\delta \leq \frac{1}{2}$, then $\hat{\alpha}$ decreases in $\delta$.

(ii) If $\delta > \frac{1}{2}$, then $\hat{\alpha}_{\text{max}}$ decreases in $\delta$.

The intuition for this result is as follows. As economic fundamentals deteriorate and expected
profitability decreases, for each level of information production, the firm invests less frequently. Then, speculators’ expected trading profits are reduced because the value of the firm is less exposed to the information about the profitability of the investment. As a result, in equilibrium, fewer speculators find it worthwhile to pay the cost of information, and the equilibrium amount of information decreases.

One manifestation of this result is shown in Figure 1. As we see there, the profit function \( \pi(\alpha) \) for the case of a positive NPV investment (the dotted curve) lies above the profit function for the case of a negative NPV investment (the solid curve). As a result, the equilibrium amount of information in case of a positive NPV investment (low \( \delta \)) is higher than the amount in even the most informative equilibrium in case of a negative NPV investment (high \( \delta \)). For example, the intersection between the dotted curve and the cost \( c' \) is obtained at \( \alpha > \alpha_2 \), where \( \alpha (\alpha_2) \) is the equilibrium amount of information when the NPV is positive (negative – considering the most informative equilibrium) and the cost is \( c' \). Proposition 3 establishes that the increase in information holds not only for the shift from a negative to a positive NPV, but rather for any increase in profitability.

The result in Proposition 3 has two empirical implications. First, it implies that the amount of information produced will vary over time in response to aggregate fluctuations in investment prospects. When prospects are poor, firms are more likely to cancel investments and this reduces available trading profits and information production incentives. One way to test this hypothesis is to look at changes in market microstructure measures of informed trading – for example the PIN measure (see Easley, Hvidkjaer, and O’Hara (2002)) – across stages of the business cycle. Another potential proxy for the amount of information produced in financial markets is analyst activity. We are not aware of any empirical studies that have related analyst activity to the business cycle, although anecdotal evidence suggests that financial firms’ employment policies are highly cyclical. In fact, they seem to be much more cyclical than employment policies of other firms, and thus the pattern cannot be completely explained by the changing fundamentals.

Second, the result suggests that information production should vary cross-sectionally with firms’ investment prospects. Taking analyst activity as a proxy for information production in financial
markets, there is some support for this hypothesis in the empirical literature. McNichols and O’Brien (1997) investigate analysts’ decisions to initiate or drop coverage of specific stocks. They find that analysts bias their coverage towards those firms about which they have more favorable expectations. Building on these findings, Sun (2003) shows that the initiation or dropping decision itself predicts future firm performance. Das, Guoh and Zhang (2006) find that among newly listed firms, analysts selectively cover those firms for which they have more positive expectations. Firms that receive more coverage perform better afterwards.

In citing these results, we do not wish to overstate the fit of our model to the data. Clearly, there may be alternative explanations for these results. One that has been put forward in the literature is that analysts attempt to please firms, and that a negative forecast reduces a firm’s willingness to communicate with an analyst and raises the cost of producing information. It would be interesting to conduct an analysis of the relation between fundamentals and information production, using more direct measures of speculative private information in price, which are not exposed to this alternative explanation. The PIN measure, which we mentioned above, is a good candidate for such an analysis. It would also be interesting to extend the existing empirical work and relate it more directly to our model by relating information not only to future stock returns of the firms, but also to their future investment behavior. Our model predicts that firms for which less information is produced (for example, firms that are dropped from coverage) will invest less in the future. Finally, as we explain in Section 6, the prediction of our model pertains only to information production on investment opportunities, not on assets in place. Thus, it would be interesting to test whether the above pattern is stronger for firms with more investment opportunities (‘growth firms’) than for firms with more assets in place (‘value firms’).

4.1.2 Implications for firm value and investment

Having studied the effect of ex-ante investment profitability on the amount of information produced in equilibrium, we now turn to analyze the effect that this has on the value of the firm. The next proposition shows that, with endogenous information acquisition, the change in the value of the firm caused by a change in fundamentals is amplified compared to a model with fixed information
production. For this comparison, let \( \alpha(\delta) \) be the equilibrium amount of information production given the threshold \( \delta \). In case of multiple equilibria, let \( \alpha(\delta) \) represent the amount of information in the most informative equilibrium. Next, define \( V(\alpha, \delta) \) to be the ex-ante value of the firm as a function of the amount of information production \( \alpha \) and the threshold \( \delta \), where \( \alpha \) is not necessarily the equilibrium value \( \alpha(\delta) \). Using (13), this is given by:

\[
\frac{1}{2} \int_{X(\alpha, \delta)}^{\infty} (R_h - I) \varphi(x - \alpha) + (R_l - I) \varphi(x + \alpha) \, dx,
\]

where \( X(\alpha, \delta) \) is derived from (4) as \( \frac{\sigma^2}{2a} \ln \frac{\delta}{1-\sigma} \).

Suppose that \( \delta \) increases from \( \delta_1 \) to \( \delta_2 \). This has two effects on firm value. First, keeping the level of information production fixed, the firm’s project is less profitable (recall that a high \( \delta \) indicates low profitability) and this reduces firm value. The reduction in firm value due to this effect can be measured by \( \frac{V(\alpha(\delta_1), \delta_2)}{V(\alpha(\delta_1), \delta_1)} \) (where a lower number indicates a larger reduction). Second, the level of information production changes endogenously, so that the total reduction in firm value that actually occurs can be measured by \( \frac{V(\alpha(\delta_2), \delta_2)}{V(\alpha(\delta_1), \delta_1)} \). The information effect exacerbates the direct effect of an increase in \( \delta \) if \( \frac{V(\alpha(\delta_1), \delta_2)}{V(\alpha(\delta_1), \delta_1)} > \frac{V(\alpha(\delta_2), \delta_2)}{V(\alpha(\delta_1), \delta_1)} \). This is indeed the case, as we show in Proposition 4.

Proposition 4 Consider two different values of \( \delta \): \( \delta_1 < \delta_2 \). Then, \( \frac{V(\alpha(\delta_1), \delta_2)}{V(\alpha(\delta_1), \delta_1)} > \frac{V(\alpha(\delta_2), \delta_2)}{V(\alpha(\delta_1), \delta_1)} \).

This result is based on the fact that the information produced by speculators increases the value of the firm. As a result, the decrease in information production caused by lower profitability amplifies the reduction in firm value. To see why information increases firm value, it is useful to inspect equation (14). Analyzing the expression inside the integral, we can see that as \( \alpha \) increases, the firm ends up undertaking the investment more frequently in the high state of the world and less frequently in the low state of the world. That is, an increase in \( \alpha \) improves the efficiency of the firm’s investment decision, and thus increases firm value. We can also see that \( \alpha \) affects the boundaries of the integral via its effect on \( X(\alpha, \delta) \). However, given that \( X(\alpha, \delta) \) is determined optimally to maximize the value of the firm, this has no effect on firm value in equilibrium (i.e., the derivative of \( V(\alpha, \delta) \) with respect to \( X(\alpha, \delta) \) is 0).

The effect of \( \delta \) on firm value is demonstrated in Figure 2. Here, the dotted line shows the effect of \( \delta \) when the amount of information remains constant, while the solid line shows the effect of \( \delta \) with
endogenous information production. Consistent with Proposition 4, the solid line is steeper, and thus the change in firm value as a result of a change in \( \delta \) is amplified by the endogenous production of information. Interestingly, Figure 2 shows that there is a point, at which a small change in \( \delta \) causes a discrete jump in firm value due to the endogenous response of information production. This happens when a change in \( \delta \) shifts speculators from an equilibrium with no production of information to an equilibrium with positive production of information (or vice versa). Consider, for example, Figure 1 and suppose that the profit function is given by the solid hump-shaped curve and the cost of producing information is \( c'' \). In this case, since \( c'' > \max_{\alpha \in \mathbb{R}^+} \pi (\alpha) \), there is a unique equilibrium with no information production: \( \hat{\alpha} = 0 \). The value of the firm is also 0 since no investment occurs. Now, since \( c'' \) is just slightly above \( \max_{\alpha \in \mathbb{R}^+} \pi (\alpha) \), a very small decrease in \( \delta \) is needed to shift the profit function upwards so that it will intersect with \( c'' \). This will generate an equilibrium with a large positive amount of information and a significantly positive firm value. Overall, a small change in fundamentals here causes a discrete jump in the equilibrium behavior of speculators and consequently in the value of the firm. This is a result of the non monotonicity of the price function, and thus it is expected to occur when the ex-ante NPV of the investment is negative.

It is also interesting to investigate the effect of \( \delta \) on the frequency of investment when information production is endogenous. This is demonstrated in Figure 3. Again, the dotted line shows the effect of \( \delta \) when the amount of information remains constant, while the solid line shows the effect of \( \delta \) with endogenous information production. We can see that, as a result of a decrease in expected profitability (an increase in \( \delta \)), there is a sharp drop in the frequency of investment only when information production is endogenous. When the production of information is not affected by profitability, the decrease in investment as a result of a decrease in profitability is very minor.

The amplification of small changes in fundamentals into large changes in firm value and investment is related to the large literature on fluctuations over the business cycle (see for example, Bernanke and Gertler (1989), Greenwald and Stiglitz (1993), Kiyotaki and Moore (1997), and Suarez and Sussman (1997)). This literature typically links these fluctuations to capital market imperfections, which limit firms’ access to capital in downturns of the business cycle. One expla-
Figure 2: The figure shows the value of the firm as a function of $\delta$ for the case where $\alpha$ is exogenous (dotted line) and for the case where $\alpha$ is endogenous (solid line). In the second case, $\alpha = \widehat{\alpha}(\delta)$, while in the first case $\alpha = \widehat{\alpha}(0.4)$, i.e., the two lines intersect when $\delta = 0.4$. The other parameters are set at $\sigma = 1$, $R_h - R_l = 1$, and $c = 0.45$. 
Figure 3: The figure shows the frequency of investment as a function of $\delta$ for the case where $\alpha$ is exogenous (dotted line) and for the case where $\alpha$ is endogenous (solid line). In the second case, $\alpha = \tilde{\alpha}(\delta)$, while in the first case $\alpha = \tilde{\alpha}(0.4)$, i.e., the two lines intersect when $\delta = 0.4$. The other parameters are set at $\sigma = 1$, $R_h - R_l = 1$, and $c = 0.45$. 
nation in the literature (the balance-sheet channel) is that during a bust a firm’s collateral is less valuable and therefore it has more restricted access to external finance. An alternative explanation (the credit channel) is that banks’ ability to lend is reduced in recessions.

We identify a different mechanism for sharply reduced investment levels. In our setting a firm’s ability to identify good investment projects is weakened during a recession because speculators’ incentives to produce information are reduced. Another way in which our result can operate – if, as we suggest in Section 2.1, we interpreted our model to have outside capital providers instead of firm managers learning from the price – is that outside capital providers will be unable to identify good investments and may cut off access to capital. In our view, it is natural to think about changing access to external capital as arising from changes in the information environment since information problems are one of the fundamental reasons for firms’ limited access to external capital. To our knowledge, the model proposed in this paper is the first that links changes in the economic outlook to endogenous changes in information production and investment behavior.

Our result is also consistent with the empirical evidence on cluster periods in IPO activity (e.g., Ibbotson and Ritter (1995), Jenkinson and Ljungqvist (2000)). This clustering seems too extreme to be explained only by variations in average project profitability. Our amplification mechanism, in which lower profitability is associated with less information production, weakens the incentives of firms to carry out an IPO in bad times. In a bust, an IPO will not lead to an informative stock price and in addition there is no need to raise capital for investment.

4.2 The effect of uncertainty

The uncertainty about the outcome of the investment can be measured by \((R_h - R_l)\). In studying the effect of expected profitability on information production we kept this uncertainty constant. We now turn to analyze the effect of the uncertainty on the amount of information being produced in equilibrium. Going back to the profit function in (10), which is the main determinant of equilibrium, we can see that \((R_h - R_l)\) has two effects on trading profits (and thus on the equilibrium amount of information). It affects the trading profits directly and also via \(\delta\). Using (3), we can rewrite \(\delta\)

\footnote{Subrahmanyan and Titman (1999) argue that obtaining an informative stock price is a major motive for IPO’s.}
as:

$$\delta = \frac{1}{2} - \frac{R_h + R_l - I}{R_h - R_l},$$

where \( \left( \frac{R_h + R_l}{2} - I \right) \) measures the expected profitability of the investment (studied in the previous subsection) and \((R_h - R_l)\) the uncertainty. Since we are interested here only in the effect of uncertainty, we will study the effect of changes in \((R_h - R_l)\), while keeping the expected profitability, measured by \( \frac{R_h + R_l}{2} - I \), constant.

Proposition 5 establishes the effect of \((R_h - R_l)\) on \(\hat{\alpha}\) (or \(\hat{\alpha}_{\text{max}}\)). As before, our comparative statics focus on the case \(\hat{\alpha} > 0\).

**Proposition 5** Suppose \(\hat{\alpha} > 0\).

(i) When \(\delta \leq \frac{1}{2}\), then \(\hat{\alpha}\) may decrease or increase in \((R_h - R_l)\).

(ii) When \(\delta > \frac{1}{2}\), then \(\hat{\alpha}_{\text{max}}\) increases in \((R_h - R_l)\).

This result is generated by two effects that the uncertainty \((R_h - R_l)\) has on the trading profits. The direct effect is standard: when uncertainty increases, private information is more valuable, and thus trading profits increase. Mathematically, this is captured by the fact that \((R_h - R_l)\) multiplies the expression for trading profits in (10). The second effect is indirect. Changes in uncertainty affect the threshold \(\delta\) above which the investment is undertaken and thus the expected frequency with which the investment is being undertaken. The direction in which this affects trading profits depends on whether the investment has a positive or a negative NPV ex ante. In case of a positive NPV \((\delta \leq \frac{1}{2})\), keeping the expected profitability constant, an increase in uncertainty makes it less likely that the investment will be undertaken, and thus decreases trading profits. To see this it is useful to inspect (15). The threshold above which the firm invests is lower than \(\frac{1}{2}\) when the ex-ante NPV is positive, but the effect of the ex-ante NPV is weaker when uncertainty is higher. Thus, high uncertainty implies less frequent investment when the ex-ante NPV is positive. The opposite is true when the ex-ante NPV is negative. As a result, the overall effect of uncertainty on trading profits when \(\delta \leq \frac{1}{2}\) is ambiguous, while when \(\delta > \frac{1}{2}\) it is positive.
5 The Role of Overinvestment

Our analysis so far assumed that the firm takes the ex-post optimal investment decision, given the information contained in market prices and order flows. A straightforward implication of Proposition 3 is that there would be more information produced in the financial market if the firm was expected to invest more than is ex-post optimal, i.e., if the firm was expected to overinvest.

Formally, let us use \( \delta_d \) to denote the threshold probability applied ex post to decide whether to undertake the investment or not (the subscript \( d \) stands for deviation because the investment decision deviates from the ex-post optimal rule). That is, the firm invests if and only if \( \theta(X, \alpha) > \delta_d \). Unlike \( \delta \), \( \delta_d \) is not constrained to satisfy ex-post optimality, so we do not require \( \delta_d \) to equal \( I - R_l - R_h \).

Given \( \delta_d \), the equilibrium amount of information \( \hat{\alpha}(\delta_d) \) is determined as before as the quantity of information such that no more speculators have an incentive to become informed; see (11) and (12). The threshold order flow above which the firm invests is then \( X(\hat{\alpha}(\delta_d), \delta_d) = \frac{\sigma^2}{2\hat{\alpha}(\delta_d)} \ln \frac{\delta_d}{1-\delta_d} \) (see (4)).

Based on Proposition 3, if \( \delta_d < \delta \), i.e., if the firm invests more often than is ex-post optimal, the equilibrium amount of information \( \hat{\alpha}(\delta_d) \) will be higher than that under optimal ex-post investment decisions \( \hat{\alpha}(\delta) \) (as before, in case of multiple equilibria, we consider the most informative equilibrium). This is because when the firm invests more often, the value of the firm ends up being more correlated with the information about the profitability of the investment, and thus the speculative value of the information increases, and speculators choose to produce more information. Note that the exercise in Proposition 3 was based on shifting parameters, such that \( \delta \) changes but everything else that determines the equilibrium remains the same. Thus the application of the proposition to the analysis of deviations from \( \delta \) to \( \delta_d \) is immediate.

The idea that more information is produced on firms that end up overinvesting is very relevant to corporate-finance theory. Indeed, many corporate-finance settings give rise to too much investment ex post. One example is when a firm is committed to pay a fee if it cancels a pre-announced investment. Such fees are very common in acquisitions and are known as break-up fees.\(^{10}\) When

\( ^{10} \)Often, break-up fees refer to fees payable by the target in the event of another bidder taking it over. Here, we refer to fees payable by the bidder in case it decides to cancel. Such fees have also become common in recent years.
the bidding firm is committed to paying a break-up fee, it may go ahead with an acquisition rather than rejecting it even when the financial markets do not react favorably to the announcement. Another example is when the firm is run by a manager who is an empire builder. Jensen’s (1986) free cash flow theory posits that managers generally want to overinvest. Overinvestment will be more pronounced in firms where shareholders have less control over the manager’s actions, for example when the manager has a lot of free cash flow – i.e., because the firm has little debt – or when corporate governance is weak.

Our paper provides a new and interesting implication about such settings. It says that when the firm has a tendency to overinvest – due to pre-committed cancellation fees, empire building managers, abundance of free cash flow, and/or weak corporate governance – speculators will have stronger incentives to produce information on its investment opportunities. Such predictions can be tested by studying the relation between information in stock price – for example, using the PIN measure – and parameters that affect the tendency to overinvest. We believe this is a promising direction for future empirical research.

Another interesting question is whether investing more often than is ex-post optimal can increase the ex-ante value of the firm. On the one hand, the analysis so far shows that information is valuable to the firm and that the firm could benefit from more information if it invested more often. However, there would be a cost to investing more often because it implies deviations from ex-post optimality. To answer this question, we investigate whether committing to deviate from \( \delta \) can increase the ex-ante firm value. Using (13), we write the value of the firm as a function of \( \delta_d \):

\[
V(\delta_d) = \frac{1}{2} \int_{X(\hat{\alpha}(\delta_d),\delta_d)}^{\infty} (R_h - I) \varphi(x - \hat{\alpha}(\delta_d)) + (R_l - I) \varphi(x + \hat{\alpha}(\delta_d)) \, dx. \tag{16}
\]

Proposition 6 establishes that, to maximize ex-ante firm value, the firm always wants to commit to set \( \delta_d \) lower than the ex-post optimal level \( \delta \), i.e., it is optimal to commit ex ante to overinvest.

**Proposition 6** In the most informative equilibrium, the value of the firm is maximized by choosing \( \delta_d < \delta \).

For example, in the HP-Compaq takeover, the merger agreement stipulated that either party would be liable to pay compensation of US$675 million if it were responsible for the failure of the transaction.
Intuitively, deviating from the ex-post optimal investment rule by committing to invest more often has two effects. First, the value of the firm falls because the investment rule no longer utilizes the information from the market in the optimal way. Second, speculators are induced to produce more information, which raises the value of the firm. However, the first effect is a second-order effect precisely because the value of the firm is maximized ex-post at that point. On the other hand, the second effect creates a first-order increase in firm value. Hence, it is always ex-ante optimal for the firm to commit to overinvest.

Importantly, the value of committing to a $\delta_d$ below the ex-post efficient level $\delta$ is particularly large when doing so allows the firm to switch from an equilibrium in which no information is acquired to one where a strictly positive amount is acquired. One example is in Figure 1, where the profit function is given by the solid hump-shaped curve and the cost of producing information is $c''$. In this case, since $c'' > \max_{\alpha \in \mathbb{R}^+} \pi (\alpha)$, there is a unique equilibrium with no information production: $\hat{\alpha} = 0$. Since $c''$ is just slightly above $\max_{\alpha \in \mathbb{R}^+} \pi (\alpha)$, the firm could commit to a $\delta_d$ that is just slightly below $\delta$ to enable an equilibrium $\hat{\alpha} (\delta_d) = \arg \max \pi (\alpha)$. This small commitment would lead to a discrete jump in the value of the firm from 0 to a significant positive number, because the amount of information produced jumps from $0$ to $\hat{\alpha} (\delta_d) >> 0$.

As we discussed above, there are several ways in which the firm can commit to an ex-post inefficient investment policy, for example by leaving free cash-flow in a firm with an empire building manager. Our theory can thus be developed further to study how financial structure can be used as a tool to achieve the ex-ante optimal level of overinvestment. Another implication is that it is optimal to give managers some compensation for expanding the firm’s operations. This is indeed consistent with empirical evidence in Bebchuk and Grinstein (2005).

The tension we identify here is one between interim efficiency and ex ante incentives. This tension exists in a variety of other settings. Brander and Lewis (1986) and Fershtman and Judd (1987) show that it is ex-ante optimal for firms in Cournot competition to create incentives for managers for aggressive behavior that is ex-post inefficient. Cremer (1995) shows that a principal may not wish to fire a high ability manager, even though he performed poorly. This reduces incentives for the manager to exert effort ex ante. Thus, the principal may wish to commit to a
tougher interim rule. Other papers have shown that the principal’s interim efficient action may be too tough (e.g., Shleifer and Summers (1988), Burkart, Gromb and Panunzi (1997), Almazan and Suarez (2003), and Kihlstrom and Wachter (2005)). In these papers, it may be ex ante optimal to retain a manager who should be sacked from the point of view of interim efficiency. In Rotemberg and Saloner (2000), top management commits to allow middle managers, who obtain private benefits from running their pet projects, to continue with marginally unprofitable projects in order to encourage them to initiate new projects. Guembel and White (2005) show that ex-ante monitoring incentives can be improved by an interim inefficient liquidation policy.

In contrast to these papers, we show that a similar tension may apply to ex ante incentives for financial market traders to acquire information for their speculative trading and the interim efficient investment decision by a firm. Our model is distinctive in that the communication between firm and speculators takes place via the stock market price. In our model, the firm would like the speculators to produce information about a random variable, when the stock price may not end up being sensitive to this random variable. The firm therefore has an incentive to commit to making the stock price reflect the variable more often. This induces the speculators to produce more information.

Our result can be interpreted in terms of the incompleteness of the securities market. There are relevant states of the world for which there is no state security. These states concern the value of a potential investment. If the firm does not make the investment, then the market is incomplete with respect to these states. But because the firm benefits from having a market for these securities it commits to making the investment and thereby makes the market more complete. Notice that it is impossible for a third party (such as a financial futures exchange) to create this state security because its value does not depend on public information. One cannot design a security whose payoff is a function of the returns on an investment opportunity that may never be undertaken.11

In parallel work, Strobl (2006) makes a point related to the one we make in this section. In his model, decision makers are fully informed about the state of the world, but the stock price still plays

---

11 One could imagine creating a futures contract whose value depends on the signals received by the speculators, but it would not be incentive compatible for them to reveal their signals truthfully.
an important role because managers receive stock-based compensation, which serves to align their incentives with the interests of shareholders. He shows that overinvestment is sometimes optimal because it increases the amount of information produced in the financial market and improves the efficiency of incentive alignments. Because of the different set up, the nature and purpose of overinvestment in his paper are different than here. First, in his model, the firm overinvests when the price is high but the private signal available to the manager indicates that the profitability is low. Thus, unlike in our model, overinvestment is not apparent to outsiders who only look at the stock price. Second, overinvestment in his model happens in a state where the speculator does not have a position in the stock. This is unlike our model, where the fact that speculators have a position is crucial since only then overinvestment enables them to get the benefit from the exposure of the value of the security to the state of the world. Thus, Strobl’s result is driven by a different force – namely, the negative impact that the firm’s ex-post sub-optimal investment policy has on the price when the speculator buys the stock.

6 Assets in Place vs. Investment Opportunities

Our paper analyzes information production on new investment opportunities. The implications we derived in the paper rely on this feature. It is interesting to compare these implications with those under the standard setting, where information is produced on assets in place and no investment decision is involved. We start by comparing the amount of information produced on assets in place with the amount produced on investment opportunities. We then go back to summarize the main empirical predictions derived in the paper and sharpen them by contrasting the case of investment opportunities with that of assets in place.

6.1 Which information is more likely to be produced?

To analyze the first question, we compare the amount of information produced on an investment opportunity, as described thus far in the paper, with the amount of information produced on an investment that has already been undertaken although the returns have not been realized yet. This means that the amount $I$ has been invested, while the expected return is still $R_h$ or $R_l$ with
equal probability. Proposition 7 shows that, holding other parameters constant, the amount of
information produced on assets in place is greater than the amount of information produced on an
investment opportunity.

**Proposition 7** If \( c < \frac{1}{2} (R_h - R_l) \), then the equilibrium level of information production is higher
for assets in place than for an investment opportunity. Otherwise, the amount of information
produced is 0 for either the assets in place or the investment opportunity.

The intuition behind this result is that for assets in place, there is no possibility that the project
will be canceled and the value of the firm will lose exposure to the information on the profitability of
the project. Thus, speculators have a greater incentive to produce information on assets in place.\(^{12}\)

We can derive an empirical prediction from this result by using book to market ratio as a proxy
for the importance of assets in place relative to investment opportunities.\(^{13}\) Our model predicts
that more information will be produced on firms with higher book to market ratios. Indeed, Hong,
Lim and Stein (2000) find that, controlling for firm size and a number of other variables, analyst
coverage, which is a measure of information production, depends positively and significantly on the
book to market ratio. It would be interesting to relate the book to market ratio to more direct
measures of private information in price, such as PIN.

The above result is related to the discussion in Hirshleifer (1971). Information on new projects
is an example of what he calls *discovery*, while information on assets in place is *foreknowledge*. Dis-
covery means learning information that will not necessarily be revealed otherwise, such as inventing
a new technology. Foreknowledge means learning information that will in any case be revealed later
on, such as learning a firm’s earnings a few days in advance. His model shows that the private
and social rewards to either kind of information production may diverge. In particular, there may

\(^{12}\)By holding other parameters constant, we focus on the difference between information production on assets in
place and on investment opportunities that is caused by the mechanism in our model. In the real world, it is likely
that other differences between assets in place and investment opportunities affect the amounts of information. For
example, the cost of producing information \( c \) and the uncertainty \( (R_h - R_l) \) are likely to be higher for investment
opportunities. These two forces may affect the amount of information in opposite directions, and thus the overall
effect of introducing them is unclear.

\(^{13}\)This is a valid proxy after the profitability of future projects is controlled for.
be a private benefit from socially useless foreknowledge. Our finding that foreknowledge is more privately profitable than discovery is consistent with his argument that economic forces do not guarantee optimal information production.

6.2 Sharpening (and summarizing) the empirical predictions

The analysis in the previous sections generated several empirical implications regarding information production on new investment opportunities. A sharper message can be obtained by contrasting these implications with those that come from the traditional framework where information is produced on assets in place. In general, the case of information production on assets in place is easy to analyze. The properties can be derived using the expression for trading profits in (27). Contrasting the two frameworks, we now summarize the main empirical implications of our paper. In testing these implications, one can use analyst coverage or market microstructure measures, such as PIN, to proxy for the amount of information. Also, growth (value) firms can be thought of as firms with mainly investment opportunities (assets in place). These are the main implications:

- Information production on investment opportunities with low expected profitability may be subject to high volatility due to multiple equilibria. This is not the case for investment opportunities with high expected profitability or for assets in place.

- Information production on investment opportunities increases in expected profitability. (This has implications for both the time series and the cross section.) Information production on assets in place is not affected by expected profitability.

- The effect of fundamentals on value is amplified for investment opportunities, but not for assets in place.

- Information production on investment opportunities increases in the amount of uncertainty when the expected profitability is low, and may increase or decrease in the amount of uncertainty when the expected profitability is high. Information production on assets in place always increases in the amount of uncertainty.
• Information production on investment opportunities increases in the tendency of the firm to overinvest (due to pre-committed break-up fees, empire-building managers, large amounts of free cash flows, and/or weak governance). This is, of course, not relevant for assets in place.

• Other things equal, there will be more information production on assets in place than on investment opportunities.

7 Conclusions

We study the incentives for information production in a financial market where prices influence real investment decisions. We derive several results on the determinants of information production and the resulting firm value and investment policy. Our model opens several directions for future research. First, as we noted throughout the paper, there are many empirical implications of our model that may guide future empirical investigations. The main predictions are summarized in Section 6.2. While some aspects of our results are consistent with existing empirical evidence, most of them call for more empirical testing.

Second, according to our model, firms always benefit from having some commitment to overinvest ex post. One way to have this commitment in place is for the firm to be run by a manager who is an empire builder. Jensen’s (1986) free cash flow theory posits that managers generally want to overinvest. He argues that leverage can be used to commit the firm to interest payments that reduce free cash flow and control the tendency to overinvest. Our model highlights a positive role for managerial overinvestment, and implies that it may be worthwhile for the firm to leave some free cash flow that will enable the manager to overinvest. This will allow the firm to benefit from the increased information production. This insight suggests that financial structure can be designed optimally to trade off the benefit of overinvestment against its cost. Studying optimal financial structure in this light is an interesting direction for future research.

Third, a natural question to ask is whether communication between speculators and the firm can improve investment efficiency in our model. Speculators may have an incentive to trade and then announce positive information to the firm so as to increase the likelihood of investment and
thereby trading profits. This, however, raises the question whether truthful announcements would be made in equilibrium. After all, speculators would like the firm to invest even when they have negative information, as long as they established a short position in the firm. Investing in this case destroys firm value in expectation, which allows speculators to close out their short positions at a lower price than if the firm had rejected the project. The precise role that communication can play is thus non-trivial and calls for further research.

Fourth, the industrial structure in which the firm operates may have important implications for the informational efficiency of its security prices. For example, when firms in an industry exhibit network externalities, the reaction of real investment to the price may be stronger. This is because the price is important, not only as a signal of fundamentals, but also as a signal of the information known to other firms, which is a strong indication of their expected actions. Given that the incentives for information production depend on the reaction of investment to the price, this implies differences in information efficiency across industries.

Fifth, in our model we studied a situation where information is produced about an investment opportunity. We compared this with a situation where the information is produced about assets in place. An interesting extension is one where the firm has both, assets in place and several investment opportunities whose payoffs may be more or less correlated with assets in place. Since the profits on information about assets in place are guaranteed, we would expect speculators to produce more information on those investment opportunities that are closer to the core business of the firm (as they are correlated with the assets in place). Also, what happens if managers are informed about some investment opportunities, but not about others? Will speculators produce information on the investment opportunities that managers are already informed about (which would be socially wasteful) or not? Finally, what are the implications of allowing divestment of assets in place for the incentives to produce information? We leave these questions for future research.
8 Appendix

Proof of Proposition 1. We first show that when $\delta \leq \frac{1}{2}$, $\pi(\alpha)$ is a strictly decreasing function. We can write

$$
\frac{d\pi(\alpha)}{d\alpha} = (R_h - R_l) \int_{X(\alpha)}^{\infty} \frac{\varphi(x - \alpha) \varphi(x + \alpha) \left(\frac{x - \alpha}{\sigma^2} \varphi(x + \alpha) - \frac{x + \alpha}{\sigma^2} \varphi(x - \alpha)\right)}{(\varphi(x - \alpha) + \varphi(x + \alpha))^2} dx

- (R_h - R_l) \frac{dX(\alpha)}{d\alpha} \frac{\varphi(X(\alpha) - \alpha) \varphi(X(\alpha) + \alpha)}{\varphi(X(\alpha) - \alpha) + \varphi(X(\alpha) + \alpha)}. \quad (17)
$$

Here, we used the fact that $\varphi'(n) = -\frac{n}{\sigma^2} \varphi(n)$.

From (4) it is clear that when $\delta \leq \frac{1}{2}$, $\frac{dX(\alpha)}{d\alpha} > 0$. Thus, the second term in (17) is negative. To show that the first term is also negative, we need to show that

$$
x(\varphi(x + \alpha) - \varphi(x - \alpha)) < \alpha(\varphi(x + \alpha) + \varphi(x - \alpha)). \quad (18)
$$

If $x < 0$, then, because $\varphi(\cdot)$ is the density function of a normal distribution with mean 0, $\varphi(x + \alpha) > \varphi(x - \alpha)$, and thus the LHS of (18) is negative while the RHS is positive, so the inequality in (18) holds. Similarly, if $x > 0$, then $\varphi(x + \alpha) < \varphi(x - \alpha)$, and again the LHS of (18) is negative while the RHS is positive, so the inequality holds. It follows that $\frac{d\pi(\alpha)}{d\alpha} < 0$.

Next, we argue that $\lim_{\alpha \to \infty} \pi(\alpha) = 0$. To see this note that because $\varphi(n)$ approaches 0, as $n$ approaches either $-\infty$ or $\infty$,

$$
\lim_{\alpha \to \infty} \frac{\varphi(x - \alpha) \varphi(x + \alpha)}{\varphi(x - \alpha) + \varphi(x + \alpha)} = 0. \quad (19)
$$

Thus, since $\pi(\alpha)$ is strictly decreasing and since it approaches 0 as $\alpha$ approaches $\infty$, there are two possible scenarios: either (i) there is a unique intersection $\pi(\alpha) = c$, or (ii) $\pi(0) < c$. Case (i) will hold when $c < \pi(0)$, whereas Case (ii) will hold when $c \geq \pi(0)$. From (11) and (12), we know that in Case (ii), the unique equilibrium is that there is no information production, i.e., $\alpha = 0$. In Case (i), there is a unique candidate $\hat{\alpha} > 0$ that satisfies $\pi(\hat{\alpha}) = c$. To verify that this is an equilibrium, we need to check that $\left.\frac{d\pi(\alpha, \hat{\alpha})}{d\alpha}\right|_{\alpha = \hat{\alpha}} < 0$. Using (9) we can calculate the derivative and
evaluate it at $\hat{\alpha}$. This yields

$$\left. \frac{d\pi(\alpha, \hat{\alpha})}{d\alpha} \right|_{\alpha = \hat{\alpha}} = \frac{1}{2}(R_h - R_l) \int_{X(\hat{\alpha})}^{\infty} \frac{-\varphi'(x - \alpha) \varphi(x + \alpha) + \varphi'(x + \alpha) \varphi(x - \alpha)}{\varphi(x - \hat{\alpha}) + \varphi(x + \hat{\alpha})} \, dx$$

(20)

which is clearly negative. ■

Proof of Proposition 2. (i) We need to show that if $\hat{\alpha} = 0$, the profit from producing information is 0. From (4), we can see that, for $\delta > \frac{1}{2}$, $\lim_{\alpha \to 0} X(\alpha) = \infty$. Plugging this in the profit function (10), and noting that $\lim_{\alpha \to 0} \frac{\varphi(x - \alpha) \varphi(x + \alpha)}{\varphi(x - \alpha) + \varphi(x + \alpha)} = \frac{1}{2} \varphi(x)$, we know that $\lim_{\alpha \to 0} \pi(\alpha) = 0$.

(ii) From (10) it is clear that $\pi(\alpha) > 0$ for all $\alpha > 0$. We showed in (19) that $\lim_{\alpha \to \infty} \pi(\alpha) = 0$, and in part (i) of this proof that (for $\delta > \frac{1}{2}$) $\lim_{\alpha \to 0} \pi(\alpha) = 0$. It follows that $\pi(\alpha)$ must have a global maximum $\max_{\alpha \in \mathbb{R}^+} \pi(\alpha)$. Consider a cost $c \leq \max_{\alpha \in \mathbb{R}^+} \pi(\alpha)$. Since $\pi(\alpha)$ is a continuous function, it follows that there is at least one point $\alpha > 0$ such that $\pi(\alpha) = c$ (note that when $c < \max_{\alpha \in \mathbb{R}^+} \pi(\alpha)$, there will be at least two such points). Moreover from (12) and (20), we know that each such $\alpha$ constitutes an equilibrium.

(iii) When $c > \max_{\alpha \in \mathbb{R}^+} \pi(\alpha)$, there is no $\alpha > 0$ such that $\pi(\alpha) = c$. Thus, following (12), there is no equilibrium with $\hat{\alpha} > 0$. This means that the equilibrium $\hat{\alpha} = 0$ identified in part (i) of this proof is unique. ■

Proof of Proposition 3. (i) Expected trading profits (see (10)) depend on $\delta$ and $\hat{\alpha}$. The dependence on $\delta$ is due to the effect of $\delta$ on $\overline{X}(\hat{\alpha})$ (see (4)). In an equilibrium with a strictly positive $\hat{\alpha}$, the dependence of $\hat{\alpha}$ on $\delta$ is implicitly given by $\pi(\hat{\alpha}) = c$. From the implicit function theorem we know that

$$\frac{\partial \hat{\alpha}}{\partial \delta} = -\frac{\frac{\partial \pi(\hat{\alpha})}{\partial \delta}}{\frac{\partial \pi(\hat{\alpha})}{\partial \hat{\alpha}}}.$$

(21)

From Proposition 1, we know that $\frac{\partial \pi(\hat{\alpha})}{\partial \alpha} < 0$. Moreover, we can calculate

$$\frac{\partial \pi(\hat{\alpha})}{\partial \delta} = (R_h - R_l) \int_{X(\hat{\alpha})}^{\infty} \frac{\varphi(x - \hat{\alpha}) \varphi(x + \hat{\alpha})}{\varphi(x - \hat{\alpha}) + \varphi(x + \hat{\alpha})} \, dx$$

$$- (R_h - R_l) \frac{\partial \overline{X}(\hat{\alpha})}{\partial \delta} \cdot \frac{\varphi(\overline{X}(\hat{\alpha}) - \hat{\alpha}) \varphi(\overline{X}(\hat{\alpha}) + \hat{\alpha})}{\varphi(\overline{X}(\hat{\alpha}) - \hat{\alpha}) + \varphi(\overline{X}(\hat{\alpha}) + \hat{\alpha})}.$$  

(22)
The first line on the RHS is 0. From (4), it follows that \( \frac{\partial X(\hat{\alpha})}{\partial \delta} > 0 \). Hence, \( \frac{\partial \pi(\hat{\alpha})}{\partial \delta} < 0 \) and therefore \( \frac{\partial \hat{\alpha}}{\partial \delta} < 0 \).

(ii) First note that at the point \( \pi(\hat{\alpha}_{\text{max}}) = c \), \( \pi(\alpha) \) cannot be strictly increasing. Suppose to the contrary that it is strictly increasing, then, because \( \pi(\alpha) \) is continuous and \( \lim_{\alpha \to \infty} \pi(\alpha) = 0 \), there must be \( \alpha > \hat{\alpha}_{\text{max}} \), for which \( \pi(\alpha) = c \). Then, by (20), \( \alpha \) constitutes another equilibrium, contradicting the fact that \( \hat{\alpha}_{\text{max}} \) is the most informative equilibrium.

Thus, we have two possible cases to consider. First, if \( \pi(\alpha) \) is strictly decreasing at the point \( \pi(\hat{\alpha}_{\text{max}}) = c \), we know from the proof in part (i) that that \( \hat{\alpha}_{\text{max}} \) is decreasing in \( \delta \). Second, if \( \pi(\alpha) \) is flat at the point \( \pi(\hat{\alpha}_{\text{max}}) = c \), then an increase in \( \delta \) will shift the function \( \pi \) down and \( \hat{\alpha}_{\text{max}} \) will fall by a discrete amount.

**Proof of Proposition 4.** We need to show that \( V(\tilde{\alpha}(\delta_1), \delta_2) > V(\tilde{\alpha}(\delta_2), \delta_2) \). From Proposition 3, we know that \( \tilde{\alpha}(\delta_1) > \tilde{\alpha}(\delta_2) \). Thus, we need to show that \( V(\alpha, \delta) \) is strictly increasing in \( \alpha \). Differentiating \( V(\alpha, \delta) \) in (14) with respect to \( \alpha \), we get:

\[
\frac{1}{2} \int_{X(\alpha, \delta)}^{\infty} \left[- (R_h - I) \varphi'(x - \alpha) + (R_l - I) \varphi'(x + \alpha) \right] dx
- \frac{1}{2} \frac{\partial X(\alpha, \delta)}{\partial \alpha} \cdot \left[ (R_h - I) \varphi(X(\alpha, \delta) - \alpha) + (R_l - I) \varphi(X(\alpha, \delta) + \alpha) \right].
\]

This can be rewritten as

\[
= \frac{1}{2} \left[ (R_h - I) \varphi(X(\alpha, \delta) - \alpha) - (R_l - I) \varphi(X(\alpha, \delta) + \alpha) \right]
- \frac{1}{2} \frac{\partial X(\alpha, \delta)}{\partial \alpha} \cdot \left[ (R_h - R_l) \varphi(X(\alpha, \delta) - \alpha) + (R_l - I) \varphi(X(\alpha, \delta) + \alpha) + \varphi(X(\alpha, \delta) - \alpha) \right].
\]

The expression in the first line is positive, while, using (4) and (3), we can see that the expression in the brackets in the second line is 0. Thus, the derivative of \( V(\alpha, \delta) \) with respect to \( \alpha \) is positive.

**Proof of Proposition 5.**

(i) Following the proof of part (i) of Proposition 3, the sign of the effect of \( (R_h - R_l) \) on \( \tilde{\alpha} \) is the same as the sign of \( \frac{\partial \pi(\tilde{\alpha})}{\partial (R_h - R_l)} \). From (10), we know that the direct effect of \( (R_h - R_l) \) on \( \pi(\tilde{\alpha}) \) is positive. From (22), we know that \( \frac{\partial \pi(\tilde{\alpha})}{\partial \delta} < 0 \), while from (15), we know that \( \frac{\partial \delta}{\partial (R_h - R_l)} > 0 \) when \( \delta \leq \frac{1}{2} \). Thus, the effect of \( (R_h - R_l) \) on \( \pi(\tilde{\alpha}) \) via \( \delta \) is negative. As a result, the overall effect of
(R_h - R_l) on \( \pi(\hat{\alpha}) \) and on \( \hat{\alpha} \) is ambiguous. Numerical examples show that the effect is sometimes positive and sometimes negative, depending on parameter values.

(ii) Following the proof of part (ii) of Proposition 3, the sign of the effect of \( (R_h - R_l) \) on \( \hat{\alpha}_{\text{max}} \) is the same as the sign of \( \frac{\partial \pi(\hat{\alpha})}{\partial (R_h - R_l)} \). As in the previous part, the direct effect of \( (R_h - R_l) \) on \( \pi(\hat{\alpha}) \) is positive. Since \( \frac{\partial \delta}{\partial (R_h - R_l)} < 0 \) when \( \delta > \frac{1}{2} \), the effect of \( (R_h - R_l) \) on \( \pi(\hat{\alpha}) \) via \( \delta \) is positive. As a result, the overall effect of \( (R_h - R_l) \) on \( \pi(\hat{\alpha}) \) and on \( \hat{\alpha}_{\text{max}} \) is positive. ■

**Proof of Proposition 6.** First note that the firm never wants to commit to a \( \delta_d > \delta \). This is because having \( \delta_d > \delta \) implies less information (by Proposition 3) and deviation from ex-post optimal investment; both forces reduce the ex-ante value of the firm. Thus, to prove the proposition, it suffices to show that \( \frac{dV(\delta_d)}{d\delta_d}|_{\delta_d=\delta} < 0 \). We can write:

\[
\frac{dV(\delta_d)}{d\delta_d} = \frac{1}{2} \int_0^\infty \left[ (R_h - I) \varphi'(x - \hat{\alpha}(\delta_d)) - (R_l - I) \varphi'(x + \hat{\alpha}(\delta_d)) \right] dx \\
- \frac{1}{2} \frac{\partial X}{\partial \delta_d} \left[ (R_h - I) \varphi'(X(\hat{\alpha}(\delta_d), \delta_d) - \hat{\alpha}(\delta_d)) + (R_l - I) \varphi'(X(\hat{\alpha}(\delta_d), \delta_d) + \hat{\alpha}(\delta_d)) \right].
\]

Using the derivations in the proof of Proposition 4, we know that the term in the square brackets at the RHS of (25) is 0 when \( \delta_d = \delta \). Then, we can write:

\[
\frac{dV(\delta_d)}{d\delta_d}|_{\delta_d=\delta} = \frac{1}{2} \frac{\partial \hat{\alpha}(\delta_d)}{\partial \delta_d} \left( (R_h - I) \varphi'(X(\hat{\alpha}(\delta_d), \delta_d) - \hat{\alpha}(\delta_d)) - (R_l - I) \varphi'(X(\hat{\alpha}(\delta_d), \delta_d) + \hat{\alpha}(\delta_d)) \right).
\]

Clearly, \( (R_h - I) \varphi'(X - \hat{\alpha}(\delta_d)) - (R_l - I) \varphi'(X + \hat{\alpha}(\delta_d)) > 0 \). Moreover, from Proposition 3, we know that \( \frac{\partial \hat{\alpha}(\delta_d)}{\partial \delta_d} < 0 \) and hence \( \frac{dV(\delta_d)}{d\delta_d}|_{\delta_d=\delta} < 0 \). ■

**Proof of Proposition 7.** Adapting the profit function \( \pi(\alpha) \) in (10) to the case of assets in place, we get:

\[
\pi_{AP}(\alpha) = (R_h - R_l) \int_{-\infty}^{\infty} \varphi(x - \alpha) \varphi(x + \alpha) dx.
\]

Essentially, since the project has already been undertaken, there is no threshold value of order flow below which the project is rejected.

The function \( \pi_{AP}(\alpha) \) is downward sloping. Thus, following (11) and (12), we know that for the case of assets in place there is a unique equilibrium \( \hat{\alpha}_{AP} \), which is greater than 0 and given by \( \pi_{AP}(\hat{\alpha}_{AP}) = c \) as long as \( c < \pi_{AP}(0) = \frac{1}{2} (R_h - R_l) \). Otherwise, \( \hat{\alpha}_{AP} = 0 \).
Comparing (27) with (10), we can see that \( \pi_{AP} (\alpha) > \pi (\alpha) \) for all \( \alpha \). This implies that, as long as \( c < \frac{1}{2} (R_h - R_l) \), \( \hat{\alpha}_{AP} > \hat{\alpha} \) (in case \( \hat{\alpha} \) is not unique, the inequality is true for all of the solutions). When \( c \geq \frac{1}{2} (R_h - R_l) \), there is no information produced for either the assets in place or the investment opportunity. ■

References


38


